How Much Inflation is Necessary to Grease the Wheels?*

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Abstract

This paper studies Tobin's proposition that inflation "greases" the wheels of the labor market. The analysis is carried out using a simple dynamic stochastic general equilibrium model with asymmetric wage adjustment costs. Optimal inflation is determined by a benevolent government that maximizes the households' welfare. The Simulated Method of Moments is used to estimate the nonlinear model based on its second-order approximation. Econometric results indicate that nominal wages are downwardly rigid and that the optimal level of grease inflation for the U.S. economy is about 1.2 percent per year, with a 95% confidence interval ranging from 0.2 to 1.6 percent.

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1 Introduction

In his presidential address to the American Economic Association in 1971, James Tobin suggests that a positive rate of inflation may be socially beneficial in an economy where nominal prices—in particular, nominal wages—are more downwardly rigid than upwardly rigid (Tobin, 1972). To illustrate Tobin's argument, suppose that the economy is hit by an exogenous shock that requires a decline in the real wage, such as a negative productivity shock. Two plausible adjustment paths are to keep the price level fixed and cut nominal wages, and to keep the nominal wages fixed and increase the price level. Tobin claims that the former path, which is characterized by a zero inflation rate, may involve significant social costs when nominal wages are downwardly rigid. Instead, the latter path, which features a positive inflation rate, may deliver the same reduction in the real wage at a lower cost. The idea that inflation eases the adjustment of the labor market by speeding the decline of real wages following an adverse shock is described in the literature by the catchphrase that inflation "greases the wheels of the labor market."

This paper uses a stylized dynamic stochastic general equilibrium model with asymmetric nominal rigidities to formally examine Tobin's proposition and to construct a theory-based estimate of the optimal amount of "grease" inflation for the U.S. economy. Optimal inflation is determined in our model by a benevolent government that maximizes the households' welfare under commitment (*i.e.*, the Ramsey policy). A nonlinear approximation of the model is estimated by the Simulated Method of Moments and an estimate of optimal grease inflation is constructed by measuring how much more expected inflation asymmetric costs yield compared to symmetric costs.

This subject matter is important because there is currently a discrepancy between economic theory—that prescribes a zero-to-negative optimal inflation rate—and monetary policy in practice—that explicitly or implicitly targets low, but strictly positive, inflation rates. The theoretical result that optimal inflation is negative is driven by Friedman's rule (Friedman, 1969). Under Friedman's rule a rate of deflation equal to the real return on capital eliminates the wedge between social marginal cost of producing money, which is essentially zero, and the private marginal cost of carrying money, which is the nominal interest rate. Additional considerations like fiscal policy and price rigidity, deliver optimal inflation rates that are larger than Friedman's rule but still negative.[1]

[1] See, among many others, Rotemberg and Woodford (1997), Chari and Kehoe (1999), Teles (2003), Khan, King and Wolman (2003), Kim and Henderson (2005), and Schmitt-Grohe and Uribe (2004, 2006), as well as references therein. In addition to the asymmetric nominal rigidities studied here, another reason because of which optimal inflation may be positive is the zero lower-bound on nominal interest rates (see, for example, Billi, 2005).

The idea that wages are more downwardly (than upwardly) rigid dates at least to Keynes (1936, Chapter 21). Empirical evidence on downward wage rigidity using micro-level data takes the form of attitude surveys and the empirical analysis of wage distributions. Bewley (1995) and Campbell and Kamlani (1997) find that employers cut wages only in cases of extreme financial distress while worrying about the effect of nominal wage cuts on the worker's morale. Kahneman, Knetsch and Thaler (1986) find that individuals dislike nominal wage cuts more than an alternative scenario even when both of them involve the same real wage cut. Researchers who study the distribution of nominal wage changes at the individual level point out that it features a peak at zero and is positively skewed with very few nominal wage cuts. See, for example, McLaughlin (1994), Akerlof *et al.* (1996), and Card and Hyslop (1997) for the United States; Kuroda and Yamamoto (2003) for Japan; Castellanos *et al.* (2004) for Mexico; and Fehr and Goette (2005) for Switzerland. Institutional characteristics, like laws forbidding nominal wage cuts (Mexico) or making the current nominal wage the default outcome when union-employer negotiations fail, may also contribute to downward nominal wage rigidity.

This paper reports four main results. First, U.S. prices and wages are rigid, but in the case of wages, rigidity is asymmetric in the sense that they are more downwardly rigid. This conclusion is based on an econometric estimate of the asymmetry in the wage adjustment cost function. Second, the Ramsey policy prescribes a positive (gross) rate of price inflation of about 1.012 (*i.e.,* optimal grease inflation of about 1.2 percent per year). This result is driven by prudence, meaning that the benevolent planner prefers the systematic, but small, price and wage adjustment costs associated with a positive inflation rate rather than taking the chance of incurring the large adjustment costs associated with nominal wage decreases. Third, asymmetry in wage adjustment costs delivers non-trivial implications for optimal responses following a productivity shocks and generates higher-order properties that are roughly in line with those found in the U.S. data. Finally, in the case where monetary policy were implemented by a strict inflation target, the optimal target is substantially larger than the unconditional inflation mean obtained under the Ramsey policy.

The paper is organized as follows. Section 2 constructs a model with imperfect competition and asymmetric price- and wage-adjustment costs and describes the Ramsey problem of the benevolent government. Section 3 presents the econometric methodology and reports estimates of the structural parameters. Section 4 reports estimates of optimal grease inflation, studies the dynamic implications of the model, and compares welfare under the Ramsey policy and under strict inflation targeting. Section 5 concludes and discusses current and future work in our research agenda on asymmetric nominal rigidities.

2 The Model

2.1 Households

The economy is populated by a continuum of infinitely-lived households indexed by $n \in [0,1]$. Households are identical, except for the fact that they have differentiated job skills which give them monopolistically competitive power over their labor supply. At time τ, household n maximizes

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{(t-\tau)} \left(\frac{(c_t^n)^{1-\rho}}{1-\rho} - \frac{(h_t^n)^{1+\chi}}{1+\chi} \right), \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, c_t^n is consumption, h_t^n is hours worked, and ρ and χ are positive preference parameters.[2] Consumption is an aggregate of differentiated goods indexed by $i \in [0,1]$

$$c_t^n = \left(\int_0^1 (c_{i,t}^n)^{1/\mu} di \right)^{\mu}, \tag{2}$$

where $\mu > 1$. In this specification, the elasticity of substitution between goods is constant and equal to $\mu/(\mu-1)$. When $\mu \to 1$, goods become perfect substitutes and the elasticity of substitution tends to infinity. When $\mu \to \infty$, the aggregator becomes the Cobb-Douglas function and the elasticity is unity.

As monopolistic competitors, households choose their wage and labor supply taking as given the firms' demand for their labor type. Labor market frictions induce a cost in the adjustment of nominal wages. This cost takes the form of the linex function (as introduced by Varian, 1974)

$$\Phi_t^n = \Phi(W_t^n/W_{t-1}^n) = \phi \left(\frac{\exp\left(-\psi\left(W_t^n/W_{t-1}^n - 1\right)\right) + \psi\left(W_t^n/W_{t-1}^n - 1\right) - 1}{\psi^2} \right), \tag{3}$$

where W_t^n is the nominal wage, and ϕ and ψ are cost parameters. This functional form is attractive for four reasons. First, the cost depends on both the magnitude and sign of the wage adjustment. Consider, for example, the case where $\psi > 0$. As W_t^n increases over W_{t-1}^n, the linear term dominates and the cost associated with wage increases rises linearly. In contrast, as W_t^n decreases below W_{t-1}^n, it is the exponential term that dominates and the cost

[2] In preliminary work, we studied a more general formulation with aggregate shocks to the disutility of work and to the overall level of utility. However, results were very similar to the ones reported here. The shock to the disutility of work behaves like the productivity shock specified in Section 2.2 but its estimated conditional variance was much smaller. The shock to the level of utility disturbs the household's Euler equation for consumption, but the Ramsey planner would adjust the nominal interest rate to perfectly undo this shock's effects. Hence, decision rules for all variables (except for the nominal interest rate) would be independent of the shock.

associated with wage decreases rises exponentially. Hence, nominal wage decreases involve a larger frictional cost than increases, even if the two percentage magnitudes are the same. The converse is true in the case where $\psi < 0$. Second, the function nests the quadratic form as a special case when ψ tends to zero.[3] Thus, the comparison between the model with asymmetric costs and a restricted version with quadratic costs is straightforward. Third, the linex function is differentiable everywhere and strictly convex for any $\phi > 0$. Finally, this function does not preclude nominal wage cuts that, although relatively rare, are observed in micro-level data. In order to develop further the readers' intuition, Figure 1 plots the quadratic and asymmetric cost functions, the latter in the case of a positive ψ.

There are two types of financial assets: one-period nominal bonds and Arrow-Debreu state-contingent securities. The household enters period t with B_{t-1} nominal bonds and a portfolio A_{t-1} of state-contingent securities, and then receives wages, interests, dividends and state-contingent payoffs. These resources are used to finance consumption and the acquisition of financial assets to be carried out to the next period. Expressed in real terms, the household's budget constraint is

$$c_t^n + \frac{\delta_{t,t+1} A_t^n - A_{t-1}^n}{P_t} + \frac{B_t^n - I_{t-1} B_{t-1}^n}{P_t} = \left(\frac{W_t^n h_t^n}{P_t}\right)(1 - \Phi_t^n) + \frac{D_t^n}{P_t},$$

for $t = \tau, \tau+1, \ldots, \infty$, where $\delta_{t,t+1}$ is a vector of prices, I_t is the gross nominal interest rate, D_t^n are dividends and

$$P_t = \left(\int_0^1 (P_{i,t})^{1/(1-\mu)} di\right)^{1/(1-\mu)}, \tag{4}$$

is an aggregate price index with $P_{i,t}$ denoting the price of good i. Without loss of generality, it is assumed that the wage adjustment cost is paid by the household. Prices are measured in terms of a unit of account called "money," but the economy is cashless otherwise.

The household's utility maximization involves choosing $\{c_t^n, A_t^n, B_t^n, W_t^n, h_t^n\}_{t=\tau}^{\infty}$ subject to the initial asset holdings and the sequence of wages, labor demand, budget constraints, and a no-Ponzi-game condition. First-order necessary conditions include

$$(c_t)^{-\rho} = \eta_t, \tag{5}$$

whereby the marginal utilities wealth and consumption are equalized at the optimum, and

$$\begin{aligned} &\frac{\eta_t}{P_t}\left(\left(\frac{\theta}{\theta-1}\right) h_t^n (1 - \Phi_t^n) + \left(\frac{W_t^n}{W_{t-1}^n}\right) h_t^n (\Phi_t^n)'\right) \\ &= \frac{\eta_t}{P_t}\left(h_t^n (1 - \Phi_t^n)\right) + \frac{\theta}{\theta-1}\left(\frac{(h_t^n)^{1+\chi}}{W_t^n}\right) + \beta E_t\left(\frac{\eta_{t+1}}{P_{t+1}}\left(\frac{W_{t+1}^n}{W_t^n}\right)^2 h_{t+1}^n \left(\Phi_{t+1}^n\right)'\right), \end{aligned} \tag{6}$$

[3] To see this, take the limit of $\Phi(\cdot)$ as $\psi \to 0$ by applying l'Hôpital's rule twice.

where $\theta/(\theta-1)$ is the elasticity of substitution between labor types (as specified below in the firms' problem), and $(\Phi_t^n)'$ denotes the derivative of the cost function with respect to its argument.[4] Condition (6), usually referred to as the wage Phillips curve, equates the marginal costs and benefits of increasing W_t^n. The costs are the decrease in hours worked as firms substitute away from the more expensive labor input, and the wage adjustment cost. The benefits are the increase in labor income per hour worked, the increase in leisure time as firms reduce their demand for type-n labor, and the reduction in the future expected wage adjustment cost. Given nominal consumption expenditures, the optimal consumption of good i satisfies

$$c_{i,t}^n = \left(\frac{P_{i,t}}{P_t}\right)^{-\mu/(\mu-1)} c_t^n. \tag{7}$$

2.2 Firms

Each firm produces a differentiated good $i \in [0,1]$ using a production function featuring decreasing returns to scale,

$$y_{i,t} = x_t h_{i,t}^{1-\alpha}, \tag{8}$$

where $y_{i,t}$ is output of good i, $h_{i,t}$ is labor input, $\alpha \in (0,1]$ is a production parameter, and x_t is an exogenous productivity shock. The productivity shock follows the stochastic process

$$\ln(x_t) = \xi \ln(x_{t-1}) + \varepsilon_t,$$

where $\xi \in (-1,1)$ and ε_t is an identically and independently distributed innovation with zero mean and variance σ^2. Labor input is an aggregate of heterogeneous labor supplied by households,

$$h_{i,t} = \left(\int_0^1 (h_{i,t}^n)^{1/\theta} dn\right)^{\theta}, \tag{9}$$

where $\theta > 1$. The price of the labor input is

$$W_{i,t} = \left(\int_0^1 (W_t^n)^{1/(1-\theta)} dn\right)^{1-\theta}, \tag{10}$$

where W_t^n is the wage demanded by the supplier of type-n labor. Product differentiation gives the firm monopolistically competitive power, so price is a choice variable. However,

[4] The other first-order conditions (not shown) price the nominal bond and the portfolio of state contingent securities.

the adjustment of nominal prices is assumed to be costly. In particular, the real cost of a price change per unit is

$$\Gamma_t^i = \Gamma(P_{i,t}/P_{i,t-1}) = \gamma \left(\frac{\exp(-\varsigma\,(P_{i,t}/P_{i,t-1}-1)) + \varsigma\,(P_{i,t}/P_{i,t-1}-1) - 1}{\varsigma^2} \right), \tag{11}$$

where $\gamma\,(>0)$ and ς are cost parameters. In what follows, we focus on the special case where $\varsigma \to 0$ (*i.e.*, the quadratic cost function proposed by Rotemberg, 1982) and price adjustment costs are, therefore, symmetric.[5]

At time τ, firm i maximizes the discounted sum of real profits

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{(t-\tau)} \frac{\eta_t}{\eta_\tau P_t} \left(\left(1-\Gamma_t^i\right) P_{i,t} c_{i,t} - \int_0^1 W_t^n h_t^n dn \right),$$

and $c_{i,t} = \int_0^1 c_{i,t}^n dn$ is total consumption demand for good i. Maximization is subject to the technology (8), the downward-sloping consumption demand function (7), and the condition that supply must meet the demand for good i at the posted price. First-order conditions equate the marginal productivity of labor with its cost,

$$(1-\alpha) x_t h_{i,t}^{-\alpha} = W_{i,t}/P_{i,t}, \tag{12}$$

and the marginal costs with the marginal benefits of increasing $P_{i,t}$,

$$\begin{aligned} & \frac{1}{P_t} \left(\left(\frac{\mu}{\mu-1} \right) c_{i,t} \left(1-\Gamma_t^i\right) + \left(\frac{P_{i,t}}{P_{i,t-1}} \right) c_{i,t} \left(\Gamma_t^i\right)' \right) \\ = \; & \frac{1}{P_t} \left(c_{i,t} \left(1-\Gamma_t^i\right) \right) + \frac{\mu}{\mu-1} \left(\frac{\Psi_t y_{i,t}}{P_{i,t}} \right) + \beta E_t \left(\frac{\eta_{t+1}}{\eta_t} \frac{c_{i,t+1}}{P_{t+1}} \left(\frac{P_{i,t+1}}{P_{i,t}} \right)^2 \left(\Gamma_{t+1}^i\right)' \right), \end{aligned} \tag{13}$$

where Ψ_t is the nominal marginal cost. On the left-hand side of this price Phillips curve, the costs are the decrease in sales, which is proportional to the elasticity of substitution between goods, and the price adjustment cost. On the right-hand side, the benefits are the increase in revenue for each unit sold, the decrease in the marginal cost, and the reduction in the

[5]In preliminary work, we considered an unrestricted version of the model with a possibly non-zero ς. However, a Wald test of $\varsigma = 0$ does not reject this hypothesis at the 5 percent significance level, and identification of the other parameters is considerably sharper when this restriction is imposed. Peltzman (2000) studies the pricing decisions of a Chicago supermarket chain at the level of individual goods and finds no asymmetry in its response to input price increases or decreases. Zbaracki *et al.* (2004) finds that customers are antagonized by price changes, even when they involve a price decrease. Price decreases are not always welcomed because passing lower prices downstream also involves adjustment costs and because current price cuts make future price increases more costly (see p. 527). In summary, the data seem to be in reasonable agreement with the assumption that price adjustment costs are symmetric.

future expected price adjustment cost. Given nominal expenditures on labor, the optimal demand of type-n labor is

$$h_t^n = \left(\frac{W_t^n}{W_t}\right)^{-\theta/(\theta-1)} h_t$$

where $-\theta/(\theta-1)$ is the elasticity of demand for the labor of household n with respect to its relative wage.

2.3 Symmetric Equilibrium

In the symmetric equilibrium, all households supply exactly the same amount of labor. This implies that $h_t^n = h_t$ and, consequently, $W_t^n = W_t$. Since households are identical in all other respects, it follows that their equilibrium choices will be same, that n subscripts can be dropped without loss of generality, and that net holdings of Arrow-Debreu securities and bonds can be neglected in the solution. Similarly, all firms are identical *ex-post* meaning that they charge the same price and produce the same quantity. Hence, all relative prices are one and the i subscripts can also be dropped. Substituting the government's budget constraint and the profits of the (now) representative firm into the budget constraint of the (now) representative household delivers the economy-wide resource constraint:

$$c_t = y_t(1-\Gamma_t) - w_t h_t \Phi_t. \tag{14}$$

where $w_t = W_t/P_t$ is the real wage.

2.4 Monetary Policy

The government follows a Ramsey policy of maximizing the households' welfare subject to the resource constraint while respecting the first-order conditions of firms and households.[6] That is, the government chooses $\{c_t,\, \eta_t,\, h_t,\, w_t,\, i_t,\, \Omega_t,\, \Pi_t\}_{t=\tau}^{\infty}$ to maximize

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{(t-\tau)} \left(\frac{(c_t)^{1-\rho}}{1-\rho} - \frac{(h_t)^{1+\chi}}{1+\chi}\right),$$

where $\Omega_t = W_t/W_{t-1}$ is gross wage inflation and $\Pi_t = P_t/P_{t-1}$ is gross price inflation, subject to conditions (5), (6), (12), and (13), and taking as given previous values for wages, goods prices, and shadow prices. Notice that in the formulation of the government's problem, it is assumed that the discount factor used to evaluate future utilities is the same as that used

[6]Admittedly, the Ramsey policy is an incomplete characterization of U.S. monetary policy. However, this policy—unlike *ad-hoc* policy rules—endogenously determines the behavior of the government, including the deterministic steady state for inflation. In Section 4.4, we compare the outcomes of the Ramsey policy with those of a simple inflation targeting rule

by households.[7] It is also assumed that the government can commit to the implementation of the optimal policy.

Since this problem does not have a closed-form solution, we use a perturbation method that involves taking a second-order Taylor series expansion of the government's decision rules as well as its constraints and characterizing local dynamics around the deterministic steady state. See Jin and Judd (2002), Kim, Kim, Schaumburg and Sims (2003), and Schmitt-Grohé and Uribe (2004) for a detailed explanation of this approach.[8]

3 Estimation

3.1 Data

The data used to estimate the model are quarterly observations of the real wage, hours worked, real consumption per capita, the price inflation rate, the wage inflation rate, and the nominal interest rate between 1964Q2 to 2006Q2. The sample starts in 1964 because aggregate data on wages and hours worked are not available prior to that year. The raw data were taken from the database available at the Federal Reserve Bank of St. Louis. The rates of price and wage inflation are measured by the percentage change in the Consumer Price Index (CPI) and the average hourly earnings for private industries. Hours worked is the total number of weekly hours worked in private industries. The nominal interest rate is the three-month Treasury Bill rate. Real consumption is measured by the Personal Consumption Expenditures in nondurable goods and services per capita divided by the CPI. The population series corresponds to the quarterly average of the mid-month U.S. population estimated by the Bureau of Economic Analysis (BEA). Except for the nominal interest rate, all data are seasonally adjusted at the source. All series were logged and linearly detrended prior to the estimation of the model.

3.2 Econometric Methodology

The second-order approximate solution of our nonlinear DSGE model is estimated using the Simulated Method of Moments (SMM). The application of SMM for the estimation

[7] In preliminary work, we relaxed this assumption and estimated the government's discount factor separately from that of households. However, econometric estimates were remarkably similar and differed only after the fifth decimal. It is interesting to note that when both factors are assumed to be different, the identification of the wage asymmetry parameter is sharper because in this case this parameter affects first-order dynamics.

[8] The codes that we employed were adapted from those originally written by Stephanie Schmitt-Grohé and Martin Uribe. The dynamic simulations of the nonlinear model are based on the pruned version of the model, as suggested by Kim, Kim, Schaumburg and Sims (2003).

of time-series models was proposed by Lee and Ingram (1991) and Duffie and Singleton (1993). Ruge-Murcia (2007) uses Monte-Carlo analysis to compare various methods used for the estimation of DSGE models and reports that moment-based estimators are generally more robust to misspecification than Maximum Likelihood (ML). This is important because economic models are stylized by definition and misspecification of an unknown form is likely.[9] Method of Moments estimators are also attractive for the estimation of nonlinear DSGE models because the numerical evaluation of its objective function is relatively cheap. This means, for example, that the researcher can afford to use genetic algorithms for its optimization. These algorithms require a larger number of function evaluations than alternative gradient-based methods, but greatly reduce the possibility of converging to a local optimum—rather than the global one.

Define $\boldsymbol{\theta}$ to be a $q \times 1$ vector of structural parameters, $\mathbf{g}_t$ to be a $p \times 1$ vector of empirical observations on variables whose moments are of our interest, and $\mathbf{g}_\iota(\boldsymbol{\theta})$ to be the synthetic counterpart of $\mathbf{g}_t$ whose elements come from simulated data generated by the model. Then, the SMM estimator, $\widehat{\boldsymbol{\theta}}$, is the value that solves

$$\min_{\{\boldsymbol{\theta}\}} \mathbf{G}(\boldsymbol{\theta})'\mathbf{W}\mathbf{G}(\boldsymbol{\theta}), \tag{15}$$

where

$$\mathbf{G}(\boldsymbol{\theta}) = (1/T)\sum_{t=1}^{T}\mathbf{g}_t - (1/\lambda T)\sum_{\iota=1}^{\lambda T}\mathbf{g}_\iota(\boldsymbol{\theta}),$$

T is the sample size, λ is a positive constant, and $\mathbf{W}$ is a $q \times q$ weighting matrix. Under the regularity conditions in Duffie and Singleton (1993),

$$\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \to N(\mathbf{0},(1+1/\lambda)(\mathbf{D}'\mathbf{W}^{-1}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}^{-1}\mathbf{S}\mathbf{W}^{-1}\mathbf{D}(\mathbf{D}'\mathbf{W}^{-1}\mathbf{D})^{-1}), \tag{16}$$

where

$$\mathbf{S} = \lim_{T\to\infty} Var\left((1/\sqrt{T})\sum_{t=1}^{T}\mathbf{g}_t\right), \tag{17}$$

and $\mathbf{D} = E(\partial\mathbf{g}_\iota(\boldsymbol{\theta})/\partial\boldsymbol{\theta})$ is a $q \times p$ matrix assumed to be finite and of full column rank.[10]

[9] On the other hand, under the assumption that the model is correctly specified, ML is statistically more efficient than the Method of Moments. This means that, even though both methods deliver consistent parameter estimates, those obtained by ML would typically have smaller standard errors.

[10] An alternative approach is to analytically compute the moments predicted by the model based on the pruned quadratic solution and use them in the objective function instead of $(1/\lambda T)\sum_{\iota=1}^{\lambda T}\mathbf{g}_\iota(\boldsymbol{\theta})$. We followed this GMM approach in preliminary work but found it problematic because, under the assumption that the private and social discount factors are the same, first-order dynamics are independent of the asymmetry parameter, and, consequently, ψ is not identified. Notice that for the pruned version of nonlinear DSGE models, SMM is not statistically equivalent to GMM as $\lambda \to \infty$. In contrast, the two are asymptotically equivalent in the case of linear models (see Ruge-Murcia, 2007)

In this application, $\boldsymbol{\theta}$ contains the discount factor (β), the curvature parameters of the utility function (ρ and χ), the parameters of the adjustment cost function (ϕ, ψ and γ), and the parameters of the productivity shock process (ξ and σ). For the simulation of the model, the productivity innovations are drawn from a Normal distribution.[11] The weighting matrix $\mathbf{W}$ is the diagonal of the inverse of the matrix with the long-run variance of the moments, $\mathbf{S}$. In turn, $\mathbf{S}$ is computed using the Newey-West estimator with a Barlett kernel. The derivatives in the Jacobian matrix $\mathbf{D}$ are numerically computed at the optimum. The moments in $\mathbf{G}(\boldsymbol{\theta})$ are five (out of six) variances, all twenty one covariances and all six autocovariances of the data series.[12]

Three parameters are weakly identified in the initial estimation and, consequently, we use additional information to fix their values to economically plausible numbers during the estimation routine. These parameters are the curvature of the production function ($1 - \alpha$) and the elasticities of substitution between goods and between labor types (μ and θ, respectively). Data from the U.S. National Income and Product Accounts (NIPA) show that the share of labor in total income is approximately 2/3 and, therefore, a plausible value for α is 1/3. The elasticities of substitution between goods and between labor types are fixed to $\mu = 1.1$ and $\theta = 1.4$, respectively. This value for μ is standard in the literature. Sensitivity analysis with respect to θ indicates that results are robust to using similarly plausible values.

3.3 SMM Estimates

SMM parameter estimates based on the second-order approximate solution of the model are reported in the first column of Table 1. Regarding the preference parameters, notice that the coefficient that determines the consumption curvature (ρ) is statistically different from zero but not from one at the 5 percent significance level. Since, in addition, its point estimate is quantitatively very close to one, it follows that consumption preferences may

[11] We also estimated a version of the model where innovations follow a t distribution. Results are very similar to those reported here because the estimated number of degrees of freedom is large and the two distributions (the t and the Normal) resemble each other.

[12] The variance of the real wage is not included in $\mathbf{G}(\boldsymbol{\theta})$ because the real wage is the ratio of nominal wages to the CPI and so it is possible to show that

$$Var(\widehat{w}_t) = (1/2)(Var(\widehat{\Pi}_t) + Var(\widehat{\Omega}_t)) - Cov(\widehat{\Pi}_t, \widehat{\Omega}_t) + Cov(\widehat{w}_t, \widehat{w}_{t-1}),$$

where the "hat" denotes deviation from the deterministic steady state. Hence, the variance of the real wage contributes no additional information beyond that contained in the variances of price inflation and wage inflation, their covariance, and the autocovariance of the real wage, all of which are included in $\mathbf{G}(\boldsymbol{\theta})$. In other words, if one were to include the variance of the real wage, then as a result of the linear combination above, the Jacobian matrix of the moments, $\mathbf{D}$, would not be of full rank and regularity conditions in Duffie and Singleton (1993) would not be satisfied.

be well approximated by a logarithmic function. On the other hand, the coefficient that determines the leisure curvature (χ) is quantitatively and statistically close to zero. Thus, the aggregate representation of the households' disutility of work is empirically consistent with the indivisible-labor model (Hansen, 1985).

Regarding the parameters of the adjustment cost functions, the hypotheses that $\phi = 0$ and $\gamma = 0$ can be rejected against the respective alternatives that $\phi > 0$ and $\gamma > 0$ at the 10 and 5 percent significance levels. In other words, the data rejects the hypothesis that U.S. nominal wages and prices are flexible in favor of the alternative hypothesis that they are rigid. Similar results are reported, among others, by Kim (2000), Ireland (2001), and Christiano, Eichenbaum and Evans (2005) using linear DSGE models that explicitly or implicitly impose symmetry in the adjustment costs of nominal variables.

The estimate of the wage asymmetry parameter is 901.4 with a standard error of 426.2. Since this estimate of positive and statistically different from zero at the 5 percent level, we conclude that U.S. nominal wages are more downwardly than upwardly rigid. Returning to Figure 1, note that the parameters used to construct the asymmetric cost function are the SMM estimates reported in Table 1, that is $\phi = 33.72$ and $\psi = 901.4$. This figure implies, for example, that an aggregate nominal-wage cut of one percent would involve frictional adjustment costs of 0.1 percent of annual labor income, while a cut of 2 percent would involve costs of 1.4 percent. Finally, estimates of the parameters of the process of the productivity shock are very similar to those reported in earlier empirical work.

In order to examine the properties of our model, it is useful to have as benchmark a restricted version of the model with quadratic wage adjustment costs. This restricted version corresponds to the special case where $\psi \to 0$. SMM estimates of this model are reported in the second column of Table 1. Note that the estimates of the preference and productivity parameters for this model are very similar to those reported for the asymmetric model. Estimates of the adjustment cost functions are imprecise but would tend to suggest that wages are substantially more rigid than prices. This implication of the quadratic cost model is not necessarily at odds with the data, except that results reported above in this paper would finesse this implication by noting that most of observed nominal wage rigidity is in the downward direction.

4 Properties of the Estimated Model

4.1 Optimal Grease Inflation

This section constructs a measure of optimal grease inflation for the U.S. economy by calculating how much asymmetric costs increase expected inflation compared with symmetric (*i.e.,* quadratic) costs. For this purpose, we compute *via* simulation the unconditional inflation mean implied by the two versions of our model, as reported in Table 1.

Consider first the model with symmetric costs (the right column). The unconditional mean of annual gross inflation is 1.000012 with the 95 percent confidence interval of [1.000008, 1.000018].[13] Since this confidence interval does not include the value of 1, the null hypothesis that optimal gross inflation is unity can be rejected at the 5 percent significance level. This result is due to the model's departure from certainty equivalence. However, given the clearly small magnitude of optimal net inflation, of about 0.12 basis points, this departure is economically insignificant.

Consider now the model with asymmetric costs. The estimate of the unconditional mean of annual gross inflation is 1.012 with 95 percent confidence interval equal to [1.002, 1.016]. As before, this confidence interval does not include 1 and, consequently, the null hypothesis that optimal gross inflation is unity can be rejected at the 5 percent significance level. However, the departure from certainty equivalence in this case is not only statistically but also economically significant. Optimal inflation is substantially larger than 1 because the monetary authority acts prudently and reduces the probability of facing highly costly downward nominal-wage adjustment by choosing an average rate of price (and wage) inflation well above unity.

This paper defines the measure of grease inflation as the difference between the two figures reported above. Subtracting optimal gross inflation under the asymmetric-cost model from its corresponding value under the symmetric-cost model delivers an estimate of optimal grease inflation for the U.S. economy at approximately 0.012, that is, 1.2 percent per year. Since the confidence interval of the symmetric-cost model is very narrow and near unity, a 95 percent confidence interval for the optimal grease inflation would range roughly from 0.2 to 1.6 percent.

[13]The lower and upper bounds of this interval are computed as follows. First, we draw 120 independent realizations of $\boldsymbol{\theta}$ from the empirical joint density function of the SMM estimates. Then, for each realization of $\boldsymbol{\theta}$, we compute the expected inflation rate. Finally, the bounds of the confidence interval are the $2.5th$ and $97.5th$ quantiles of the simulated expected inflation rates.

4.2 Impulse Responses

This section examines how the economy responds to shocks. Starting at the stochastic steady state, the economy is subjected to an unexpected temporary shock, and the responses of consumption, hours worked, price inflation, wage inflation, the real wage, and the interest rate are then plotted as a function of time. In linear models, the responses to a shock of size ϵ are one-half those to a shock size 2ϵ and the mirror image of those to a shock of size $-\epsilon$. Thus, any convenient normalization (*e.g.*, $\epsilon = 1$) summarizes all relevant information about dynamics. However, in nonlinear models like ours, responses will typically depend on both the sign and the size of the shock.[14] Thus, we plot responses to innovations of size $+1$, $+2$, -1, and -2 standard deviations. Responses to productivity shocks when $\psi = 0$ and 901.4 are reported in Figures 2 and 3, respectively. The vertical axis is the percentage deviation from the deterministic steady state and the flat line is the level of the stochastic steady state. The distance between this line and zero represents the effect of uncertainty on the unconditional first-moments of the variables and, thus, the model's departure from certainty equivalence.

First, consider the responses in Figure 2, where $\psi = 0$. Following a negative shock, consumption, hours, wage inflation, and the real wage decrease, while price inflation and the nominal interest rate increase. The converse happens following a positive shock. There is very little asymmetry between positive and negative, and between small and large shocks.

Now, consider the responses in Figure 3, where $\psi = 901.4$. A negative productivity shock decreases the marginal productivity of labor and consequently the real wage must fall. This is an example of the type of shock that Tobin had in mind in his presidential address to the American Economic Association. From Figures 2 and 3, it is apparent that the real wage does indeed fall as required but that the optimal adjustment depends on the size of the asymmetry parameter ψ.

When $\psi = 0$, the nominal wage decreases and price level increases (Figure 2). When $\psi > 0$, the Ramsey policy involves positive average rates of price and wage inflation. Hence, in Figure 3, the nominal wage still increases or decreases by very little. Wage inflation is initially larger than its steady state when the shock is large and most of the reduction in the real wage is achieved by an increase in the price level. Thus, the response of price inflation is larger when $\psi > 0$ than when $\psi = 0$ and more than proportional when the shock is large. Hours and consumption decrease following a negative shock, and their response to a large shock of -2σ is more than twice of that to a smaller shock of $-\sigma$.

[14] See Gallant, Rossi and Tauchen (1993), and Koop, Pesaran, and Potter (1996) for more complete treatments of impulse-response analysis in nonlinear systems.

Consider also the effect of a positive productivity shock. In this case, the real wage increases but again the adjustment depends on the value of ψ. In general, the adjustment takes place with a decrease of the price level and an increase in the nominal wage. However, the decrease of the price level is smaller and the increase in the nominal wage is larger than in the case where $\psi = 0$. This effect increases when the productivity shock is larger. The increase of hour and consumption and the decrease in the nominal rate is much smaller when $\psi > 0$ and the response to large and small shocks are quantitatively similar.

4.3 Higher-Order Moments

This section derives and evaluates the model predictions for higher-order moments of the variables. This exercise is important for three reasons. First, in contrast to linear DSGE models that inherit their higher-order properties directly from the shock innovations, the nonlinear propagation mechanism in our model means that economic variables may be non-Gaussian, even if the productivity innovations are Gaussian. Second, this observation means that up to the extent that actual data has non-Gaussian features, comparing the higher-order moments predicted by the model with those of the data may be a useful tool in model evaluation. Finally, since previous literature on downward wage rigidity documents the positive skewness of individual nominal wage changes, it is interesting to examine whether the same is true for the representative household in our model.

The skewness and kurtosis predicted by the models with asymmetric and quadratic wage adjustment costs are computed on the basis of 10000 simulated observations and reported in Table 2, along with their respective counterparts computed using U.S. data. In the U.S. data, the nominal interest rate and the rates of price and wage inflation are positively skewed and leptokurtic, consumption is negatively skewed and leptokurtic, and hours worked and the real wage are mildly skewed but platykurtic. (Leptokurtic distributions are characterized by a sharp peak at the mode and fat tails, while platykurtic distributions are characterized by flatter peaks around the mode and thin tails.)

The model with quadratic wage adjustment costs generally predicts distributions with little or no skewness and kurtosis similar to that of the Normal distribution. In contrast, the model with asymmetric costs predicts positively skewed and leptokurtic rates of nominal interest, price inflation and wage inflation. The prediction of leptokurtic wage inflation is in agreement with microeconomic studies based on individual wage changes (see, among others, Akerlof, Dickens and Perry, 1996). Predictions regarding consumption are relatively more accurate than those of the quadratic model in that consumption is leptokurtic is negatively skewed, though in the latter case not as much as in the data. On the other hand, both models deliver rather imperfect predictions regarding hours worked and the real wage. In particular, the asymmetric cost model predicts negatively skewed hours and thick-tailed distributions for hours and real wage than in the data. These results are summarized in Figure 4 that plots the histograms for price and wage inflation in the data and for both models.

4.4 Comparison with Strict Inflation Targeting

In order to better understand the degree of optimal grease inflation under the Ramsey policy, this section computes the inflation rate that delivers the highest (unconditional) welfare when the monetary authority follows a simple rule that strictly hits the inflation target. Figure 5 plots unconditional welfare for different values of the inflation target and indicates that, given the estimated parameters, the optimal inflation target would be around 3 percent per year. This value is more than twice as large as that of the Ramsey policy. The reason is that positive inflation in a model with downward wage rigidity is driven by prudence. With limited knowledge and less flexibility with respect to shocks, the inflation targeting government needs a larger buffer above zero inflation to eschew paying the costs associated with nominal wage cuts.

5 Conclusion

This paper investigates Tobin's proposition that inflation greases the wheels of the labor market in the context of a simple but fully-specified dynamic general equilibrium model. Previous research based on linearized DSGE models did not examine this issue because, by construction, linearization eliminates the asymmetries of the underlying model. Although microeconomic research documents asymmetries in the raw wage data, the micro data itself contains elements of both economic structure and individual behavior and cannot fully reveal the mechanism through which downward wage rigidity may generate aggregate implications. Furthermore, the question is important because of the current discrepancy between theory—that prescribes zero-to-negative inflation rates—and actual practice—where central banks target low, but positive, inflation rates.

SMM estimates based on the second order approximation of model indicate that U.S. nominal wages are downwardly rigid, that optimal grease inflation is approximately 1.2 percent per year, and that downward wage rigidity has nontrivial implications for the dynamics of aggregate variables. Needless to say, the estimate of optimal grease inflation may depend on the model specification. For example, in a model with *ex-post* heterogeneity, optimal grease inflation may be larger because productivity growth (and hence real wages) would vary across agents. In contrast, in a model with technological growth, optimal grease inflation may be smaller because a positive trend growth in real wages would decrease the need for nominal wage cuts. In ongoing and future work, we study these questions in the context of a fully-fledged monetary economy, examine the role of asymmetric shocks, and derive the business cycle implications of asymmetric nominal rigidities.

Table 1. SMM Estimates

Parameter	Description	Wage Adjustment Costs: Asymmetric	Wage Adjustment Costs: Quadratic
		(1)	(2)
β	Discount rate	0.998^{*}	0.998^{*}
		(0.166)	(0.402)
ρ	Consumption curvature	1.093^{*}	0.874^{*}
		(0.108)	(0.222)
χ	Leisure curvature	3.6×10^{-6}	1.2×10^{-6}
		(0.418)	(1.095)
ϕ	Wage adjustment cost	$33.72^{\dagger}$	71.98
		(25.08)	(136.4)
γ	Price adjustment cost	35.23^{*}	7.687
		(20.08)	(7.215)
ψ	Wage asymmetry	901.40^{*}	0
		(426.2)	–
ξ	Autoregressive coefficient	0.927^{*}	0.922^{*}
		(0.018)	(0.027)
σ	Standard deviation	0.012^{*}	0.012^{*}
		(0.002)	(0.002)

Notes: The figures in parenthesis are standard errors. The superscripts $*$ and † denote the rejection of the hypothesis that the true parameter value is zero at the 5 and 10 percent significance level, respectively.

Table 2. Higher-Order Moments

Variable	U.S. Data	Wage Adjustment Costs Asymmetric	Wage Adjustment Costs Quadratic
	(1)	(2)	(3)
		A. Skewness	
Consumption	−1.049	−0.101	0.016
Hours	0.215	−1.027	−0.073
Price Inflation	1.115	0.992	0.028
Wage Inflation	0.821	1.655	−0.083
Real Wage	0.233	0.081	0.039
Nominal Interest	1.030	0.576	0.155
		B. Kurtosis	
Consumption	4.030	3.023	2.965
Hours	2.152	4.776	3.078
Price Inflation	4.872	4.640	3.119
Wage Inflation	4.515	7.826	3.153
Real Wage	1.820	2.916	2.937
Nominal Interest	4.333	3.679	3.153

Notes: The skewness and kurtosis predicted by the asymmetric and quadratic cost models were computed using 10000 simulated observations. The skewness and kurtosis of the Normal distribution are 0 and 3, respectively.

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Figure 1: Adjustment Cost Functions

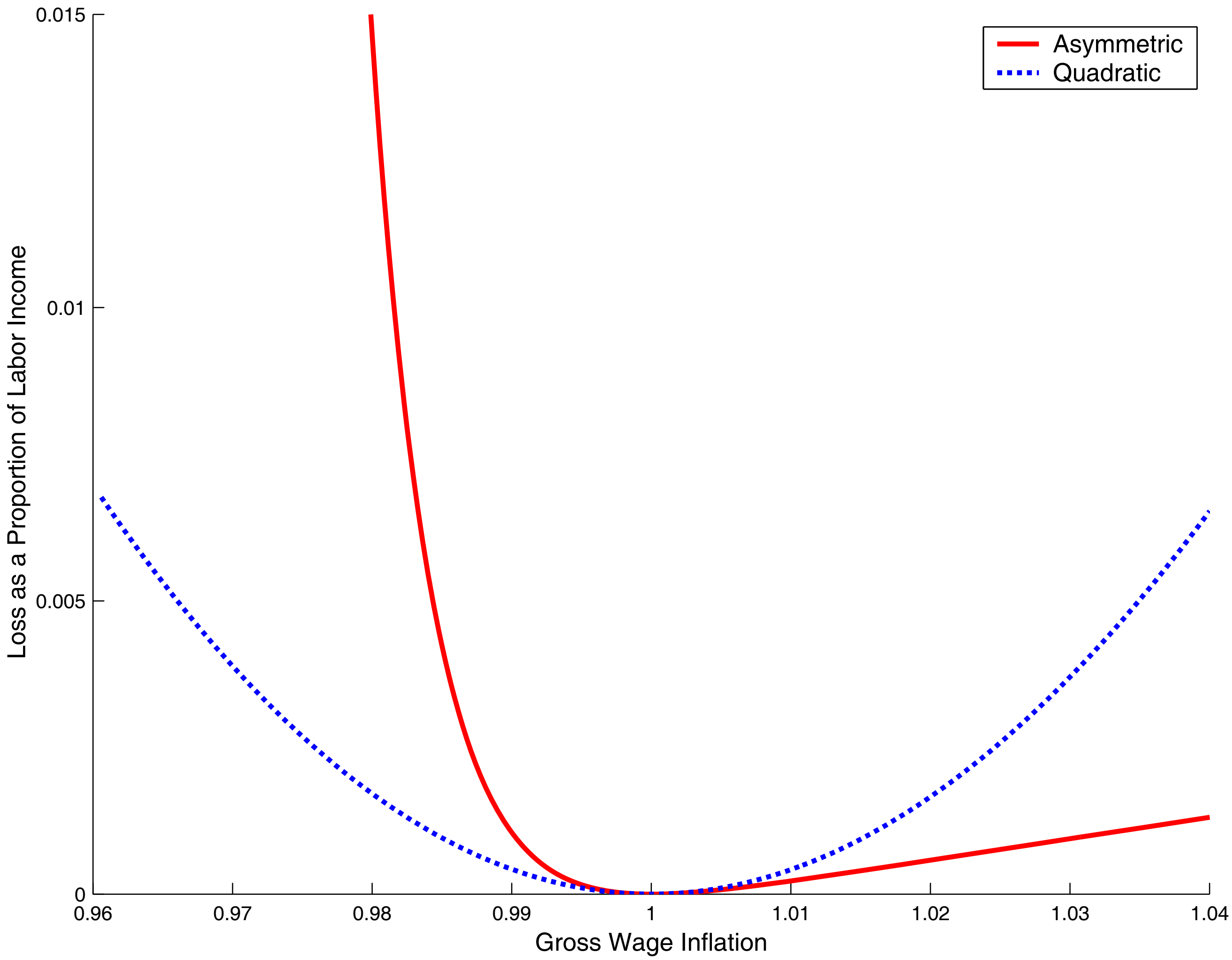

Figure 2: Responses to a Productivity Shock
Quadratic Wage Adjustment Costs

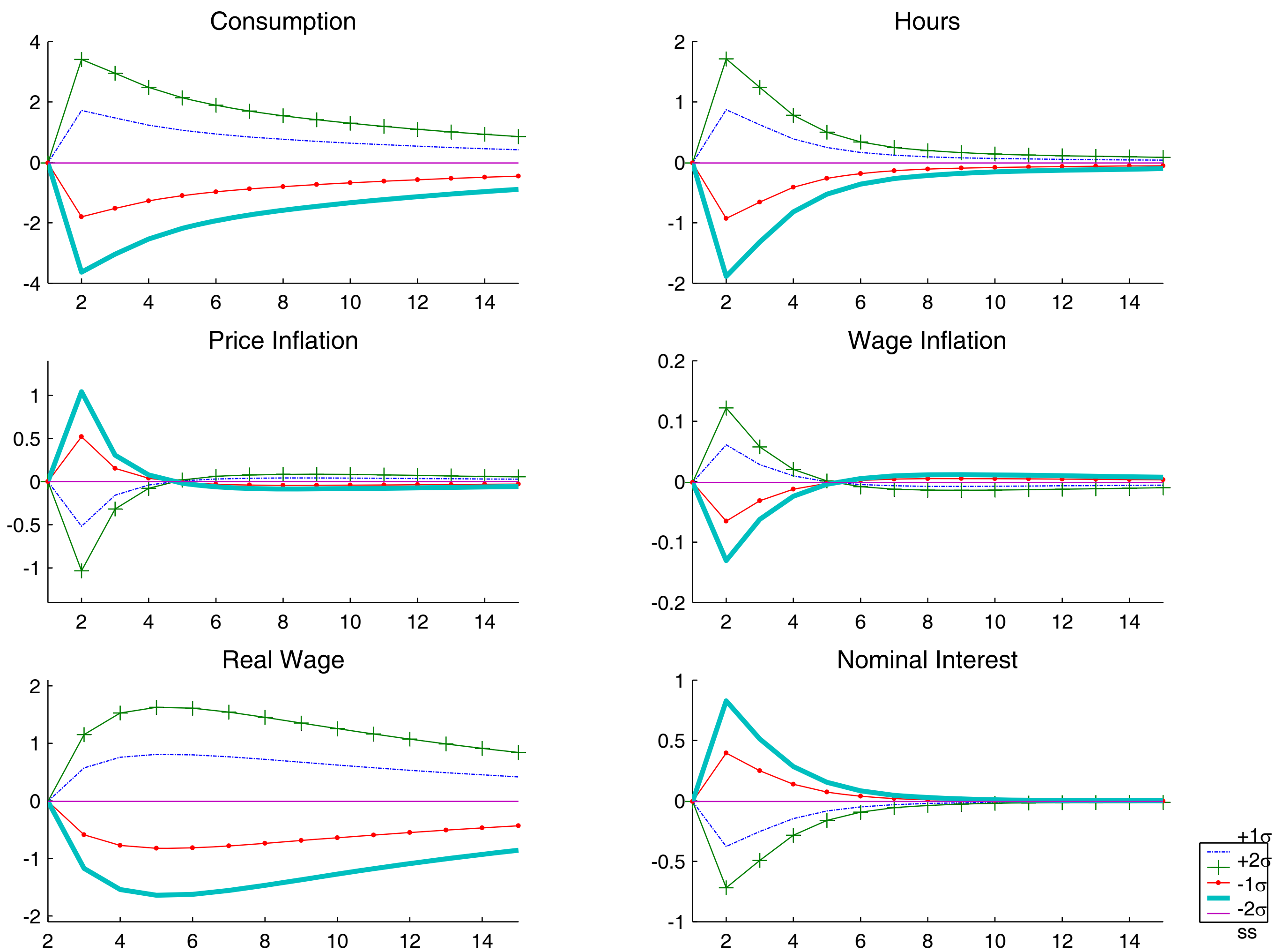

Figure 3: Responses to a Productivity Shock
Asymmetric Wage Adjustment Costs

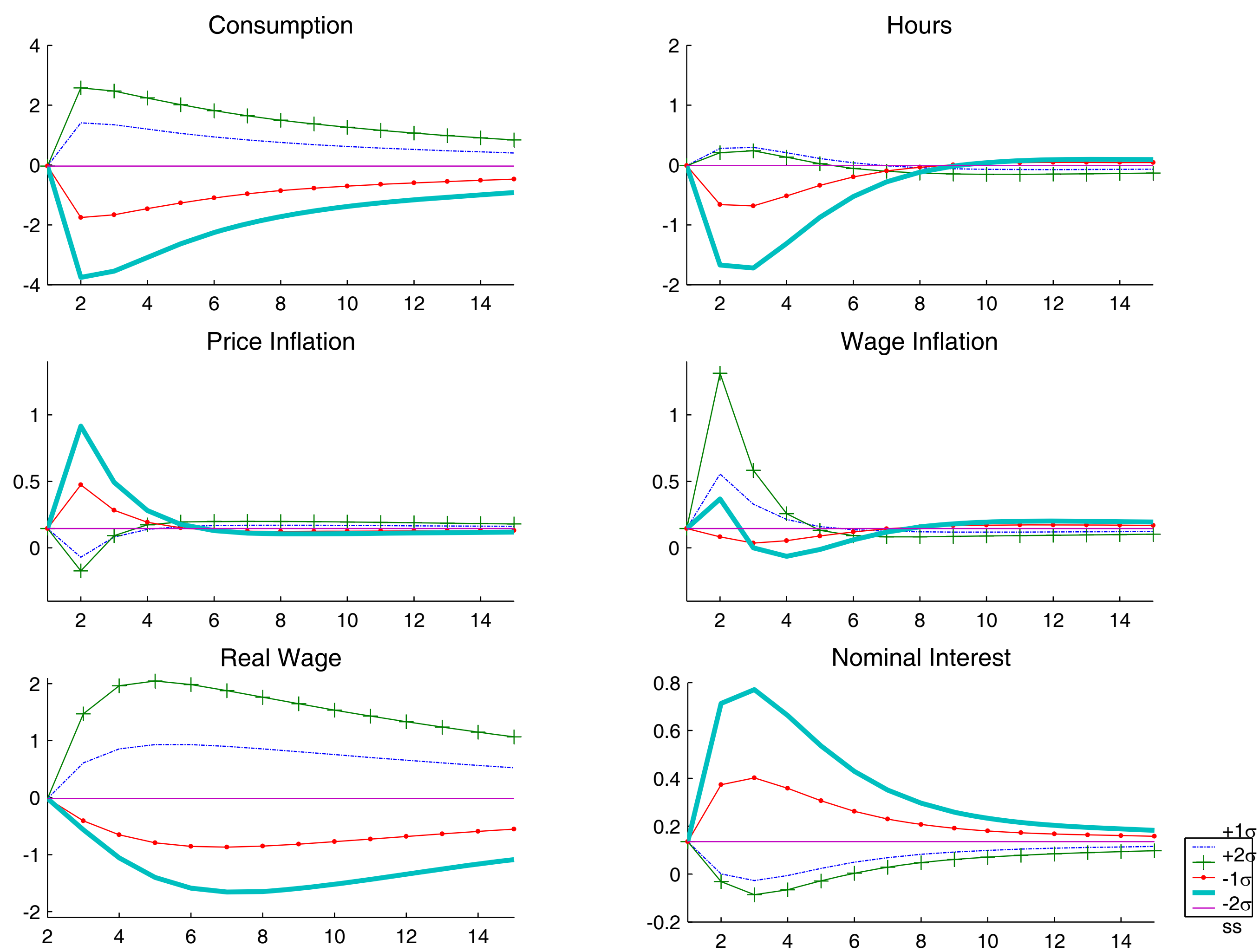

Figure 4: Frequency Histograms

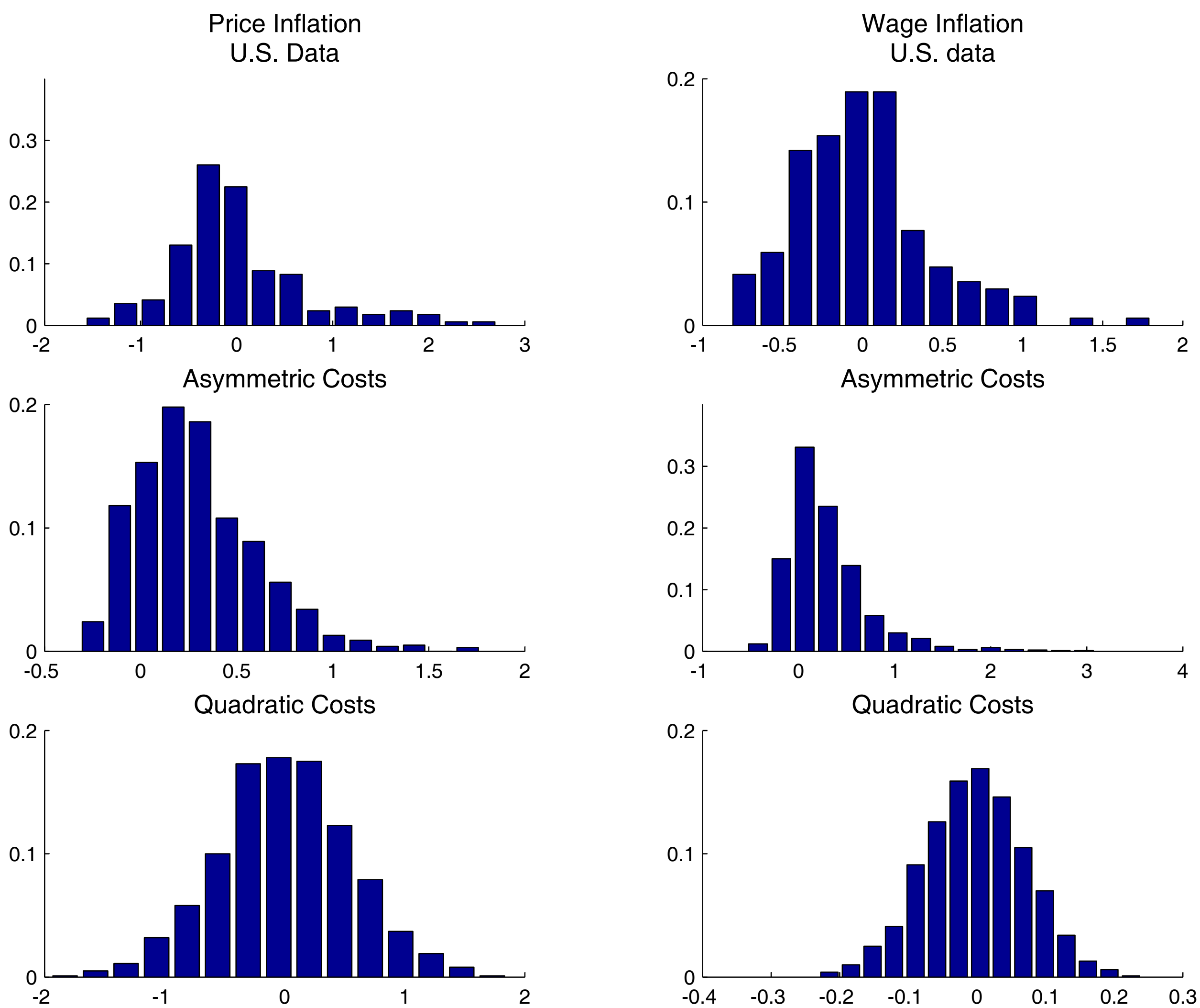

Figure 5: Welfare Under Inflation Targeting