Learning and Time-Varying Macroeconomic Volatility

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Introduction

- Strong evidence of changes in macro volatility over time (The Great Moderation)
Figure: Conditional Standard Deviation series for Inflation and Output Gap
Introduction

- Need to correctly model volatility
- Sims and Zha (AER 2006): BVAR, Regime changes in volatilities of shocks
Introduction

- In DSGE Models?
  Exogenous shocks with constant variance

- DSGE with Stochastic Volatility

- Time variation in the volatility of exogenous shocks
But what explains the changing volatility?
Scope of the paper

- Present a simple model with learning
- The learning speed (gain coefficient) of the agents is endogenous: it responds to previous forecast errors
- **Endogenous** Time-Varying Volatility
Results:

1. The changing gain induces endogenous time variation in the volatilities of the macroeconomic variables the agents try to learn.
2. Evidence of time variation in endogenous gain from estimated model.
3. The econometrician can spuriously find evidence of stochastic volatility if learning is not taken into account.
The Model

- **Stylized New Keynesian Model**

\[
\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa x_t + u_t \quad (1)
\]

\[
x_t = \hat{E}_t x_{t+1} - \sigma (i_t - \hat{E}_t \pi_{t+1}) + g_t \quad (2)
\]

\[
i_t = \rho_t i_{t-1} + (1 - \rho_t) (\chi_{\pi,t} \pi_{t-1} + \chi_{x,t} x_{t-1}) + \varepsilon_t \quad (3)
\]

- Learning instead of RE
- TV Monetary Policy
VAR to form inflation and output expectations

Perceived Law of Motion (VAR(1)):

\[ Z_t = a_t + b_t Z_{t-1} + \eta_t \]  \hspace{1cm} (4)

where \( Z_t \equiv [\pi_t, x_t, i_t]' \)

\( \approx \) Minimum State Variable solution
Learning

Coefficient Updating

\[ \hat{\phi}_t = \hat{\phi}_{t-1} + g_{t,y} R_t^{-1} X_t (Z_t - X'_t \hat{\phi}_{t-1}) \]  
\[ R_t = R_{t-1} + g_{t,y} (X_{t-1} X'_{t-1} - R_{t-1}) \]  

where \( \hat{\phi}_t = (a'_t, \text{vec}(b_t)')' \) and \( X_t \equiv \{1, Z_{t-1}\}_0^{t-1} \).
Endogenous Time-Varying Gain

- Decreasing Gain if Forecast Errors are small
- Switch to Constant Gain if Forecast Errors become large

\[ g_{t,y} = \begin{cases} 
  t^{-1} & \text{if } \frac{\sum_{j=0}^{J}(|y_{t-j} - E_{t-j-1}y_{t-j}|)}{J} < \nu_{t}^y \\
  \frac{1}{g_{y}^{-1} + t} & \text{if } \frac{\sum_{j=0}^{J}(|y_{t-j} - E_{t-j-1}y_{t-j}|)}{J} \geq \nu_{t}^y, 
\end{cases} \quad (7) \]

where \( y = \pi, \chi, i \). (Decr. Gain reset to \( \frac{1}{g_{y}^{-1} + t} \))

- Similar to Marcet-Nicolini (\( \nu_t \) is m.a.d. of forecast errors)
- Constant Gain is estimated
- Which situations?
Questions:

1. Does the gain coefficient affect volatility? Can the model generate time-varying volatility in inflation and in the output gap?

2. Does the model fit U.S. data? Is there evidence of changes in the gain over time?

3. Does the omission of learning imply that researchers spuriously find stochastic volatility in the structural shocks?

4. Does the model-implied stochastic volatility resemble the SV estimated from the data?

5. What are the effects of MP on the estimated Volatility?
1. Endogenous Gain and TV Volatility

Figure: Volatility of simulated Inflation and Output Gap as a function of the constant gain coefficient.
1. Endogenous Gain and TV Volatility

- Volatility typically increases in the gain
- Simulation (10,000 periods)
- Gain switches endogenously according to previous forecast errors
1. Endogenous Gain and TV Volatility

Figure: Time-Varying Volatility with Time-Varying Endogenous Gain Coefficient.
2. Bayesian Estimation

- Gain switches from decreasing to constant
- Constant Gain jointly estimated in the system
- Metropolis-Hastings
- Uniform priors for gains
## 2. Bayesian Estimation: Priors

<table>
<thead>
<tr>
<th>Description</th>
<th>Param.</th>
<th>Range</th>
<th>Distr.</th>
<th>Mean</th>
<th>95% Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse IES</td>
<td>$\sigma^{-1}$</td>
<td>$\mathbb{R}^+$</td>
<td>$G$</td>
<td>1</td>
<td>[.12, 2.78]</td>
</tr>
<tr>
<td>Slope PC</td>
<td>$\kappa$</td>
<td>$\mathbb{R}^+$</td>
<td>$G$</td>
<td>.25</td>
<td>[.03, .7]</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>$\beta$</td>
<td></td>
<td>$-$</td>
<td>.99</td>
<td></td>
</tr>
<tr>
<td>Interest-Rate Smooth</td>
<td>$\rho_{\text{pre79}}$</td>
<td>[0, 1]</td>
<td>$B$</td>
<td>.8</td>
<td>[.46, .99]</td>
</tr>
<tr>
<td>Feedback to Infl.</td>
<td>$\chi_{\pi,\text{pre79}}$</td>
<td>$\mathbb{R}$</td>
<td>$N$</td>
<td>1.5</td>
<td>[.51, 2.48]</td>
</tr>
<tr>
<td>Feedback to Output</td>
<td>$\chi_{x,\text{pre79}}$</td>
<td>$\mathbb{R}$</td>
<td>$N$</td>
<td>.5</td>
<td>[.01, .99]</td>
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<td>$N$</td>
<td>.5</td>
<td>[.01, .99]</td>
</tr>
<tr>
<td>Std. MP shock</td>
<td>$\sigma_{\varepsilon}$</td>
<td>$\mathbb{R}^+$</td>
<td>$IG$</td>
<td>1</td>
<td>[.34, 2.81]</td>
</tr>
<tr>
<td>Std. $g_t$</td>
<td>$\sigma_g$</td>
<td>$\mathbb{R}^+$</td>
<td>$IG$</td>
<td>1</td>
<td>[.34, 2.81]</td>
</tr>
<tr>
<td>Std. $u_t$</td>
<td>$\sigma_u$</td>
<td>$\mathbb{R}^+$</td>
<td>$IG$</td>
<td>1</td>
<td>[.34, 2.81]</td>
</tr>
<tr>
<td>Constant Gain infl.</td>
<td>$\bar{g}_{\pi}$</td>
<td>[0, 0.3]</td>
<td>$U$</td>
<td>.15</td>
<td>[.007, .294]</td>
</tr>
<tr>
<td>Constant Gain gap</td>
<td>$\bar{g}_{x}$</td>
<td>[0, 0.3]</td>
<td>$U$</td>
<td>.15</td>
<td>[.007, .294]</td>
</tr>
<tr>
<td>Constant Gain FFR</td>
<td>$\bar{g}_{i}$</td>
<td>[0, 0.3]</td>
<td>$U$</td>
<td>.15</td>
<td>[.007, .294]</td>
</tr>
</tbody>
</table>

**Table 1 - Prior Distributions.**
2. Bayesian Estimation: Results

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Mean</th>
<th>95% Post. Prob. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse IES</td>
<td>$\sigma^{-1}$</td>
<td>6.04</td>
<td>[4.17-9.14]</td>
</tr>
<tr>
<td>Slope PC</td>
<td>$\kappa$</td>
<td>0.021</td>
<td>[0.0026-0.054]</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>-</td>
</tr>
<tr>
<td>IRS pre-79</td>
<td>$\rho_{pre79}$</td>
<td>0.937</td>
<td>[0.85-0.99]</td>
</tr>
<tr>
<td>Feedback Infl. pre79</td>
<td>$\chi_{\pi,pre-79}$</td>
<td>1.30</td>
<td>[0.83-1.81]</td>
</tr>
<tr>
<td>Feedback Gap pre79</td>
<td>$\chi_{x,pre-79}$</td>
<td>0.66</td>
<td>[0.29-1.13]</td>
</tr>
<tr>
<td>IRS post-79</td>
<td>$\rho_{post79}$</td>
<td>0.93</td>
<td>[0.88-0.97]</td>
</tr>
<tr>
<td>Feedback Infl. post79</td>
<td>$\chi_{\pi,post-79}$</td>
<td>1.66</td>
<td>[1.19-2.11]</td>
</tr>
<tr>
<td>Feedback Gap post79</td>
<td>$\chi_{x,post-79}$</td>
<td>0.48</td>
<td>[0.07-0.85]</td>
</tr>
<tr>
<td>Autoregr. Cost-push shock</td>
<td>$\rho_u$</td>
<td>0.39</td>
<td>[0.27-0.49]</td>
</tr>
<tr>
<td>Autoregr. Demand shock</td>
<td>$\rho_g$</td>
<td>0.85</td>
<td>[0.78-0.92]</td>
</tr>
<tr>
<td>Std. Cost-push shock</td>
<td>$\sigma_u$</td>
<td>0.89</td>
<td>[0.81-0.98]</td>
</tr>
<tr>
<td>Std. Demand shock</td>
<td>$\sigma_g$</td>
<td>0.65</td>
<td>[0.59-0.72]</td>
</tr>
<tr>
<td>Std. MP shock</td>
<td>$\sigma_\epsilon$</td>
<td>0.97</td>
<td>[0.88-1.07]</td>
</tr>
<tr>
<td>Constant gain (Infl.)</td>
<td>$\bar{g}_\pi$</td>
<td><strong>0.082</strong></td>
<td>[0.078-0.09]</td>
</tr>
<tr>
<td>Decreasing gain (Infl.)</td>
<td>$\bar{g}_\pi t^{-1}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Constant gain (Gap)</td>
<td>$\bar{g}_x$</td>
<td><strong>0.073</strong></td>
<td>[0.06-0.082]</td>
</tr>
<tr>
<td>Decreasing gain (Gap)</td>
<td>$\bar{g}_x t^{-1}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Constant gain (FFR)</td>
<td>$\bar{g}_i$</td>
<td>0.003</td>
<td>[0.0023]</td>
</tr>
<tr>
<td>Decreasing gain (FFR)</td>
<td>$\bar{g}_i t^{-1}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2 - Posterior Distributions: baseline case with $J = 4$. 
2. Bayesian Estimation: Time-Varying Gain

Figure: Endogenous Time-Varying Gain Coefficients (estimated constant gain). Baseline Case
Is it a good idea to use this learning rule?

- **Is it dominated by alternatives?**

<table>
<thead>
<tr>
<th></th>
<th>Endogenous TV Gain</th>
<th>Decreasing Gain</th>
<th>Constant Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.94</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.88</td>
<td>1.00</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 6 - RMSEs.

- **Optimality Tests.**

\[ l_{t+1,t} \equiv 1(Y_{t+1,t} < \hat{Y}_{t+1,t}) = \alpha + \beta \hat{Y}_{t+1,t} + u_{t+1} \] (8)

- **Back out Loss Function**
2. Bayesian Estimation: Time-Varying Gain

**Figure:** Endogenous Time-Varying Gain Coefficients (estimated constant gain). Case with $J = 20$
2. Bayesian Estimation: Time-Varying Gain

**Figure:** Constant Gain Coefficients: Prior and Posterior Distributions.
2. Bayesian Estimation: Time-Varying Gain

Figure: Endogenous Time-Varying Gain Coefficients (Case with low and high constant gain coefficients only).
2. Bayesian Estimation: Forecast Errors

Figure: Forecast errors for inflation, output gap, and federal funds rate (absolute values).
2. Bayesian Estimation: Forecast Errors

Figure: Rolling Mean Absolute Forecast errors vs. Updated $\nu_t$ for inflation, output gap, and federal funds rate series.
3. If learning is neglected:

- The volatility of shocks may be overestimated
- Possible to spuriously find Stochastic Volatility
3. Test for ARCH/GARCH Effects

Table 7 - Test for the existence of ARCH/GARCH effects (5% significance): proportion of rejections of the null hypothesis of no ARCH/GARCH effects.

<table>
<thead>
<tr>
<th></th>
<th>Endogenous TV Gain</th>
<th>No Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$J = 4$</td>
<td>$J = 20$</td>
</tr>
<tr>
<td>ARCH(1) GARCH(1,1)</td>
<td>0.517 0.61</td>
<td>0.48 0.56</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output Gap</td>
<td>0.785 0.89</td>
<td>0.85 0.90</td>
</tr>
</tbody>
</table>
Figure: Maximum rolling Standard Deviation of residuals across simulations: Kernel Density Estimation.
4. The Great Moderation

<table>
<thead>
<tr>
<th>Ratio ( \frac{\text{Std. Infl. 1985−2006}}{\text{Std. Infl. 1960−1984}} )</th>
<th>Endogenous TV Gain</th>
<th>No Learning</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J = 20 ) CG</td>
<td>0.39</td>
<td>0.43</td>
<td>1.00</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.42</td>
<td>0.54</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 8 - The Great Moderation: ratio of standard deviations for inflation and output gap in the second versus the first part of the simulated samples (median across simulations).
5. Monetary Policy, Learning, and Volatility

- Simulation for $\chi_\pi = [0, \ldots, 5]$
- Related: Benati-Surico (2007)

Figure: Effects of Monetary Policy on Volatility.
I am not convinced that the decline in macroeconomic volatility of the past two decades was primarily the result of good luck.

Changes in monetary policy could conceivably affect the size and frequency of shocks hitting the economy, at least as an econometrician would measure those shocks.

Changes in inflation expectations, which are ultimately the product of the monetary policy regime, can also be confused with truly exogenous shocks in conventional econometric analyses.

Some of the effects of improved monetary policies may have been misidentified as exogenous changes in economic structure or in the distribution of economic shocks.
6. TV Volatility: Learning or Exogenous Shocks?

Test ARCH/GARCH in DSGE Model Innovations now

<table>
<thead>
<tr>
<th></th>
<th>Output Gap</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSGE-RE</td>
<td>ARCH</td>
<td>ARCH</td>
</tr>
<tr>
<td>DSGE-TV Gain</td>
<td>ARCH</td>
<td>No ARCH</td>
</tr>
</tbody>
</table>
6. TV Volatility: Learning or Exogenous Shocks?

Figure: Rolling Std. estimated innovations under RE and Learning
Conclusions

- Strong Evidence of Stochastic Volatility in the economy
  Usually Exogenous
- Learning with endogenous TV gain (depends on previous forecast errors) ⇒ Endogenous Stochastic Volatility
- Gain often larger in pre-1984 sample
- Overestimation of TV in volatility of exogenous shocks.
Future Directions

- How much volatility can learning explain? (estimate DSGE model with learning and TV volatility).
- More serious attempt to match volatility series in the data.
- Different ways to model endogenous gain/ Optimality
- Interactions Policy/Learning/Volatility