Inflation expectations, adaptive learning and optimal monetary policy

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Key developments in monetary economics

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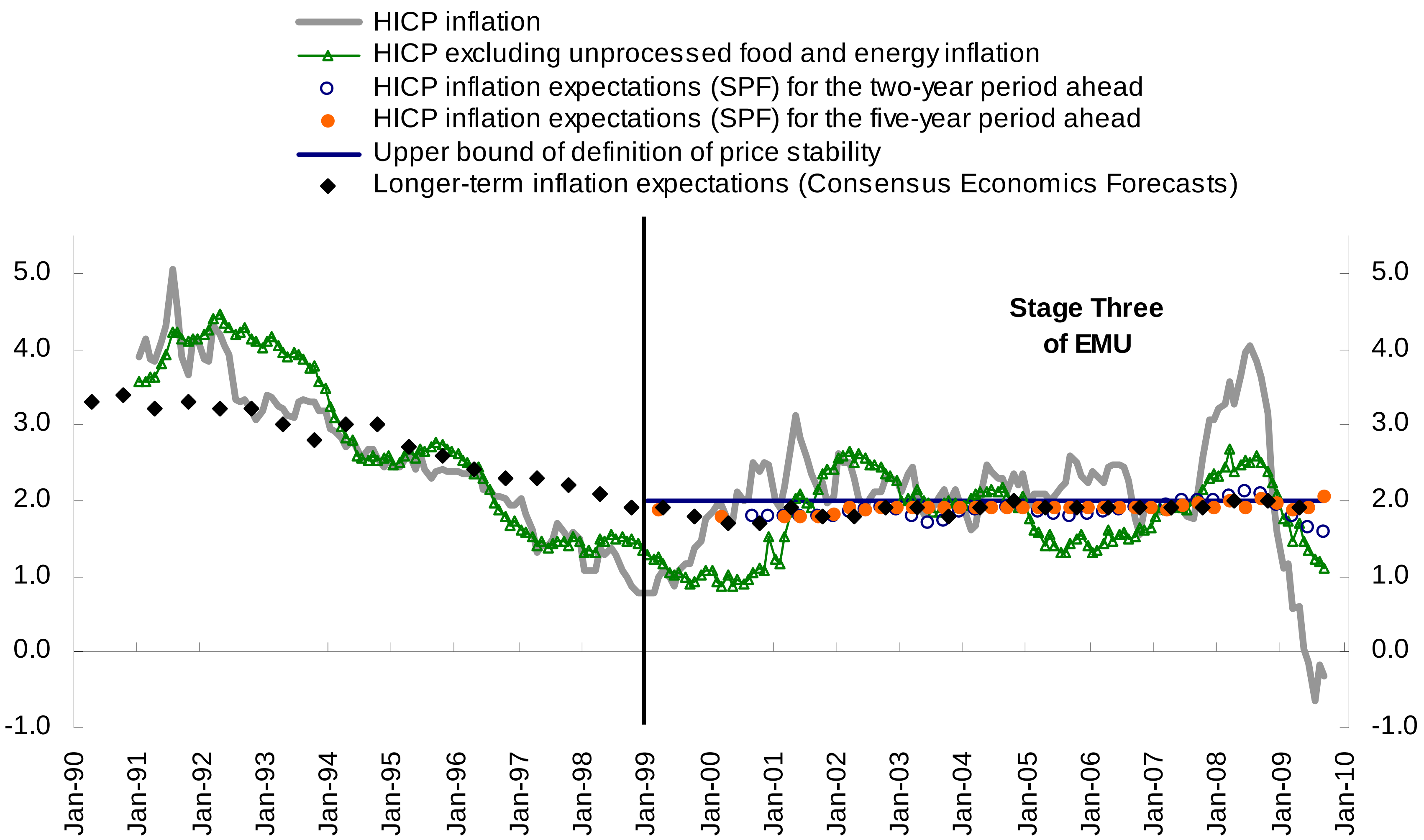
Anchoring inflation expectations…

… has become the modern mantra of central banking:

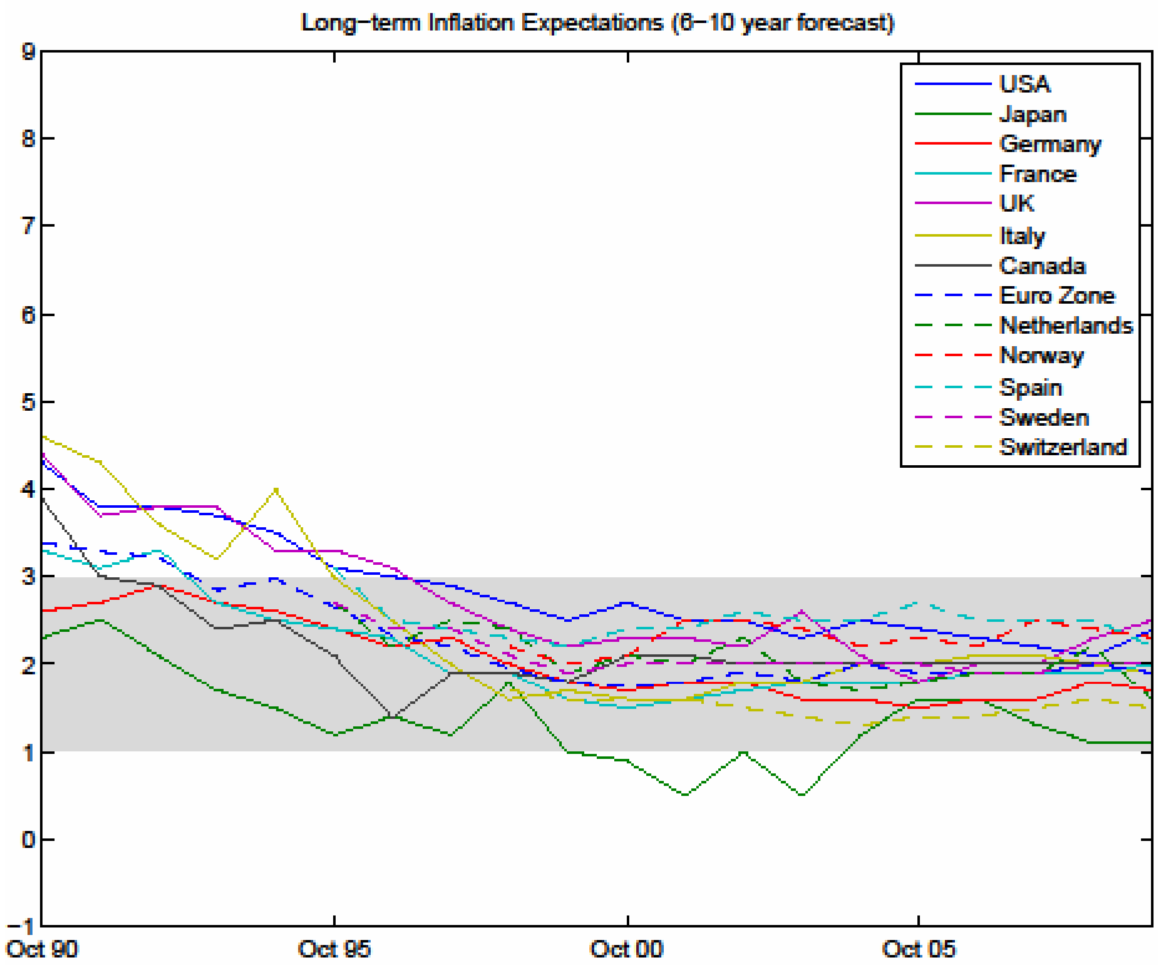
- **Trichet (2009): "It is absolutely essential to ensure that inflation expectations remain firmly anchored in line with price stability over the medium term"**
- **Bernanke (2007): "The extent to which inflation expectations are anchored has first-order implications for the performance of inflation and the economy more generally"**
- **Volcker (2006): "I have one lesson indelible in my brain: don't let inflation get ingrained. Once that happens, there is too much agony in stopping momentum"**

Anchoring has been successful…

… in the euro area



… and in many other industrial countries



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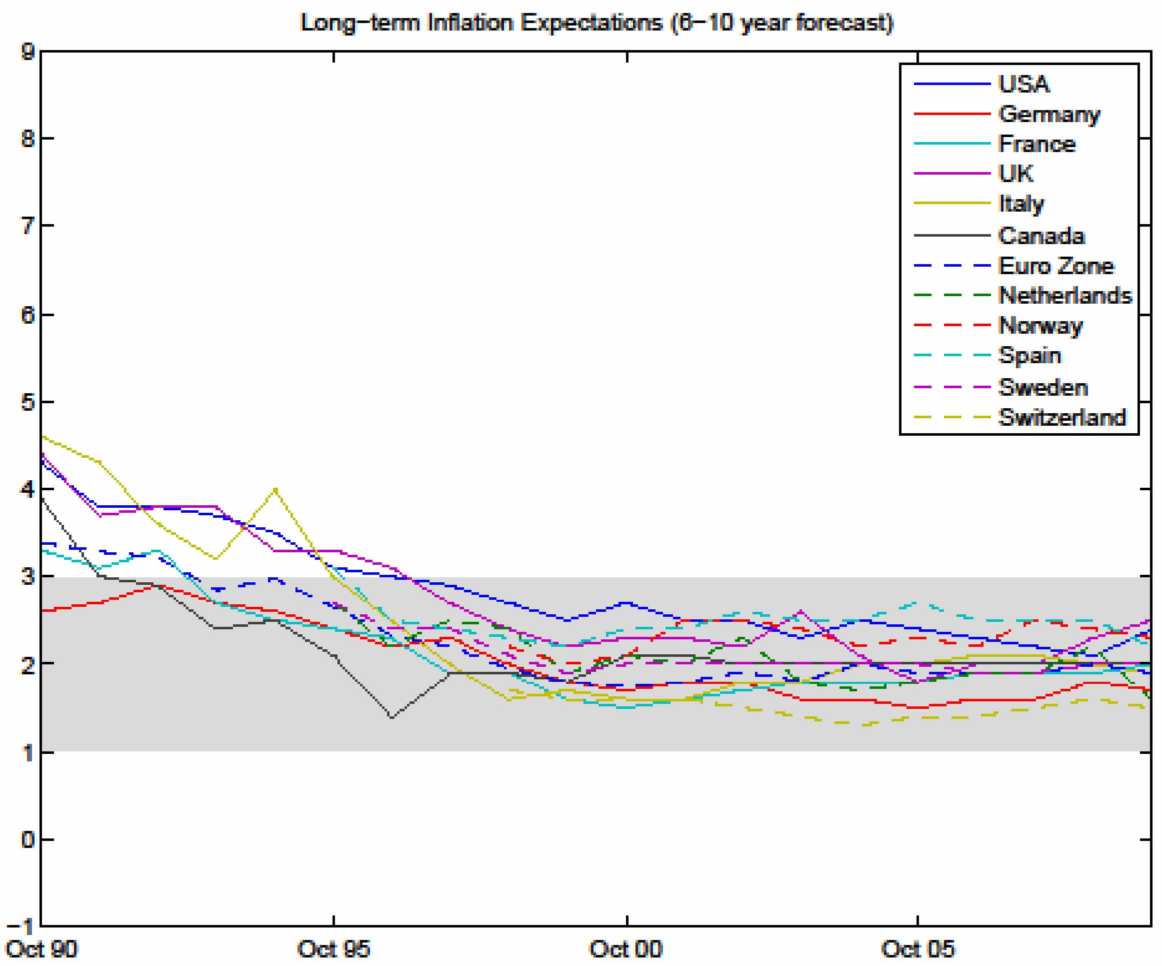
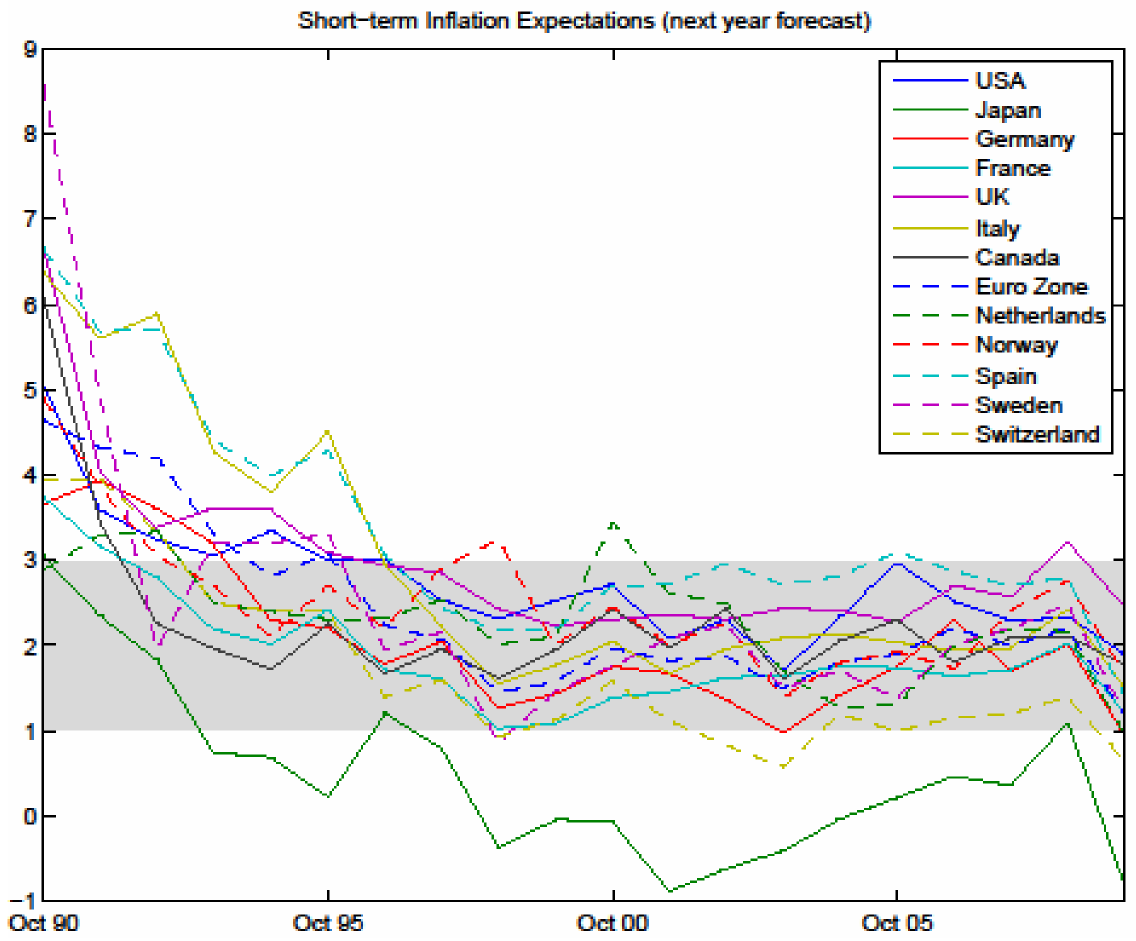


… even at shorter horizons.



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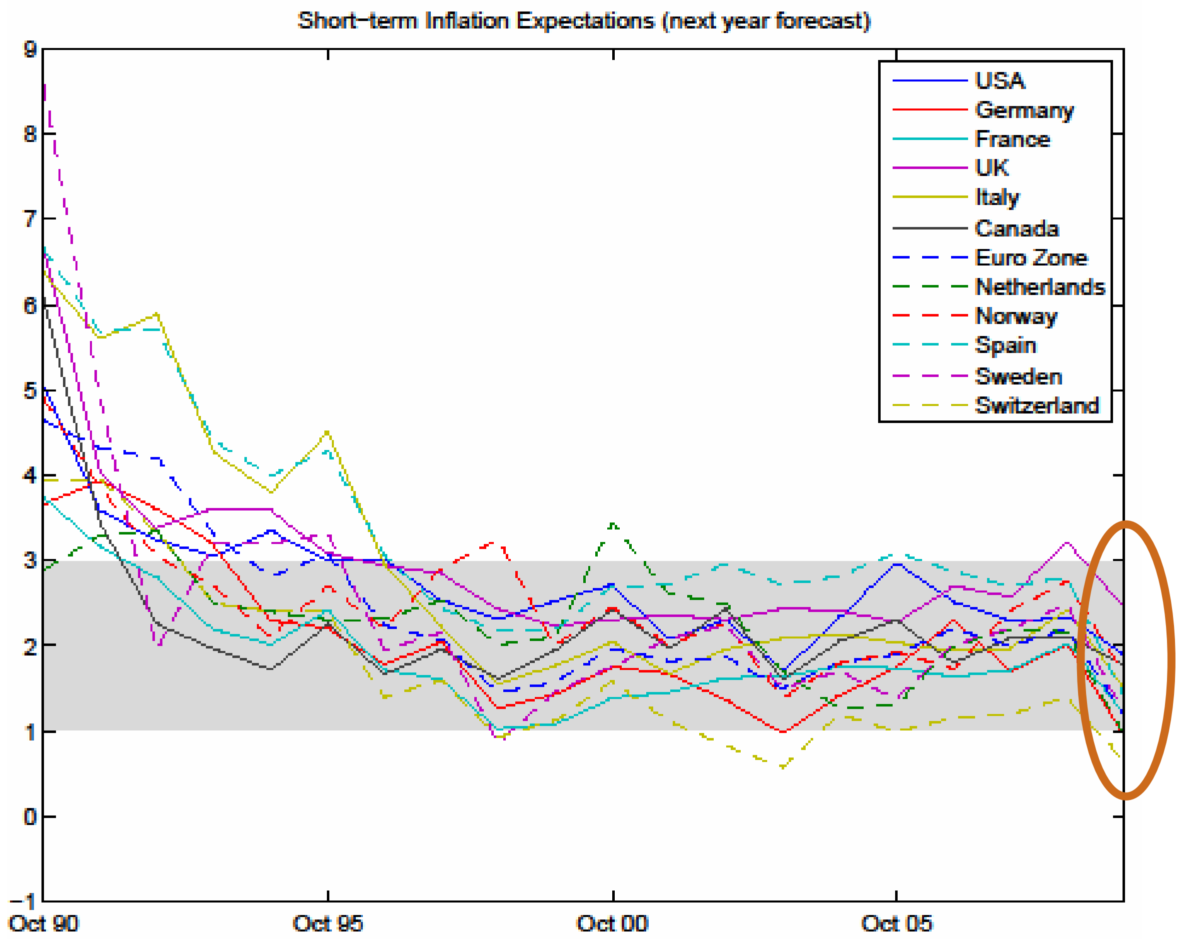


Modelling inflation expectations

- Standard approach is to assume rational or "model-consistent" expectations: e.g. HB chapter by Mike Woodford;
- Important benchmark, but
  - Credibility (distinction between discretion and commitment) is a binary variable: the central bank either has the credibility to commit to future policy actions or not (HB Chapter by Bob King);
  - Some implications are counterfactual: e.g. costless disinflations;
  - RE is an extreme assumption given the pervasive model uncertainty economic agents are facing.

Modelling inflation expectations

- **There are RE alternatives:**
  - **Limited processing power and rational inattention (e.g. HB chapter by Sims, Mackowiac and Wiederholt, Adam, …)**
  - **Limited information and signal extraction (e.g. Erceg and Levin, Schorfheide, …)**
- **But:**
  - **Even trivial models become extremely cumbersome to solve;**
  - **Assumes quite knowledgeable individuals (not even trained economists can work out the optimal price-setting plans… without considerable effort!)**

Modelling inflation expectations

- A reasonable alternative is "adaptive learning" or "constant-gain least squares learning":
  - Agents are endowed with an econometric specification, which may be consistent with the reduced form of the rational expectations equilibrium;
  - As time goes by, they update their knowledge and the associated forecasting rule;
  - Constant gain takes into account the possibility of breaks and time-varying parameters.
- Moreover, such specifications work empirically:
  - Orphanides and Williams (2004),
  - Milani (2005),
  - Slobodyan and Wouters (2009)

Objective of this chapter

- **Characterize optimal monetary policy responses when agents use adaptive learning to form inflation expectations;**
- **Assess the robustness of policy rules that are optimal under rational expectations (RE) to small deviations from RE.**
- **The chapter builds on the work by:**
  - **Orphanides and Williams (2005, …)**
  - **Gaspar, Smets and Vestin (2005)**
  - **Molnar and Santoro (2006)**

Related literature

- **Evans and Honkapohja (2001) and many related papers:**
  - **Analyze how least-squares learning affects the stability and determinacy of macro-economic equilibria under various monetary policy interest rate rules**
  - **See Evans and Honkapohja (2008) for an excellent survey.**
- **This chapter focuses on the implications of targeting rules taking the non-linearity of the expectation formation process into account.**

Related literature

- **Large literature on monetary policy making under uncertainty (e.g. Hansen and Sargent, Taylor and Williams)**
- **A few papers analyse the interaction with learning by the private sector:**
  - **Orphanides and Williams (2007) study the interaction of imperfect knowledge by the central bank and constant gain learning by private agents.**
  - **Evans and Honkapohja (2003a,b)**
  - **Woodford (2005) analyses optimal policy when agents' expectations may be distorted away from rational expectations.**

Related literature

- **Alternative types of learning:**
  - Branch and Evans (2007) and Brazier, Harrison, King and Yates (2006) assume that private agents may use different forecast models, with the proportion changing over time with relative forecast performance.
  - Arifovic, Bullard and Kostyshyna (2007) and De Grauwe (2007) use social learning
  - Bullard, Evans and Honkapohja (2007)

Outline

- **New Keynesian model and its solution under rational expectations.**
- **Adaptive learning consistent with discretionary RE equilibirum: Simple rules and optimal policy.**
  - **Model calibration and results.**
  - **The workings of optimal policy.**
  - **Sensitivity analysis.**
- **Conclusions.**

The New Keynesian model

- **Monopolistic competition**
- **Sticky prices with partial indexation**

$$\pi_t - \gamma\pi_{t-1} = \beta \mathrm{E}_t(\pi_{t+1} - \gamma\pi_t) + \kappa x_t + u_t$$

- **Loss function**

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2$$

- **Use output gap (x(t)) as policy instrument**

The New Keynesian model: RE

- **Under RE, there is a distinction between optimal policy under discretion and commitment:**

- **Under discretion:**

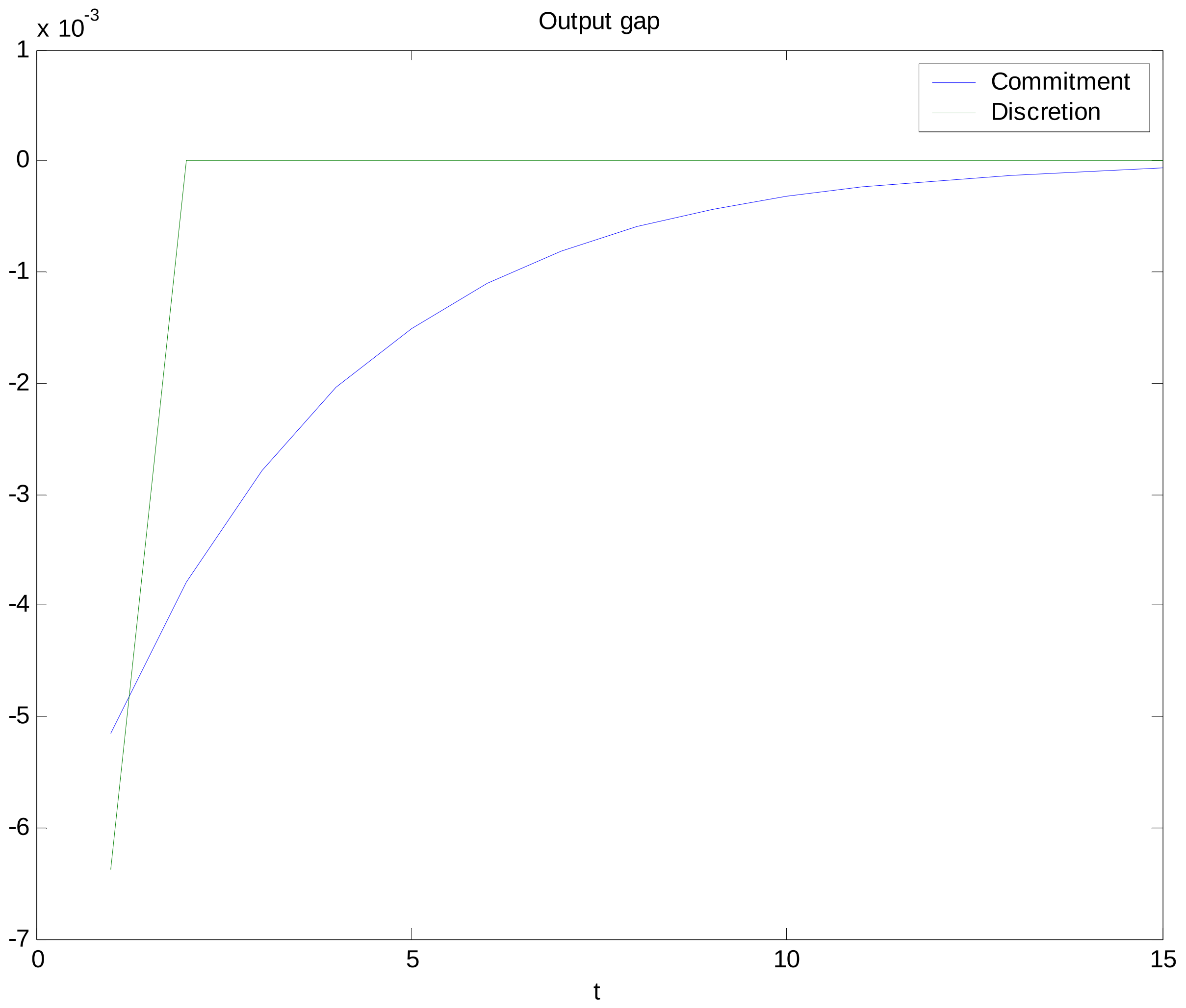
$$x_t = -\frac{\kappa}{\kappa^2 + \lambda} u_t \qquad \pi_t = \gamma \pi_{t-1} + \frac{\lambda}{\kappa^2 + \lambda} u_t$$

- **Under commitment:**

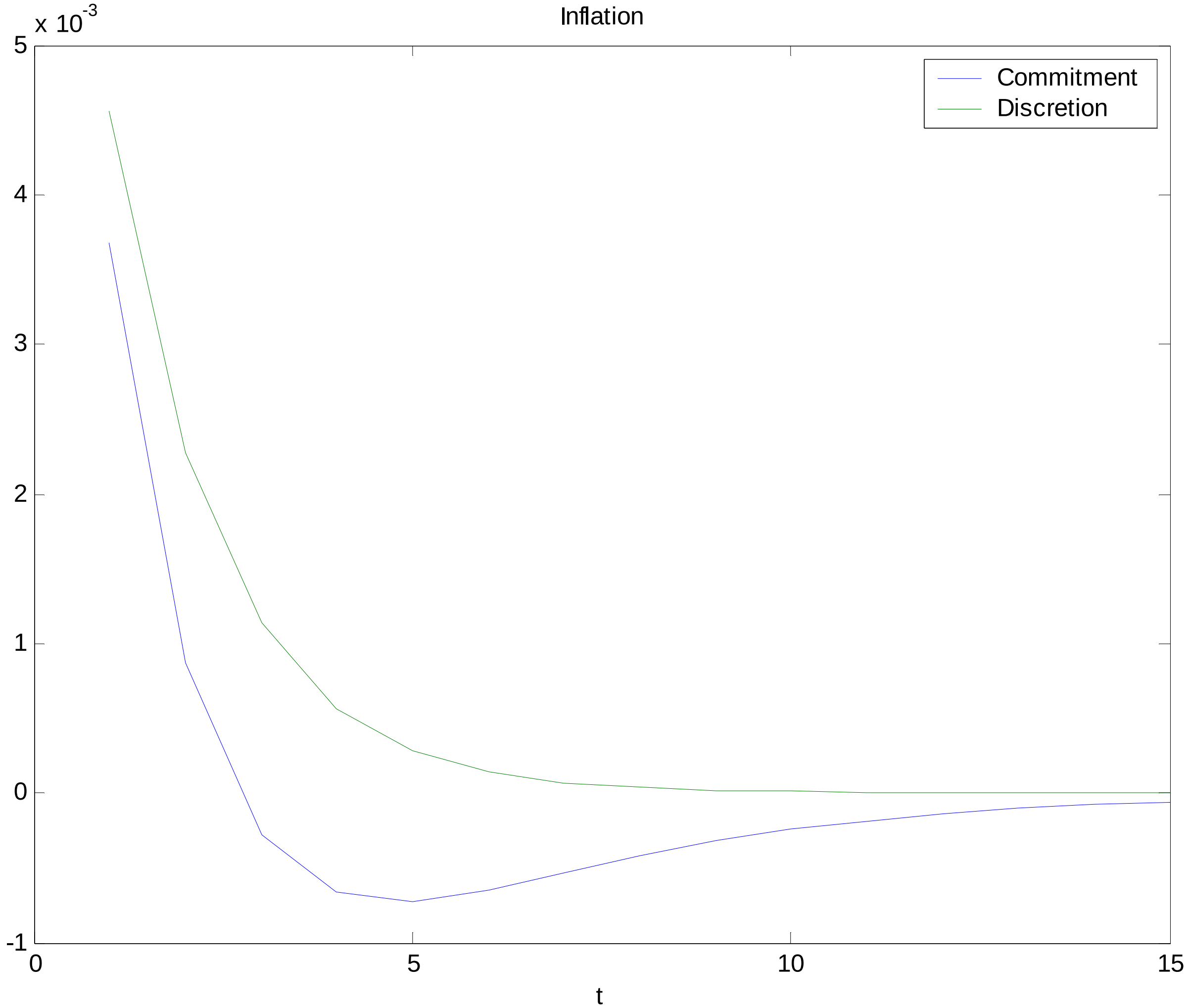
$$z_t = -\frac{\lambda}{\kappa}\left(x_t - x_{t-1}\right) \text{ with } z_t = \pi_t - \gamma \pi_{t-1}$$

$$x_t = \delta x_{t-1} - \frac{\kappa}{\lambda} u_t \qquad \pi_t = \gamma \pi_{t-1} + \frac{\lambda(1-\delta)}{\kappa} x_{t-1} + \delta u_t$$

Impulse response to a cost-push shock



Impulse response to a cost-push shock



Adaptive learning

- **Agents estimate AR(1) for inflation:**

$$\pi_t = c_t \pi_{t-1} + \varepsilon_t$$

- **Consistent with reduced-form of RE equilibrium under discretion**
- **Recursive updating:**

$$c_t = c_{t-1} + \phi R_t^{-1} \pi_{t-1} (\pi_t - \pi_{t-1} c_{t-1})$$

$$R_{t+1} = R_t + \phi(\pi_t^2 - R_t)$$

Adaptive learning

- **Which info is available to agents when?**
- **Simultaneity problem in forward-looking models.**
- **One solution: lagged information**
- **Here: agents use current inflation in the forecast but not in the updating of the parameters:**

$$\mathrm{E}_t \pi_{t+1} = c_{t-1} \pi_t$$

- **Implies:** $\pi_t = \frac{1}{1+\beta(\gamma - c_{t-1})}(\gamma \pi_{t-1} + \kappa x_t + u_t)$

Optimal targeting

- **Formulate value function**

$$V(u_{t,}\pi_{t-1,}c_{t-1,}R_t) = \max_{x_t} - \frac{(\pi_{t,} - \gamma\pi_{t-1})^2 + \lambda x_t^2}{2} + \beta \mathrm{E}_t V(u_{t+1,}\pi_{t,}R_{t+1,}c_t),$$

- **Maximize subject to Phillips curve, learning equations and forecasting equation**

Calibration

Key parameters							
β	γ	λ	θ	α	ϕ	κ	σ
0.99	0.5	0.002	10	0.66	0.02	0.019	0.004
Discount rate	Degree of indexation	Weight on output gap	Elast. of subst.	Fraction of non-optimal prices	Constant gain	Slope of Phillips curve	Std. of shocks

$$\pi_t - \gamma\pi_{t-1} = \beta \mathrm{E}_t(\pi_{t+1} - \gamma\pi_t) + \kappa x_t + u_t$$

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2$$

Results: losses

RE		Adaptive Learning		
Com.	Disc.	Optimal	Com.	Disc.
1.00	1.29	1.09	1.11 (!!)	1.37

- Optimal policy about 20% improvement
- RE commitment rule comes close to optimal policy outcome

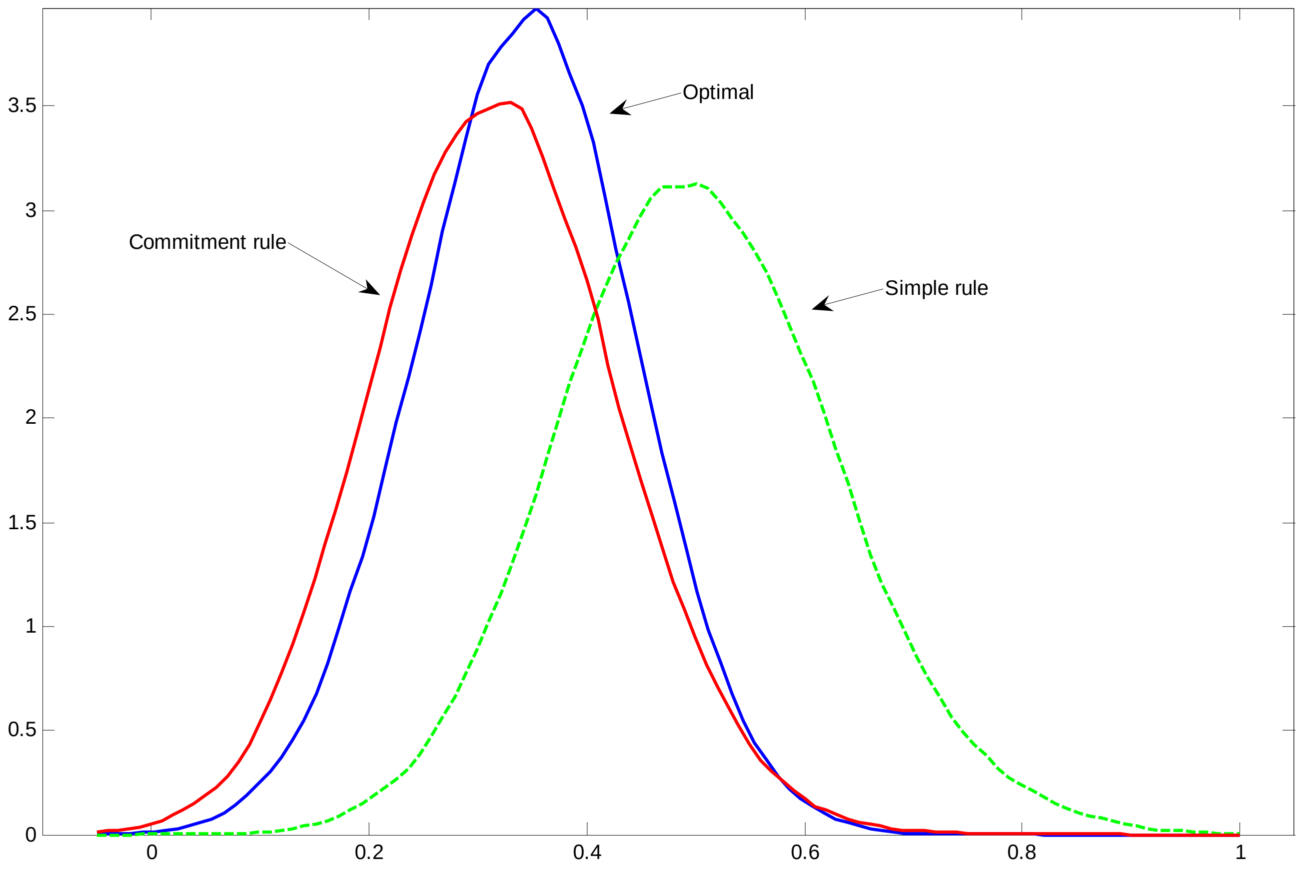
Results: Variances and autocorrelations

	RE		Learning		
	Com.	Disc.	Optimal	C-rule	D-rule
Var(Output)	1.00	0.95	1.02	1.00	0.95
Var(Inflation)	1.00	1.85	1.23	1.27	2.18
Cor(Output)	0.66	0.00	0.54	0.66	0.00
Cor(Inflation)	0.24	0.50	0.33	0.34	0.56

- Lower persistence of inflation under C-rule helps reducing inflation volatility at low cost in terms of output volatility
- C-rule and optimal yield similar outcomes but different mechanism at play

Outcomes

Distribution of inflation persistence



Characterising optimal policy

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda\chi_t^2} u_t + \frac{\kappa\gamma(\chi_t - 1) + \beta\kappa\chi_t \phi R_t^{-1} E_t V_c}{\kappa^2 + \lambda\chi_t^2} \pi_{t-1} + \beta \frac{\kappa\chi_t}{\kappa^2 + \lambda\chi_t^2} E_t V_\pi$$

If $\pi_{t-1} = 0$

then:

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda\chi_t^2} u_t$$

where $\chi_\mathrm{t} = 1 + \beta(\gamma - \mathrm{c}_{\mathrm{t}-1})$

If $\gamma = \mathrm{c}_{\mathrm{t}-1}$ then equal to discretionary rule: intratemporal trade-off.

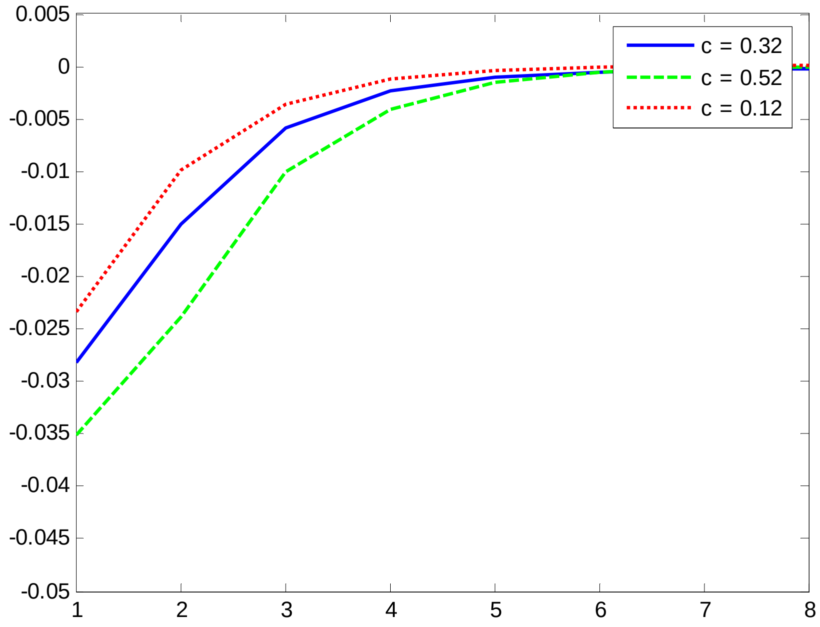
Characterising optimal policy

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda\chi_t^2} u_t + \frac{\kappa\gamma(\chi_t - 1) + \beta\kappa\chi_t\phi R_t^{-1} E_t V_c}{\kappa^2 + \lambda\chi_t^2} \pi_{t-1} + \beta \frac{\kappa\chi_t}{\kappa^2 + \lambda\chi_t^2} E_t V_\pi$$

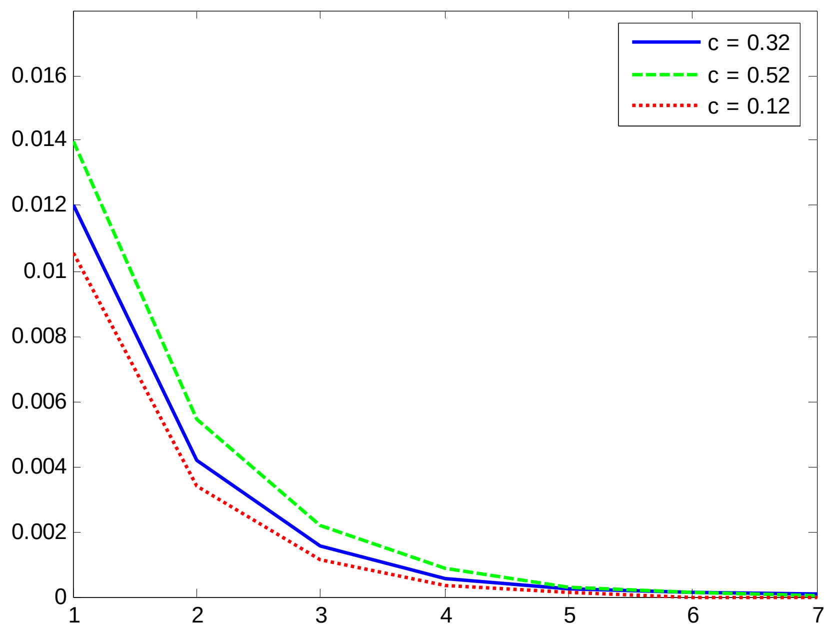
- However, in general optimal policy under learning will also respond to lagged inflation – resembles history dependence;
- If $\gamma = c_{t-1}$ then: $$x_t = +\frac{\beta\kappa\phi E_t V_c}{\kappa^2 + \lambda} \pi_{t-1} + \ldots$$
- Captures intertemporal trade-off between stabilising output gap and steering the perceived degree of inflation persistence by responding more agressively to inflation

Mean dynamic response of output gap

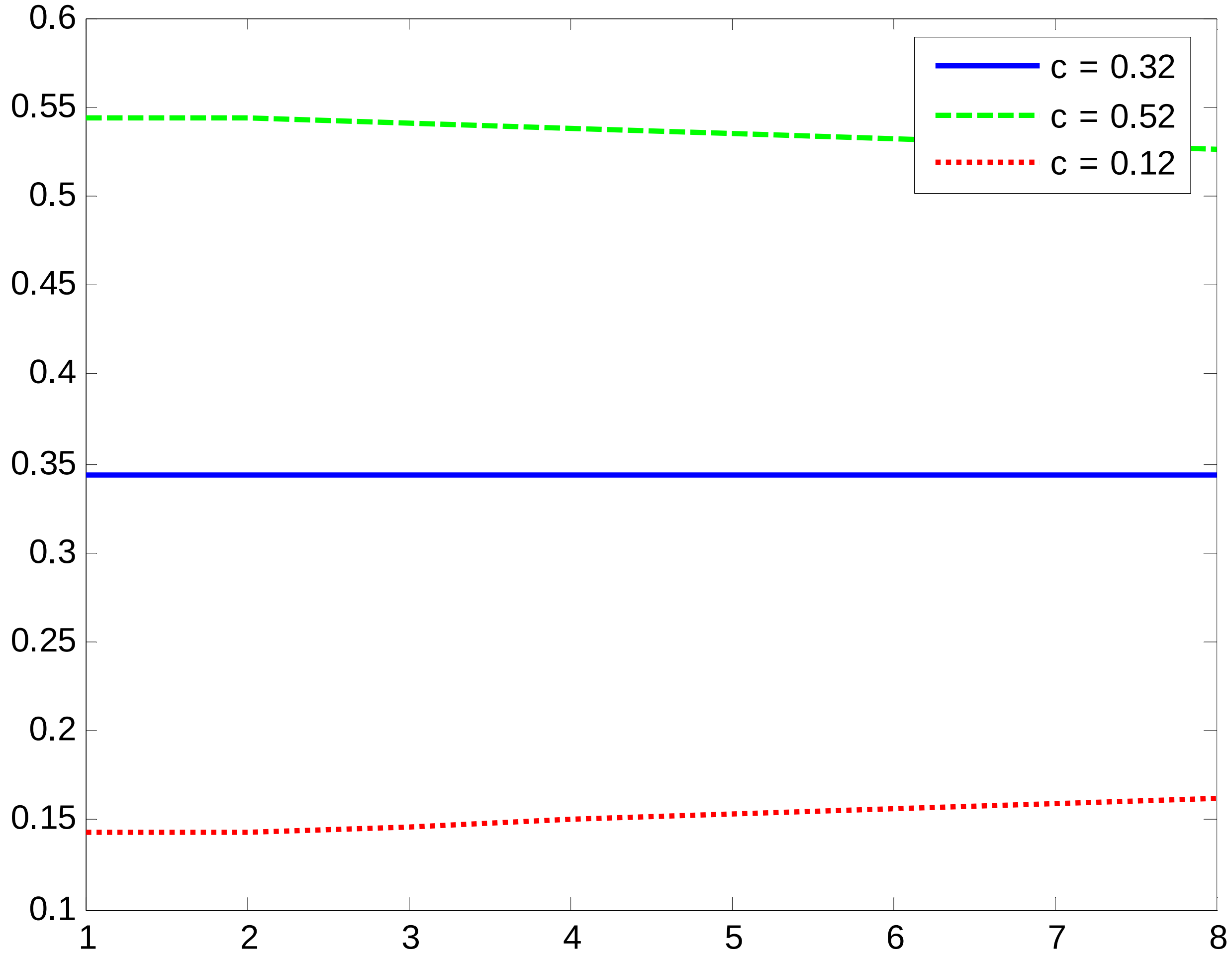
Output gap



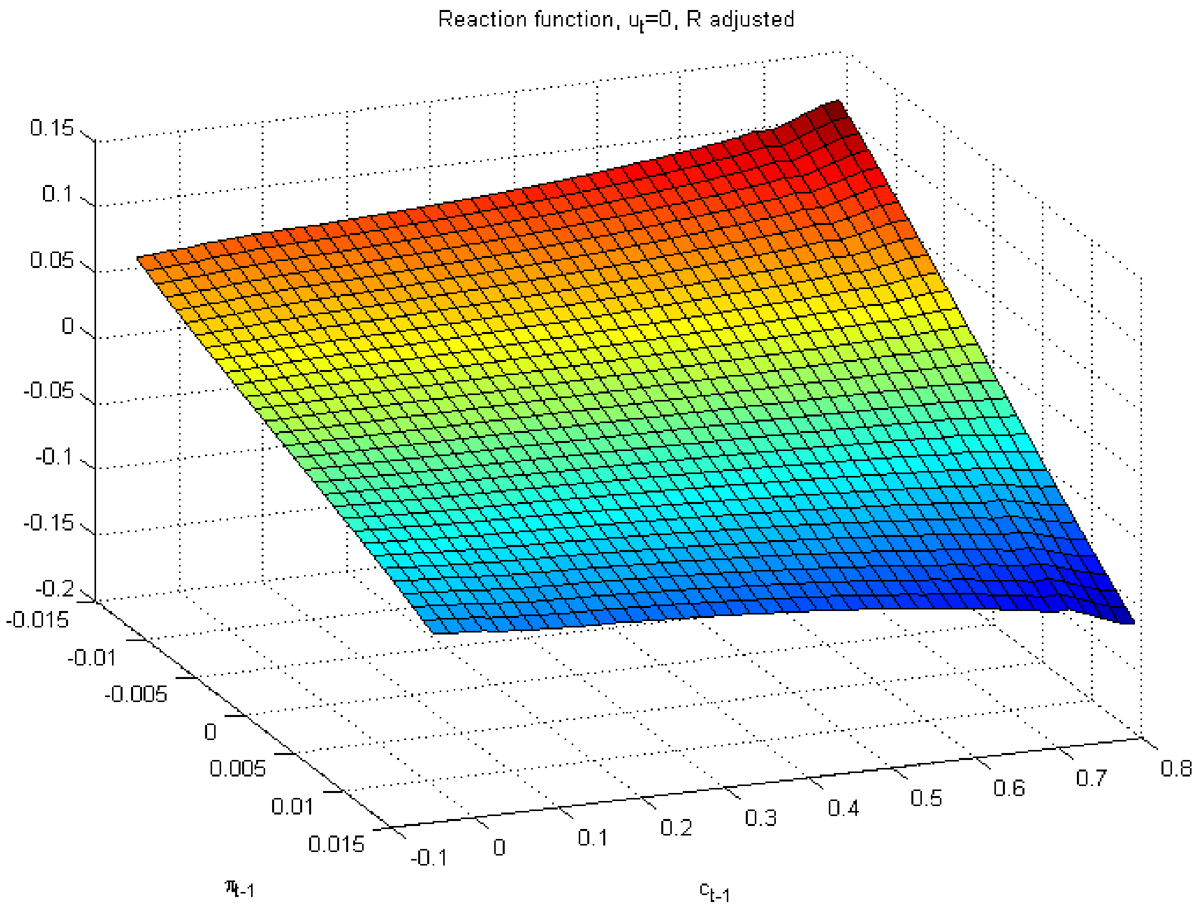
Inflation



Mean dynamic response of persistence

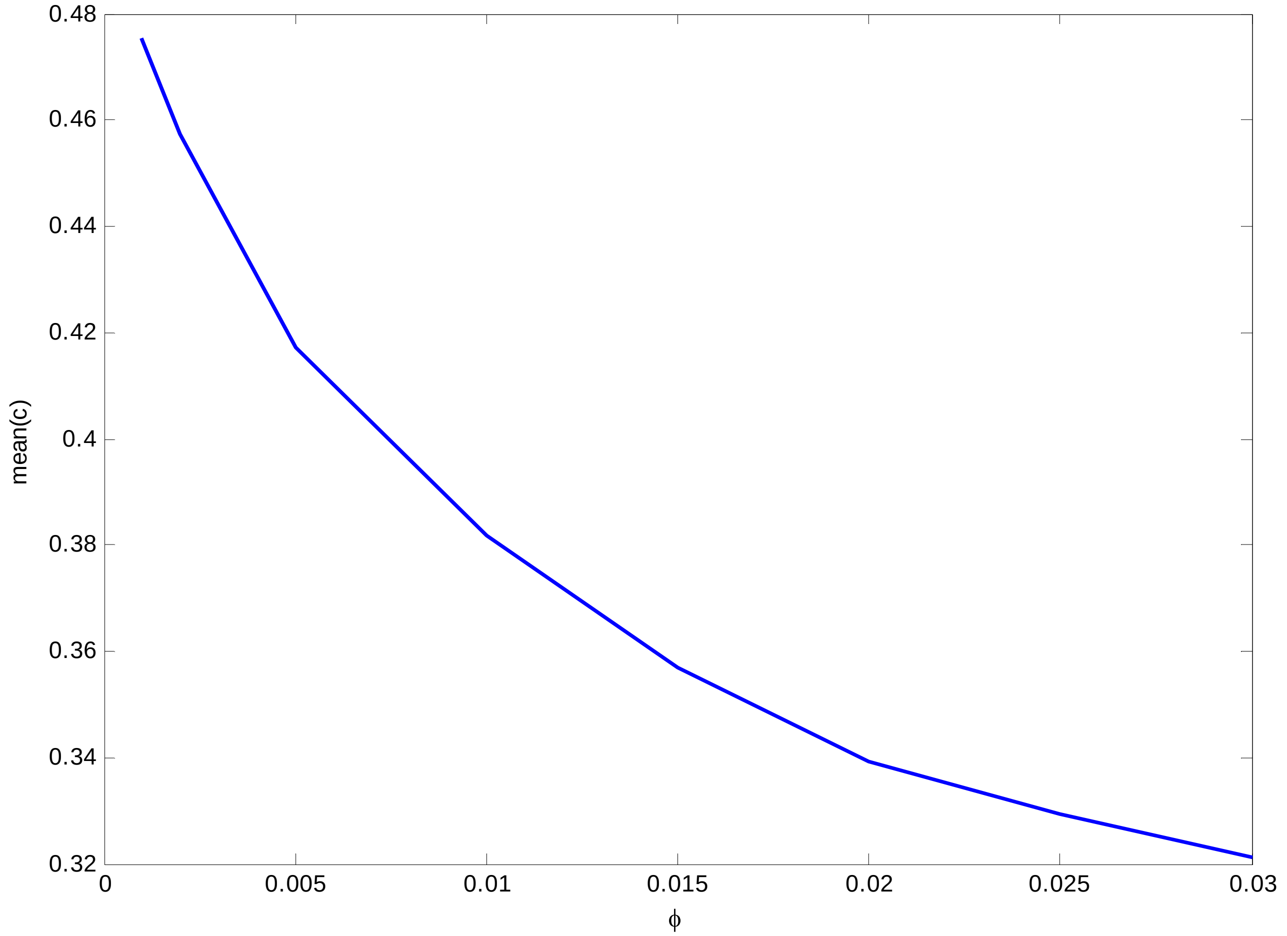


Reaction function: no shock



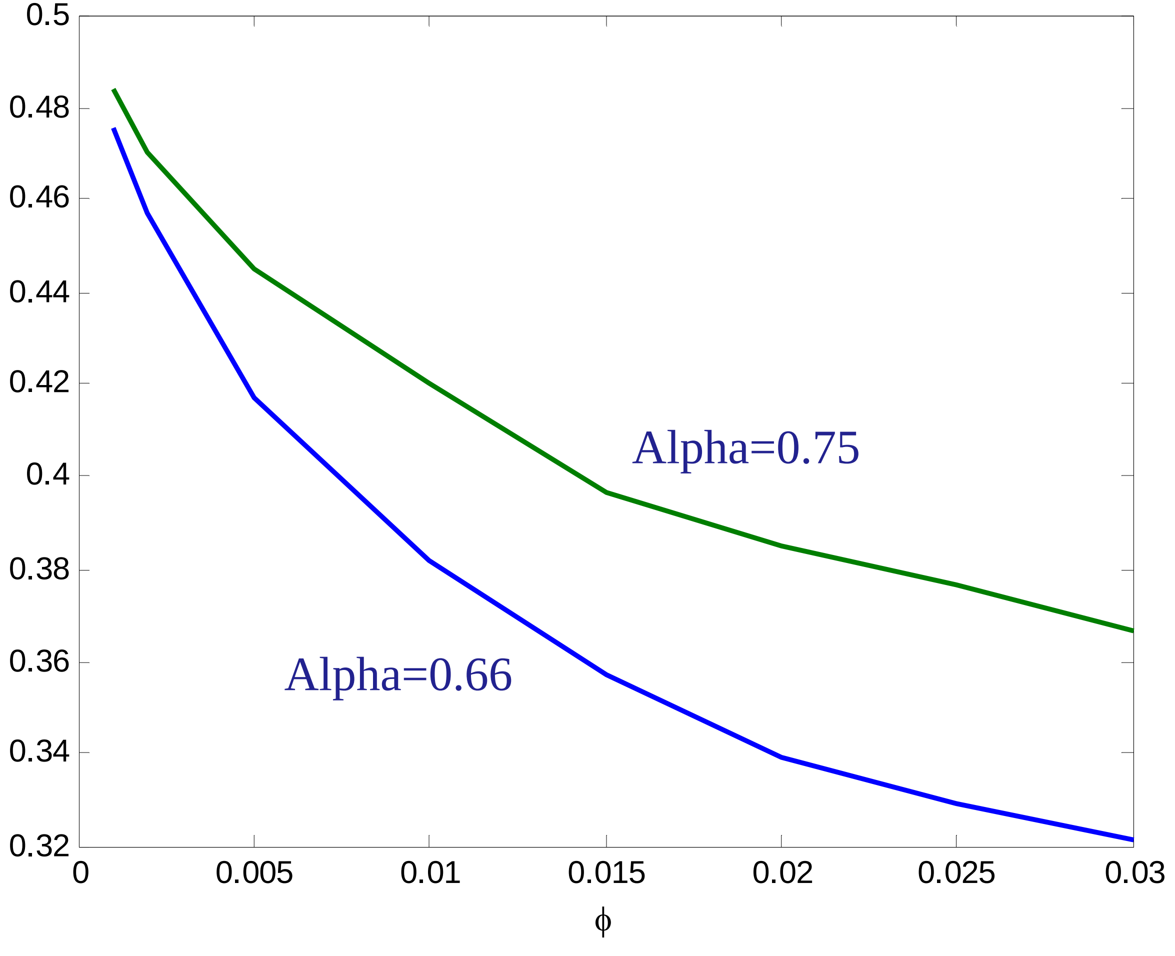
Sensitivity analysis

Average estimated persistence as a function of the gain…



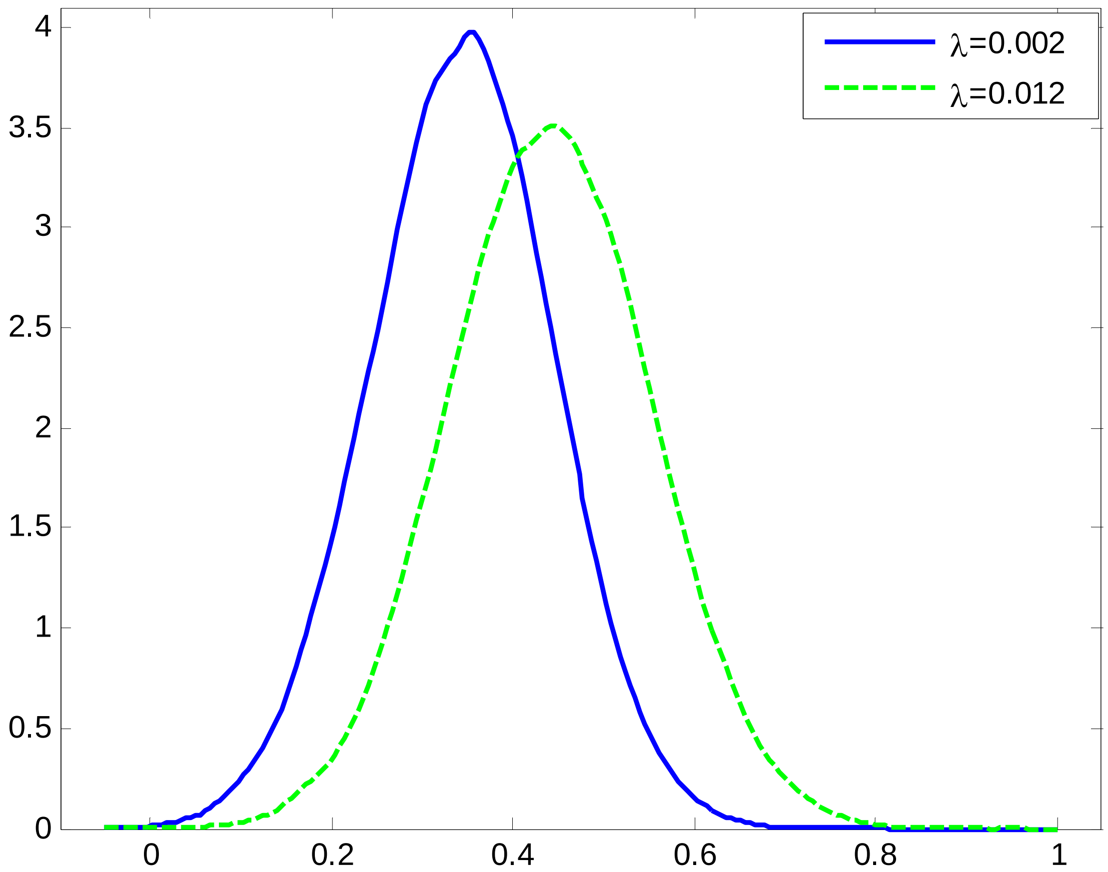
Sensitivity analysis

… and as a function of degree of price stickiness



Sensitivity analysis

Estimated persistence as function of weight on output gap



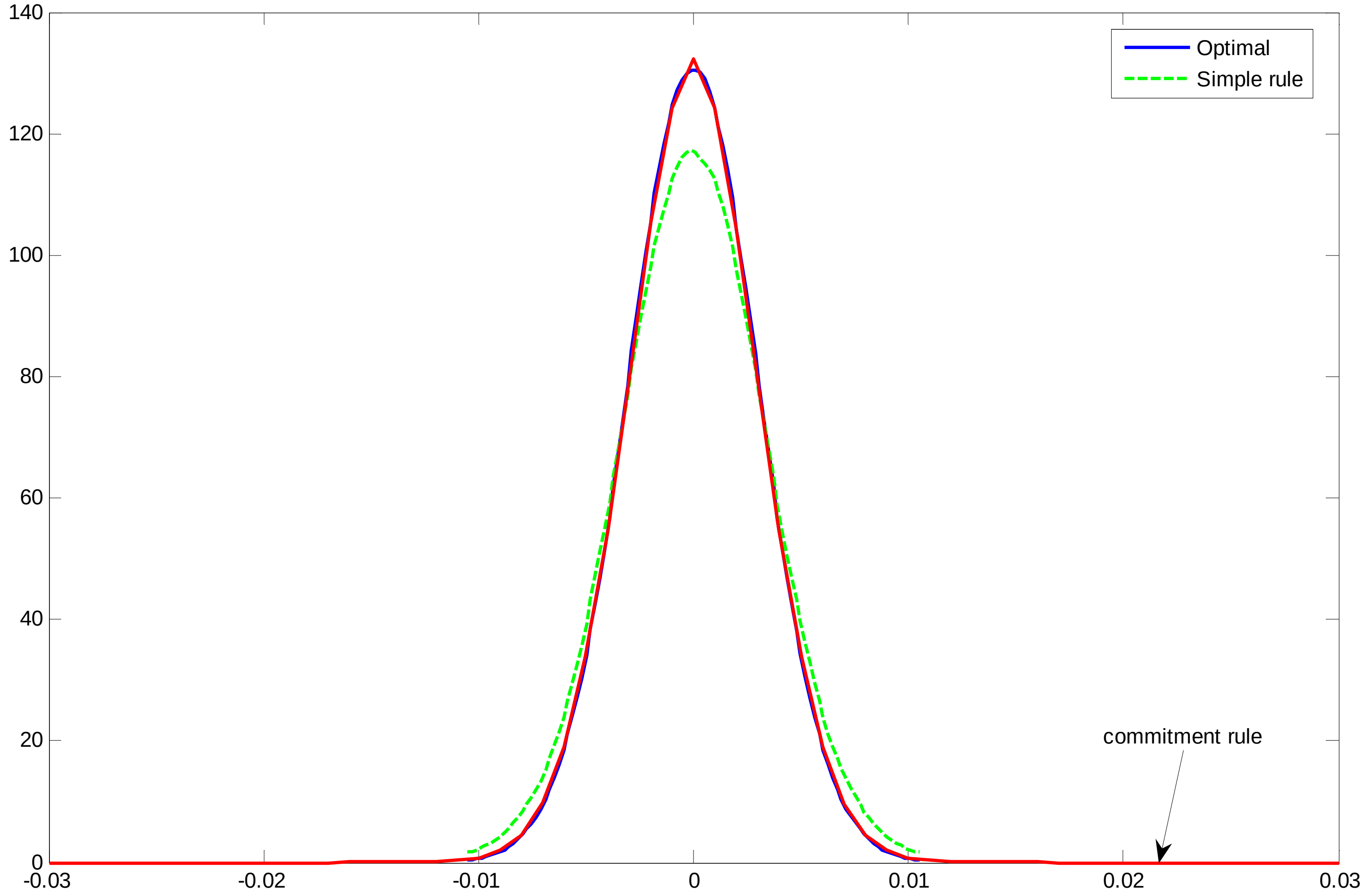
Tentative conclusions

- **Management of inflation expectations remains important under adaptive learning;**
  - **In addition to the usual intratemporal trade-off, central banks face an intertemporal trade-off between current output stabilisation and management of future inflation expectations;**
  - **The mechanism is different: In case of RE it works through expectations of future policy; in the case of AL, it works through perceived inflation persistence as function of past and current policy.**
  - **Both policy responses are history dependent; but AL optimal policy response is time-varying depending on the perceived persistence of inflation.**
- **Nevertheless, commitment rule under RE comes very close to optimal policy under learning.**

To do list

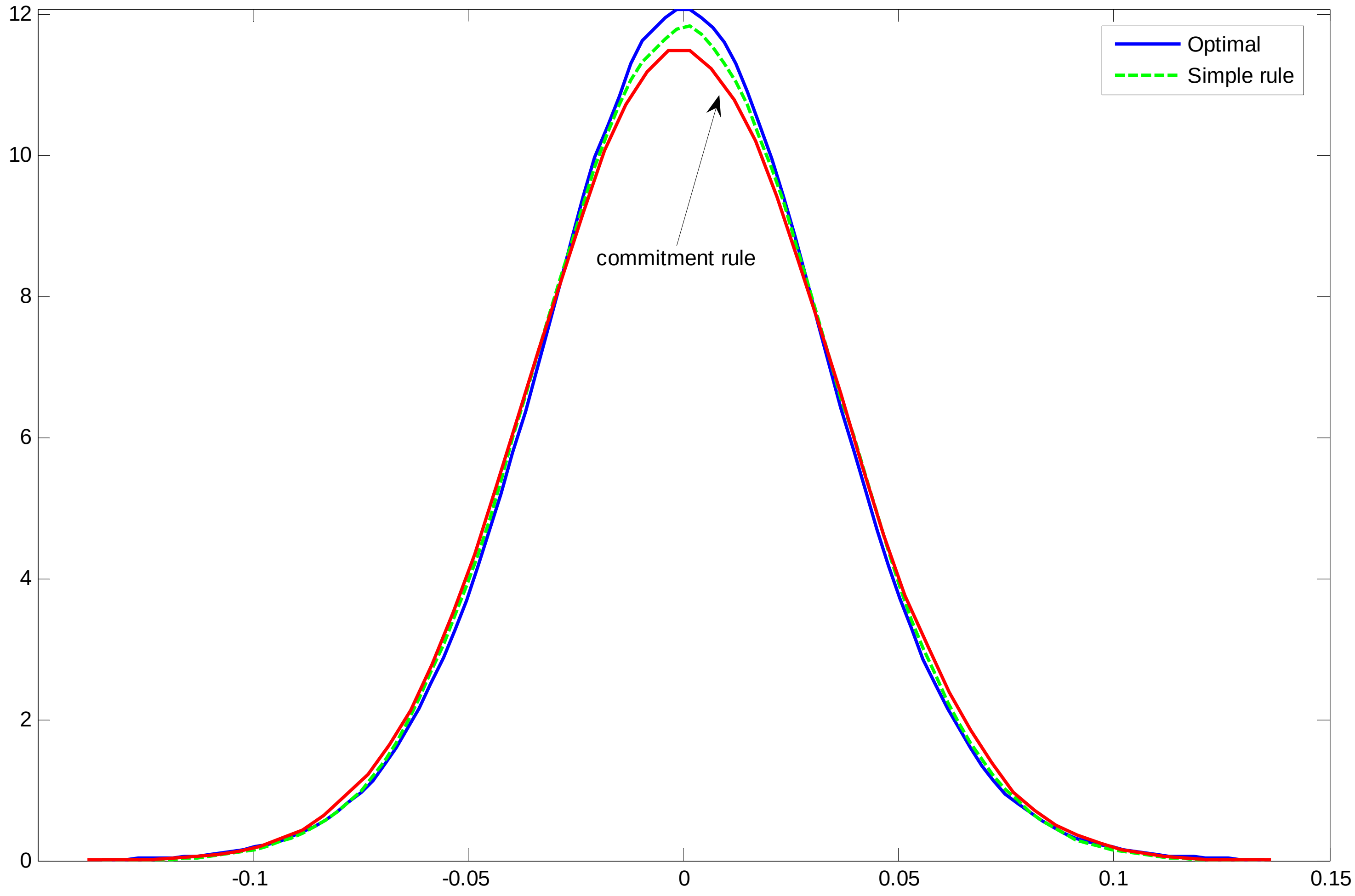
- **Encompass the commitment equilibrium in the learning specification:**
  - **Simplify model to forward-looking specification and analyse learning about price level persistence;**
  - **Introduce output gap in learning model; use stochastic gradient learning to limit state space.**
- **Investigate the transition from a discretion rule to the optimal regime under adaptive learning;**
- **Embed the analysis in the learning literature.**

Outcomes



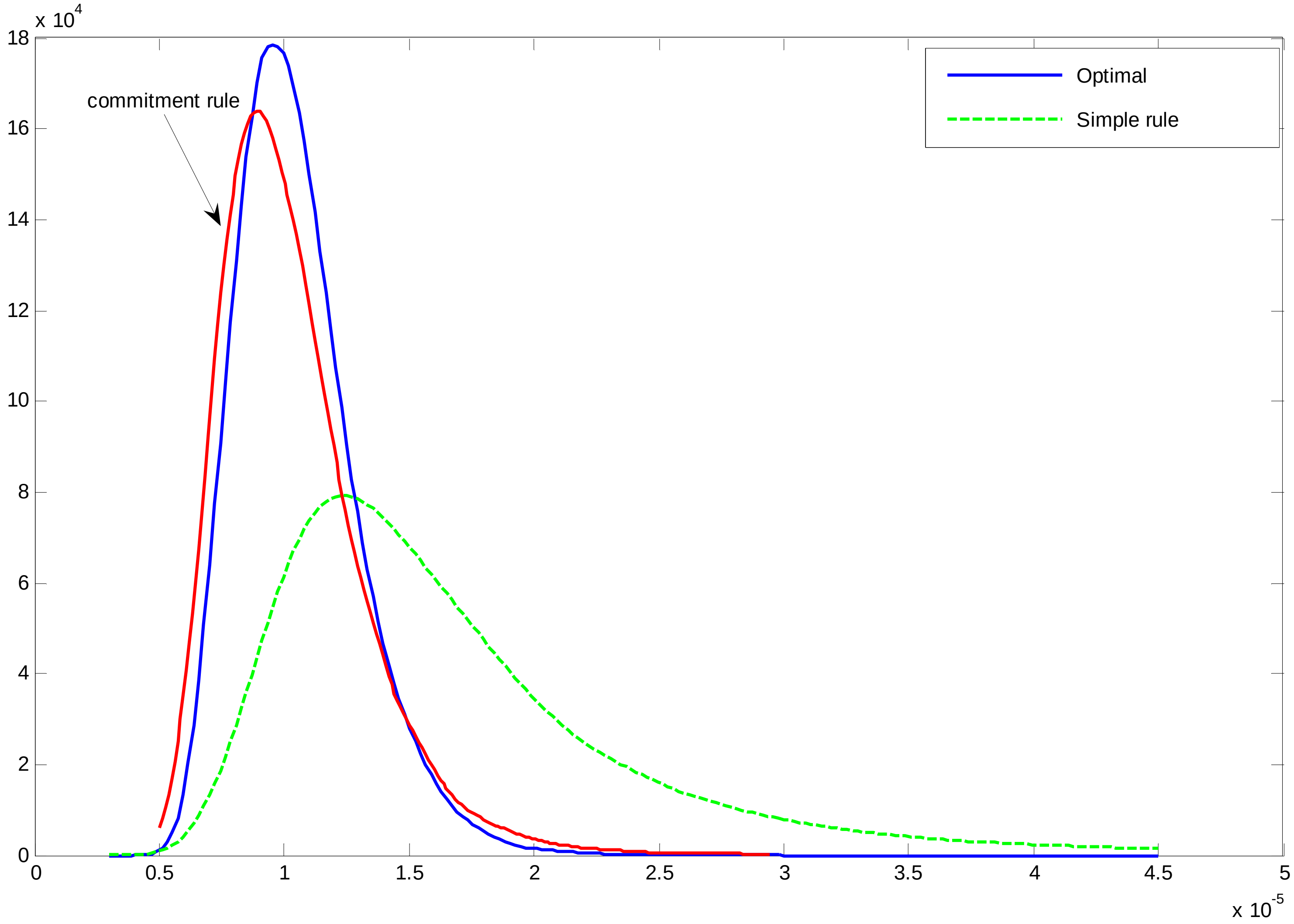
Optimal
Simple rule
commitment rule
140
120
100
80
60
40
20
0
-0.03
-0.02
-0.01
0
0.01
0.02
0.03

Outcomes



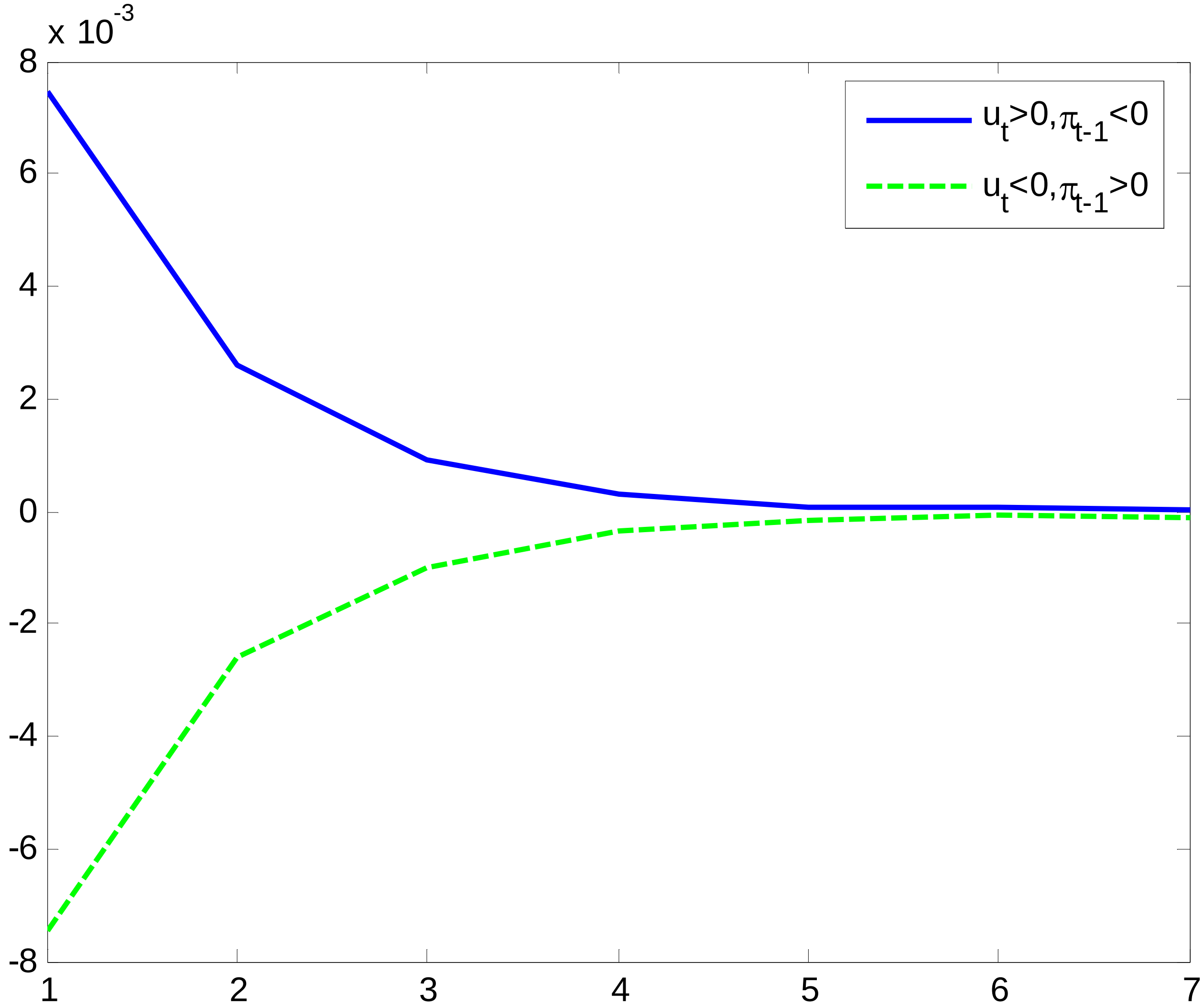
12
10
8
6
4
2
0
-0.1
-0.05
0
0.05
0.1
0.15
Optimal
Simple rule
commitment rule

Outcomes

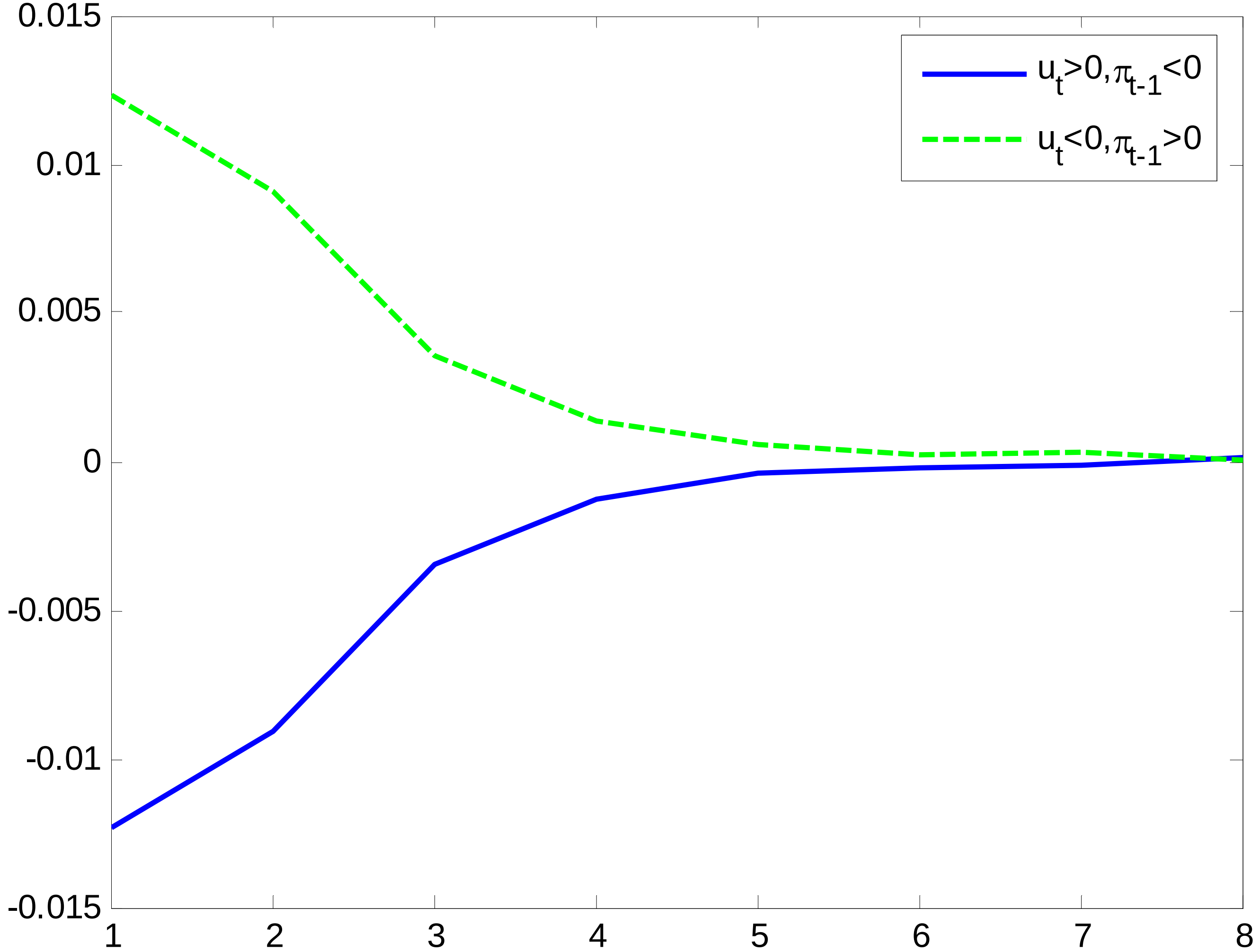


x 10^4
18
16
14
12
10
8
6
4
2
0
commitment rule
Optimal
Simple rule
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x 10^-5

Symmetry: response of inflation



Symmetry: response of output



Reaction function: response to shock