Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model*

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Abstract

In this paper, we study Ramsey-optimal fiscal and monetary policy in a medium-scale model of the U.S. business cycle. The model features a rich array of real and nominal rigidities that have been identified in the recent empirical literature as salient in explaining observed aggregate fluctuations. The main result of the paper is that price stability appears to be a central goal of optimal monetary policy. The optimal rate of inflation under an income tax regime is half a percent per year with a volatility of 1.1 percent. This result is surprising given that the model features a number of frictions that in isolation would call for a volatile rate of inflation—particularly nonstate-contingent nominal public debt, no lump-sum taxes, and sticky wages. Under an income-tax regime, the income tax rate is quite stable, with a mean of 30 percent and a standard deviation of 1.1 percent. Simple monetary and fiscal rules are shown to implement a competitive equilibrium that mimics well the one induced by the Ramsey policy. When the fiscal authority is allowed to tax capital and labor income at different rates, optimal fiscal policy is characterized by a large and volatile subsidy on capital.
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1 Introduction

This paper addresses a classic question in macroeconomics, namely, how should a benevolent government conduct stabilization policy. A central characteristic of all existing studies is that optimal policy is derived in highly stylized environments. Typically, optimal policy is characterized for economies with a single or a very small number of deviations from the frictionless neoclassical paradigm. A case in point are the numerous recent studies concerned with optimal stabilization policy within the context of the two-equation, one-friction, neo-Keynesian model without capital accumulation.[1] Other cases in which the optimal policy design problem is studied within theoretical frameworks featuring a small number of rigidities include models with distorting income taxes (Lucas and Stokey, 1983; Schmitt-Grohé and Uribe, 2004b), and models with sticky product and factor prices (Erceg, et al., 2000). An advantage of this stylized approach is that it facilitates understanding the ways in which policy should respond to mitigate the distortionary effects of a particular friction in isolation.

An important drawback of studying optimal stabilization policy one distortion at a time is that highly simplified models are unlikely to provide a satisfactory account of cyclical movements for more than just a few macroeconomic variables of interest. For this reason, the usefulness of this strategy to produce policy advice for the real world is necessarily limited.

The approach to optimal policy that we propose in this paper departs from the literature extant in that it is based on a rich medium-scale theoretical framework capable of explaining observed business cycle fluctuations for a wide range of nominal and real variables. Following the lead of Kimball (1995), the model emphasizes the importance of combining nominal as well as real rigidities in explaining the propagation of macroeconomic shocks. Specifically, the model features four nominal frictions, sticky prices, sticky wages, a demand for money by households, and a cash-in-advance constraint on the wage bill of firms, and five sources of real rigidities, investment adjustment costs, variable capacity utilization, habit formation, imperfect competition in product and factor markets, and distortionary taxation. Aggregate fluctuations are driven by supply shocks, which take the form of stochastic variations in total factor productivity, and demand shocks stemming from exogenous innovations to the level of government purchases and the level of government transfers. Altig et al. (2004) and Christiano, Eichenbaum, and Evans (2005) argue that the model economy for which we seek to design optimal policy can indeed explain the observed responses of inflation, real wages, nominal interest rates, money growth, output, investment, consumption, labor productivity,

[1] Examples of this line of research include Ireland (1997), Rotemberg and Woodford (1997), Woodford (2003), and Clarida, Galí, and Gertler (1999), among many others.

and real profits to productivity and monetary shocks in the postwar United States. In this respect, the present paper aspires to be a step ahead in the research program of generating policy evaluation that is of relevance for the actual policy making.

The government is assumed to be benevolent in the Ramsey sense, that is, it seeks to bring about the competitive equilibrium that maximizes the lifetime utility of the representative agent and has access to a commitment technology that allows it to honor its promises. The policy instruments available to the government are assumed to be taxes on income, possibly, differentiated across different sources of income, and the short-term nominal interest rate. Public debt is assumed to be nominal and non-state contingent.

A key finding of the paper is that price stability appears to be a central goal of optimal monetary policy. The optimal rate of inflation under an income tax regime is 0.5 percent per year with a volatility of 1.1 percent. In this sense, price stickiness emerges as the single most important distortion shaping optimal policy. This result is surprising given that the model features a number of other frictions that in isolation would call for a volatile rate of inflation with a mean different from zero.

Consider first the forces calling for an optimal inflation rate that is different from zero. As is well known, the presence of a demand for money by households provides an incentive to drive inflation down to a level consistent with the Friedman rule. In this paper, we identify two additional reasons why the Ramsey planner may want to deviate from price stability. First, under an income tax regime, i.e., when all sources of income are taxed at the same rate, the Ramsey planner has an inflationary bias originating from the fact that it is less distorting to tax labor income than it is to tax capital income. With a cash-in-advance constraint on the wage bill of firms, inflation acts as a tax on labor income. Second, the Ramsey planner has an incentive to tax away transfers as they represent pure rents accruing to households. Without a direct instruments to tax transfers, the government imposes an indirect levy on this source of household income via the inflation tax.

Optimal policy calls for low inflation volatility in spite of the following two distortions that by themselves call for high inflation volatility. First, the fact that nominal government debt is non-state contingent and that regular taxes are distortionary, makes it attractive for the Ramsey planner to use unexpected variations in inflation as a lump-sum tax on nominal asset holdings. This is the reason why in flexible price environments the optimal inflation volatility is very high (Chari et al. 1995). Second, the fact that nominal wages are sticky provides an incentive for the government to set the price level so as to engineer the efficient real wage. This practice, when studied in isolation, also makes high inflation volatility optimal.

When the fiscal authority is allowed to tax capital and labor income at different rates, optimal fiscal policy is characterized by a large and volatile subsidy on capital. It is well known from the work of Judd (2002) that in the presence of imperfect competition in product markets optimal taxation calls for a subsidy on capital of a magnitude approximately equal to the markup of prices over marginal cost. However, our results suggest that the optimal capital subsidy is much larger than the one identified in the work of Judd. The reason for this discrepancy is that capital depreciation, which is ignored in the work of Judd, exacerbates the need to subsidize capital. This is because the markup distorts the gross rate of return on capital whereas the subsidy applies to the return on capital net of depreciation.

In our model, the optimal capital subsidy is extremely volatile. Its standard deviation is 150 percent. The high volatility of capital income taxes emerges for the familiar reason that capital is a fixed factor of production in the short run, so the fiscal authority uses unexpected changes in the capital income tax rate as a shock absorber for innovations in its budget. We identify two frictions capable of driving this high volatility down significantly. One is time to tax. When tax rates are determined four quarters in advance the optimal volatility of the capital income tax rate falls to about 50 percent. This is because the tax elasticity of the demand for capital increases with the number of periods between the announcement of the tax rate and its application. The second friction that is important in understanding the volatility of capital taxes is investment adjustment cost. Intuitively, the higher are the impediments to adjust the level of investment, the lower is the elasticity of capital with respect to temporary changes in tax rates. In the absence of investment adjustment costs, the optimal volatility of the capital income tax rate falls to 65 percent. Furthermore, in an environment with 4 periods of time to tax and no capital adjustment cost, the optimal capital income tax has a volatility of 25 percent.

Ramsey outcomes are mute on the issue of what policy regimes can implement them. The information on policy one can extract from the solution to the Ramsey problem is limited to the equilibrium behavior of policy variables such as tax rates, the nominal interest rate, etc. as a function of the state of the economy. Even if the policymaker could observe the state of the economy, using the equilibrium process of the policy variables to define a policy regime would not guarantee the Ramsey outcome as the competitive equilibrium. The problem is that such a policy regime could give rise to multiple equilibria. We address the issue of implantation of optimal policy by limiting attention to simple monetary and fiscal rules. These rules are defined over a small set of readily available macro indicators and are designed to ensure local uniqueness of the rational expectations equilibrium. We find parameterizations of such policy rules capable of inducing equilibrium dynamics in output, consumption, investment, and hours fairly close to those associated with the Ramsey equilibrium.

Finally, a methodological contribution of this paper is the development of a set of numerical tools that allow the computation of Ramsey policy in a general class of stochastic dynamic general equilibrium models.

2 The Model

2.1 Households

The economy is populated by a large representative family with a continuum of members. Consumption and hours worked are identical across family members. The household's preferences are defined over per capita consumption, c_t, and per capita labor effort, h_t, and are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - b c_{t-1}, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave. Preferences display internal habit formation, measured by the parameter $b \in [0,1)$. The consumption good is assumed to be a composite made of a continuum of differentiated goods c_{it} indexed by $i \in [0,1]$ via the aggregator

$$c_t = \left[\int_0^1 {c_{it}}^{1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{2}$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods.

For any given level of consumption of the composite good, purchases of each individual variety of goods $i \in [0,1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t. The demand for goods of variety i is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t, \tag{3}$$

where P_t is a nominal price index defined as

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}. \tag{4}$$

This price index has the property that the minimum cost of a bundle of intermediate goods yielding c_t units of the composite good is given by $P_t c_t$.

Labor decisions are made by a central authority within the household, a union, which supplies labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in [0,1]$.[2] In each labor market j, the union faces a demand for labor given by $\left(W_t^j/W_t\right)^{-\tilde{\eta}} h_t^d$. Here W_t^j denotes the nominal wage charged by the union in labor market j at time t, W_t is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. We postpone a formal derivation of this labor demand function until we consider the firm's problem. In each particular labor market, the union takes W_t and h_t^d as exogenous. The case in which the union takes aggregate labor variables as endogenous can be interpreted as an environment with highly centralized labor unions. Higher-level labor organizations play an important role in some European and Latin American countries, but are less prominent in the United States. Given the wage charged in each labor market $j \in [0,1]$, the union is assumed to supply enough labor, h_t^j, to satisfy demand. That is,

$$h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d, \tag{5}$$

where $w_t^j \equiv W_t^j/P_t$ and $w_t \equiv W_t/P_t$. In addition, the total number of hours allocated to the different labor markets must satisfy the resource constraint $h_t = \int_0^1 h_t^j dj$. Combining this restriction with equation (5), we obtain

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{6}$$

The household is assumed to own physical capital, k_t, which accumulates according to the following law of motion

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{7}$$

[2]This setup departs slightly from most existing expositions of models with nominal wage inertia. It avoids the need to assume separability of preferences in leisure and consumption to ensure homogeneity of consumption across households.

where i_t denotes gross investment and δ is a parameter denoting the rate of depreciation of physical capital. The function $\mathcal{S}$ introduces investment adjustment costs and is assumed to satisfy $\mathcal{S}(1) = \mathcal{S}'(1) = 0$ and $\mathcal{S}''(1) > 0$. These assumptions imply the absence of adjustment costs up to first order in the vicinity of the deterministic steady state.

Owners of physical capital can control the intensity at which this factor is utilized. Formally, we let u_t measure capacity utilization in period t. We assume that using the stock of capital with intensity u_t entails a cost of $a(u_t)k_t$ units of the composite final good. The function a is assumed to satisfy $a(1) = 0$, and $a'(1), a''(1) > 0$. Both the specification of capital adjustment costs and capacity utilization costs are somewhat peculiar. More standard formulations assume that adjustment costs depend on the level of investment rather than on its growth rate, as is assumed here. Also, costs of capacity utilization typically take the form of a higher rate of depreciation of physical capital. The modeling choice here is guided by the need to fit the response of investment and capacity utilization to a monetary shock in the US economy. For further discussion of this point, see Christiano, Eichenbaum, and Evans (2003, section 6.1) and Altig et al. (2004).

Households rent the capital stock to firms at the real rental rate r_t^k per unit of capital. Thus, total income stemming from the rental of capital is given by $r_t^k u_t k_t$. The investment good is assumed to be a composite good made with the aggregator function (2). Thus, the demand for each intermediate good $i \in [0,1]$ for investment purposes, i_{it}, is given by $i_{it} = i_t \left(P_{it}/P_t\right)^{-\eta}$.

As in earlier related work (Schmitt-Grohé and Uribe, 2004a), we motivate a demand for money by households by assuming that purchases of consumption are subject to a proportional transaction cost that is increasing in consumption based money velocity. Formally, the purchase of each unit of consumption entails a cost given by $\ell(v_t)$. Here,

$$v_t \equiv \frac{c_t}{m_t^h} \tag{8}$$

is the ratio of consumption to real money balances held by the household, which we denote by m_t^h. The transaction cost function ℓ satisfies the following assumptions: (a) $\ell(v)$ is nonnegative and twice continuously differentiable; (b) There exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $\ell(\underline{v}) = \ell'(\underline{v}) = 0$; (c) $(v - \underline{v})\ell'(v) > 0$ for $v \neq \underline{v}$; and (d) $2\ell'(v) + v\ell''(v) > 0$ for all $v \geq \underline{v}$. Assumption (b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption (c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level. As will become clear shortly, assumption (d) ensures that the demand for money is decreasing in the nominal interest rate. (Note that assumption (d) is weaker than the more common assumption of strict convexity of the transaction cost function.)

Households are assumed to have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment X^h_{t+1} in period $t+1$ at the dollar cost $E_t r_{t,t+1} X^h_{t+1}$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t+1$. Households must pay taxes on labor income, capital income, and profits. We denote by τ^h_t, τ^k_t, and τ^ϕ_t, respectively, the labor income tax rate, the capital income tax rate, and the profit tax rate in period t A tax allowance is assumed to apply to costs due to depreciation. Households receive real lump-sum transfers from the government in the amount n_t per period. The household's period-by-period budget constraint is given by:

$$\begin{aligned} E_t r_{t,t+1} x^h_{t+1} + c_t[1 + \ell(v_t)] + i_t + m^h_t &= (1-\tau^k_t)[r^k_t u_t - a(u_t)]k_t + (1-\tau^h_t) h^d_t \int_0^1 w^j_t \left(\frac{w^j_t}{w_t}\right)^{-\tilde{\eta}} \\ &\quad + (1-\tau^\phi_t)\phi_t + \tau^k_t q_t \delta k_t + n_t + \frac{x^h_t + m^h_{t-1}}{\pi_t}. \end{aligned} \tag{9}$$

The variable $x^h_t/\pi_t \equiv X^h_t/P_t$ denotes the real payoff in period t of nominal state-contingent assets purchased in period $t-1$. The variable ϕ_t denotes dividends received from the ownership of firms, q_t denotes the price of capital in terms of consumption, and $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of consumer-price inflation.

We introduce wage stickiness in the model by assuming that each period the household (or union) cannot set the nominal wage optimally in a fraction $\tilde{\alpha} \in [0,1)$ of randomly chosen labor markets. In these markets, the wage rate is indexed to the previous period's consumer-price inflation according to the rule $W^j_t = W^j_{t-1}\pi^{\tilde{\chi}}_{t-1}$, where $\tilde{\chi}$ is a parameter measuring the degree of wage indexation. When $\tilde{\chi}$ equals 0, there is no wage indexation. When $\tilde{\chi}$ equals 1, there is full wage indexation to past consumer price inflation. In general, $\tilde{\chi}$ can take any value between 0 and 1.

The household chooses processes for c_t, h_t, x^h_{t+1}, w^j_t, k_{t+1}, i_t, u_t, and m^h_t so as to maximize the utility function (1) subject to (6)-(9), the wage stickiness friction, and a no-Ponzi-game constraint, taking as given the processes w_t, r^k_t, h^d_t, $r_{t,t+1}$, q_t, π_t, ϕ_t, τ^h_t, τ^k_t, and τ^ϕ_t and the initial conditions x^h_0, k_0, and m^h_{-1}. Of course, the household's optimal plan must satisfy constraints (6)-(9). In addition, letting $\beta^t \lambda_t w_t(1-\tau^h_t)/\tilde{\mu}_t$, $\beta^t \tilde{q}_t \lambda_t$, and $\beta^t \lambda_t$ denote Lagrange multipliers associated with constraints (6), (7), and (9), respectively, the

Lagrangian associated with the household's optimization problem is

$$\begin{aligned}
\mathcal{L} \;=\; & E_t \sum_{j=0}^{\infty} \beta^j \left\{ U(c_{t+j} - bc_{t+j-1}, h_{t+j}) \right. \\
& + \lambda_{t+j} \left[(1-\tau^h_{t+j}) h^d_{t+j} \int_0^1 w^i_{t+j} \left(\frac{w^i_{t+j}}{w_{t+j}} \right)^{-\tilde{\eta}} di + (1-\tau^k_{t+j})[r^k_{t+j} u_{t+j} - a(u_{t+j})] k_{t+j} + (1-\tau^\phi_{t+j})\phi_{t+j} \right. \\
& \left. - c_{t+j} \left[1 + \ell \left(\frac{c_{t+j}}{m^h_{t+j}} \right) \right] - i_{t+j} + \tau^k_{t+j} q_{t+j} \delta k_{t+j} - r_{t+j,t+j+1} x^h_{t+j+1} - m^h_{t+j} + \frac{m^h_{t+j-1} + x^h_{t+j}}{\pi_{t+j}} \right] \\
& + \frac{\lambda_{t+j}(1-\tau^h_{t+j}) w_{t+j}}{\tilde{\mu}_{t+j}} \left[h_{t+j} - h^d_{t+j} \int_0^1 \left(\frac{w^i_{t+j}}{w_{t+j}} \right)^{-\tilde{\eta}} di \right] \\
& \left. + \lambda_{t+j} \tilde{q}_{t+j} \left[(1-\delta) k_{t+j} + i_{t+j} \left[1 - \mathcal{S}\left(\frac{i_{t+j}}{i_{t+j-1}} \right) \right] - k_{t+j+1} \right] \right\}.
\end{aligned}$$

The first-order conditions with respect to c_t, x^h_{t+1}, h_t, k_{t+1}, i_t, m^h_t, u_t, and w^j_t, in that order, are given by

$$U_c(c_t - bc_{t-1}, h_t) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}) = \lambda_t [1 + \ell(v_t) + v_t \ell'(v_t)], \tag{10}$$

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{11}$$

$$-U_h(c_t - bc_{t-1}, h_t) = \frac{\lambda_t (1-\tau^h_t) w_t}{\tilde{\mu}_t}, \tag{12}$$

$$\lambda_t \tilde{q}_t = \beta E_t \lambda_{t+1} \left[(1-\tau^k_{t+1})[r^k_{t+1} u_{t+1} - a(u_{t+1})] + \tilde{q}_{t+1}(1-\delta) + \delta q_{t+1} \tau^k_{t+1} \right], \tag{13}$$

$$\lambda_t = \lambda_t \tilde{q}_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}} \right) - \left(\frac{i_t}{i_{t-1}} \right) \mathcal{S}'\left(\frac{i_t}{i_{t-1}} \right) \right] + \beta E_t \lambda_{t+1} \tilde{q}_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t} \right) \tag{14}$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}. \tag{15}$$

$$r^k_t = a'(u_t) \tag{16}$$

$$w^i_t = \begin{cases} \tilde{w}_t & \text{if } w^i_t \text{ is set optimally in } t \\ w^i_{t-1} \pi^{\tilde{\chi}}_{t-1} / \pi_t & \text{otherwise} \end{cases}$$

where $\tilde{w}_t$ denotes the real wage prevailing in the $1-\tilde{\alpha}$ labor markets in which the union can set wages optimally in period t. Let $\tilde{h}_t$ denote the level of employment supplied to those markets. Note that because the labor demand curve faced by the union is identical across all labor markets, and because the cost of supplying labor is the same for all markets, one can assume that wage rates, $\tilde{w}_t$, and employment, $\tilde{h}_t$, will be identical across all labor markets updating wages in a given period. By equation (5), we have that $\tilde{w}_t^{\tilde{\eta}}\tilde{h}_t = w^{\tilde{\eta}} h_t^d$. It is of use to track the evolution of real wages in a particular labor market. In any labor market j where the wage is set optimally in period t, the real wage in that period is $\tilde{w}_t$. If in period $t+1$ wages are not reoptimized in that market, the real wage is $\tilde{w}_t \pi_t^{\tilde{\chi}}/\pi_{t+1}$. This is because the nominal wage is indexed by $\tilde{\chi}$ percent of past price inflation. In general, s period after the last reoptimization, the real wage is $\tilde{w}_t \prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\tilde{\chi}}}{\pi_{t+k}}\right)$. To derive the household's first-order condition with respect to the wage rate in those markets where the wage rate is set optimally in the current period, it is convenient to reproduce the parts of the Lagrangian given above that are relevant for this purpose,

$$\mathcal{L}^w = E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s}(1-\tau_{t+s}^h) h_{t+s}^d w_{t+s}^{\tilde{\eta}} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\tilde{w}_t^{1-\tilde{\eta}} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{-1} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \tilde{w}_t^{-\tilde{\eta}}\right].$$

The first-order condition with respect to $\tilde{w}_t$ is:

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} w_{t+s}^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{(1-\tau_{t+s}^h)\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}(1-\tau_{t+s}^h)}{\tilde{\mu}_{t+s}}\right]$$

Using equation (12) to eliminate $\tilde{\mu}_{t+s}$, we obtain that the real wage $\tilde{w}_t$ must satisfy

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{(1-\tau_{t+s}^h)\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{-U_{ht+s}}{\lambda_{t+s}}\right]$$

This expression states that in labor markets in which the wage rate is reoptimized in period t, the real wage is set so as to equate the union's future expected average after-tax marginal revenue to the average marginal cost of supplying labor. The union's after-tax marginal revenue s periods after its last wage reoptimization is given by $\frac{\tilde{\eta}-1}{\tilde{\eta}}(1-\tau_{t+s}^h)\tilde{w}_t \prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\tilde{\chi}}}{\pi_{t+k}}\right)$. Here, $\tilde{\eta}/(\tilde{\eta}-1)$ represents the markup of after-tax wages over marginal cost of labor that would prevail in the absence of wage stickiness. The factor $\prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\tilde{\chi}}}{\pi_{t+k}}\right)$ in the expression for marginal revenue reflects the fact that as time goes by without a chance to reoptimize, the real wage declines as the price level increases when wages are imperfectly indexed. In turn, the marginal cost of supplying labor is given by the marginal rate of substitution between consumption and leisure, or $\frac{-U_{ht+s}}{\lambda_{t+s}} = \frac{w_{t+s}(1-\tau^h_{t+s})}{\tilde{\mu}_{t+s}}$. The variable $\tilde{\mu}_t$ is a wedge between the disutility of labor and the average after-tax real wage prevailing in the economy. Thus, $\tilde{\mu}_t$ can be interpreted as the average markup that unions impose on the labor market. The weights used to compute the average difference between marginal revenue and marginal cost are decreasing in time and increasing in the amount of labor supplied to the market.

We wish to write the wage-setting equation in recursive form. To this end, define

$$f^1_t = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right) \tilde{w}_t E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s}(1-\tau^h_{t+s}) \left(\frac{w_{t+s}}{\tilde{w}_t}\right)^{\tilde{\eta}} h^d_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi^{\tilde{\chi}}_{t+k-1}}\right)^{\tilde{\eta}-1}$$

and

$$f^2_t = -\tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s w^{\tilde{\eta}}_{t+s} h^d_{t+s} U_{ht+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi^{\tilde{\chi}}_{t+k-1}}\right)^{\tilde{\eta}}.$$

One can express f^1_t and f^2_t recursively as

$$f^1_t = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right) \tilde{w}_t \lambda_t (1-\tau^h_t) \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h^d_t + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{\pi^{\tilde{\chi}}_t}\right)^{\tilde{\eta}-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f^1_{t+1}, \tag{17}$$

$$f^2_t = -U_{ht} \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h^d_t + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{\pi^{\tilde{\chi}}_t}\right)^{\tilde{\eta}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f^2_{t+1}. \tag{18}$$

With these definitions at hand, the wage-setting equation becomes

$$f^1_t = f^2_t. \tag{19}$$

The household's optimality conditions imply a liquidity preference function featuring a negative relation between real balances and the short-term nominal interest rate. To see this, we first note that the absence of arbitrage opportunities in financial markets requires that the gross risk-free nominal interest rate, which we denote by R_t, be equal to the reciprocal of the price in period t of a nominal security that pays one unit of currency in every state of period $t+1$. Formally, $R_t = 1/E_t r_{t,t+1}$. This relation together with the household's optimality condition (11) implies that

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{20}$$

which is a standard Euler equation for pricing nominally risk-free assets. Combining this expression with equations (10) and (15), we obtain

$$v_t^2 \ell'(v_t) = 1 - \frac{1}{R_t}.$$

The right-hand side of this expression represents the opportunity cost of holding money, which is an increasing function of the nominal interest rate. Given the assumptions regarding the form of the transactions cost function ℓ, the left-hand side is increasing in money velocity. Thus, this expression defines a liquidity preference function that is decreasing in the nominal interest rate and unit elastic in consumption.

2.2 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using as factor inputs capital services, k_{it}, and labor services, h_{it}. The production technology is given by

$$z_t F(k_{it}, h_{it}) - \psi,$$

where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments. The variable z_t denotes an aggregate, exogenous, and stochastic productivity shock whose law of motion is given by

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{21}$$

where $\rho_z \in (-1, 1)$, and ϵ_t^z is an i.i.d. innovation with mean zero, standard deviation σ_{ϵ^z}, and bounded support. The parameter $\psi > 0$ introduces fixed costs of operating a firm in each period. It implies that the production function exhibits increasing returns to scale. We model fixed costs to ensure a realistic profit-to-output ratio in steady state.

Aggregate demand for good i, which we denote by y_{it}, is given by

$$y_{it} = (P_{it}/P_t)^{-\eta} y_t,$$

where

$$y_t \equiv c_t[1 + \ell(v_t)] + i_t + g_t + a(u_t)k_t, \tag{22}$$

denotes aggregate absorption. The variable g_t denotes government consumption of the composite good in period t.

We rationalize a demand for money by firms by imposing that wage payments be subject to a cash-in-advance constraint of the form:

$$m_{it}^f \geq \nu w_t h_{it}, \tag{23}$$

where m_{it}^f denotes the demand for real money balances by firm i in period t and $\nu \geq 0$ is a parameter indicating the fraction of the wage bill that must be backed with monetary assets.

Firms must pay capital income taxes on nondistributed operational profits, $\left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{it} - w_t h_{it} - \phi_{it} - (1 - R_t^{-1}) m_{it}^f$. Here, the variable ϕ_{it} denotes real dividend payments that firm i makes to households in period t. To avoid double taxation, distributed dividends are tax exempt at the firm level. Note that the firm incurs in financial costs in the amount $(1 - R_t^{-1}) m_{it}^f$ stemming from the need to hold money to satisfy the working-capital constraint. The period-by-period budget constraint of firm i can then be written as

$$E_t r_{t,t+1} x_{it+1}^f + m_{it}^f - \frac{x_{it}^f + m_{it-1}^f}{\pi_t} = (1 - \tau_t^k) \left[\left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{it} - w_t h_{it} - \phi_{it} \right] + \tau_t^k (1 - R_t^{-1}) m_{it}^f,$$

where $E_t r_{t,t+1} x_{it+1}^f$ denotes the total real cost of one-period state-contingent assets that the firm purchases in period t in terms of the composite good.[3] We assume that the firm must satisfy demand at the posted price. Formally, we impose

$$z_t F(k_{it}, h_{it}) - \psi \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t. \tag{24}$$

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, k_{it}, x_{it+1}^f, and m_{it}^f so as to maximize the present discounted value of after tax dividend payments, given by

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} (1 - \tau_{t+s}^k) \phi_{it+s},$$

where $r_{t,t+s} \equiv \prod_{k=1}^{s} r_{t+k-1,t+k}$, for $s \geq 1$, denotes the stochastic nominal discount factor between t and $t+s$, and $r_{t,t} \equiv 1$. Firms are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games.

Clearly, because $r_{t,t+s}$ represents both the firm's stochastic discount factor and the market pricing kernel for financial assets, and because the firm's objective function is linear in asset

[3]Implicit in this specification of the firm's budget constraint is the assumption that firms rent capital services from a centralized market. This is a common assumption in the related literature (e.g., Christiano et al., 2003; Kollmann, 2003; Carlstrom and Fuerst, 2003; and Rotemberg and Woodford, 1992). A polar assumption is that capital is sector specific, as in Woodford (2003, chapter 5.3) and Sveen and Weinke (2003). Both assumptions are clearly extreme. A more realistic treatment of investment dynamics would incorporate a mix of firm-specific and homogeneous capital.

holdings, it follows that any asset accumulation plan of the firm satisfying the no-Ponzi constraint is optimal. Suppose, without loss of generality, that the firm manages its portfolio so as to ensure that its financial position at the beginning of each period is nil. Formally, assume that $x^f_{it+1} + m^f_{it} = 0$ at all dates and states. Note that this financial strategy makes x^f_{it+1} state-noncontingent. In this case, distributed dividends take the form

$$\phi_{it} = \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r^k_t k_{it} - w_t h_{it} - (1 - R_t^{-1}) m^f_{it}. \tag{25}$$

For this expression to hold in period zero, we impose the initial condition $x^f_{i0} + m^f_{i-1} = 0$. The last term on the right-hand side of this expression represents the firm's financial costs associated with the cash-in-advance constraint on wages. This financial cost is increasing in the opportunity cost of holding money, $1 - R_t^{-1}$, which is an increasing function of the short-term nominal interest rate R_t.

Throughout our analysis, we will focus on equilibria featuring a strictly positive nominal interest rate. This implies that the cash-in-advance constraint (23) will always bind. Then, letting $r_{t,t+s} P_{t+s} \text{mc}_{it+s}$ be the Lagrange multiplier associated with constraint (24), the first-order conditions of the firm's maximization problem with respect to capital and labor services are, respectively,

$$\text{mc}_{it} z_t F_h(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{26}$$

and

$$\text{mc}_{it} z_t F_k(k_{it}, h_{it}) = r^k_t. \tag{27}$$

It is clear from these optimality conditions that the presence of a working-capital requirement introduces a financial cost of labor that is increasing in the nominal interest rate. We note also that because all firms face the same factor prices and because they all have access to the same production technology with the function F being linearly homogeneous, marginal costs, mc_{it}, are identical across firms. Indeed, because the above first-order conditions hold for all firms independently of whether they are allowed to reset prices optimally or not, marginal costs are identical across all firms in the economy.

Prices are assumed to be sticky à la Calvo (1983) and Yun (1996). Specifically, each period $t \geq 0$ a fraction $\alpha \in [0, 1)$ of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. Instead, these firms index their prices to past inflation according to the rule $P_{it} = P_{it-1} \pi^{\chi}_{t-1}$. The interpretation of the parameter χ is the same as that of its wage counterpart $\tilde{\chi}$. The remaining $1 - \alpha$ firms choose prices optimally. Consider the price-setting problem faced by a firm that gets to reoptimize the price in period

t. This price, which we denote by $\tilde{P}_t$, is set so as to maximize the expected present discounted value of profits. That is, $\tilde{P}_t$ maximizes the following Lagrangian:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left\{ \left(\frac{\tilde{P}_t}{P_t}\right)^{1-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{1-\eta} y_{t+s} - r_{t+s}^k k_{it+s} - w_{t+s} h_{it+s}[1+\nu(1-R_{t+s}^{-1})] \right.$$
$$\left. +\text{mc}_{it+s}\left[z_{t+s} F(k_{it+s}, h_{it+s}) - \psi - \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \right] \right\}.$$

The first-order condition with respect to $\tilde{P}_t$ is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \left[\frac{\eta-1}{\eta} \left(\frac{\tilde{P}_t}{P_t}\right) \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{it+s} \right] = 0. \tag{28}$$

According to this expression, optimizing firms set nominal prices so as to equate average future expected marginal revenues to average future expected marginal costs. The weights used in calculating these averages are decreasing with time and increasing in the size of the demand for the good produce by the firm. Under flexible prices ($\alpha = 0$), the above optimality condition reduces to a static relation equating marginal costs to marginal revenues period by period.

It will prove useful to express this first-order condition recursively. To that end, let

$$x_t^1 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} mc_{it+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta-1} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{(1+\eta)/\eta}}\right)^{-\eta}$$

and

$$x_t^2 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{\eta/(\eta-1)}}\right)^{1-\eta}.$$

Express x_t^1 and x_t^2 recursively as

$$x_t^1 = y_t \text{mc}_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} (\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta} x_{t+1}^1, \tag{29}$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2. \tag{30}$$

Then we can write the first-order condition with respect to $\tilde{P}_t$ as

$$\eta x_t^1 = (\eta - 1)x_t^2. \tag{31}$$

The labor input used by firm $i \in [0,1]$, denoted h_{it}, is assumed to be a composite made of a continuum of differentiated labor services, h_{it}^j indexed by $j \in [0,1]$. Formally,

$$h_{it} = \left[\int_0^1 h_{it}^{j\;1-1/\tilde{\eta}} dj\right]^{1/(1-1/\tilde{\eta})}, \tag{32}$$

where the parameter $\tilde{\eta} > 1$ denotes the intratemporal elasticity of substitution across different types of activities. For any given level of h_{it}, the demand for each variety of labor $j \in [0,1]$ in period t must solve the dual problem of minimizing total labor cost, $\int_0^1 W_t^j h_{it}^j dj$, subject to the aggregation constraint (32), where W_t^j denotes the nominal wage rate paid to labor of variety j at time t. The optimal demand for labor of type j is then given by

$$h_{it}^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_{it}, \tag{33}$$

where W_t is a nominal wage index given by

$$W_t \equiv \left[\int_0^1 W_t^{j\,1-\tilde{\eta}} dj\right]^{\frac{1}{1-\tilde{\eta}}}. \tag{34}$$

This wage index has the property that the minimum cost of a bundle of intermediate labor inputs yielding h_{it} units of the composite labor is given by $W_t h_{it}$.

2.3 The Government

Each period, the government consumes g_t units of the composite good. We assume that the variable g_t is exogenous and that its logarithm follows a first-order autoregressive process of the form

$$\ln(g_t/\bar{g}) = \rho_g \ln(g_{t-1}/\bar{g}) + \epsilon_t^g, \tag{35}$$

where $\rho_g \in (-1,1)$ and $\bar{g} > 0$ are parameters, and ϵ_t^g is an i.i.d. innovation with mean zero, standard deviation σ_{ϵ^g}, and bounded support. The parameter $\bar{g}$ represents the nonstochastic steady-state level of government absorption. We assume that the government minimizes the cost of producing g_t. As a result, public demand for each variety $i \in [0,1]$ of differentiated goods g_{it} is given by $g_{it} = (P_{it}/P_t)^{-\eta} g_t$. A second source of government expenditures are transfer payments to households in the amount n_t, measured in units of the composite good. Like government consumption, transfers are assumed to be exogenous and to follow the law of motion

$$\ln(n_t/\bar{n}) = \rho_n \ln(n_{t-1}/\bar{n}) + \epsilon_t^n, \tag{36}$$

where $\rho_n \in (-1, 1)$ and $\bar{n} > 0$ are parameters, and ϵ_t^n is an i.i.d. innovation with mean zero, standard deviation σ_{ϵ^n}, and bounded support. The parameter $\bar{n}$ represents the nonstochastic steady-state level of government transfers.

The government levies labor, capital, and profit income taxes, allowing for tax exemptions for the cost of depreciation. Total tax revenues are given by $\tau_t \equiv \tau_t^k[r_t^k u_t - a(u_t) - q_t\delta]k_t + \tau_t^h h_t^d \int_0^1 w_t^j (w_t^j/w_t)^{-\tilde{\eta}} dj + \tau_t^\phi \phi_t$. The government issues money given in real terms by $m_t \equiv m_t^h + \int_0^1 m_{it}^f di$. The fiscal authority covers deficits by issuing one-period, nominally risk-free bonds, B_t. The period-by-period budget constraint of the consolidated government is then given by $b_t - (R_{t-1}/\pi_t)b_{t-1} + m_t - m_{t-1}/\pi_t = g_t + n_t - \tau_t$. Letting $a_t \equiv R_t b_t + m_t$, we can write the government's budget constraint as

$$\frac{a_t}{R_t} + m_t(1 - R_t^{-1}) + \tau_t = \frac{a_{t-1}}{\pi_t} + g_t + n_t. \tag{37}$$

We postpone the presentation of the monetary and fiscal policy regime.

2.4 Aggregation

We limit attention to a symmetric equilibrium in which all firms that get to change their price optimally at a given time indeed choose the same price. It then follows from (4) that the aggregate price index can be written as $P_t^{1-\eta} = \alpha(P_{t-1}\pi_{t-1}^\chi)^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}$. Dividing this expression through by $P_t^{1-\eta}$ one obtains

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}. \tag{38}$$

In equilibrium, the shadow value of capital, $\tilde{q}_t$, must equal the market value of capital, q_t,

$$q_t = \tilde{q}_t. \tag{39}$$

2.4.1 Market Clearing in the Final Goods Market

Naturally, the set of equilibrium conditions includes a resource constraint. Such a restriction is typically of the type $z_t F(k_t, h_t) - \psi = c_t[1 + \ell(v_t)] + i_t + g_t + a(u_t)k_t$. In the present model, however, this restriction is not valid. This is because the model implies relative price dispersion across varieties. This price dispersion, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. To see this, consider the following expression stating that supply must equal demand at the firm level:

$$z_t F(k_{it}, h_{it}) - \psi = [c_t + \ell(v_t)c_t + i_t + g_t + a(u_t)k_t] \left(\frac{P_{it}}{P_t}\right)^{-\eta}.$$

Integrating over all firms and taking into account that (a) the capital-labor ratio is common across firms, (b) that the aggregate demand for the composite labor input, h_t^d, satisfies

$$h_t^d = \int_0^1 h_{it} di,$$

and that (c) the aggregate effective level of capital, $u_t k_t$ satisfies

$$u_t k_t = \int_0^1 k_{it} di,$$

we obtain

$$h_t^d z_t F\left(\frac{u_t k_t}{h_t^d}, 1\right) - \psi = [c_t + \ell(v_t)c_t + i_t + g_t + a(u_t)k_t] \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di.$$

Let $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$. Then we have

$$\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \\
&= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} + (1-\alpha)\alpha\left(\frac{\tilde{P}_{t-1}\pi_{t-1}^{\chi}}{P_t}\right)^{-\eta} + (1-\alpha)\alpha^2\left(\frac{\tilde{P}_{t-2}\pi_{t-1}^{\chi}\pi_{t-2}^{\chi}}{P_t}\right)^{-\eta} + \ldots \\
&= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\tilde{P}_{t-j}\prod_{s=1}^{j}\pi_{t-j-1+s}^{\chi}}{P_t}\right)^{-\eta} \\
&= (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}.
\end{aligned}$$

Summarizing, the resource constraint in the present model is given by the following two expressions

$$z_t F(u_t k_t, h_t^d) - \psi = [c_t + \ell(v_t)c_t + i_t + g_t + a(u_t)k_t] s_t \tag{40}$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\chi}}\right)^{\eta} s_{t-1}, \tag{41}$$

with s_{-1} given. The state variable s_t summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model in equilibrium. Three observations are in order about the price dispersion measure s_t. First, s_t is bounded below by 1. That is, price dispersion is always a costly distortion in this model. To see that s_t is bounded below by 1, let $v_{it} \equiv (P_{it}/P_t)^{1-\eta}$. It follows from the definition of the price index given in equation (4) that $\left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} = 1$. Also, by definition we have $s_t = \int_0^1 v_{it}^{\eta/(\eta-1)}$. Then, taking into account that $\eta/(\eta-1) > 1$, Jensen's inequality implies that $1 = \left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} \leq \int_0^1 v_{it}^{\eta/(\eta-1)} = s_t$. Second, in an economy where the non-stochastic level of inflation is nil (i.e., when $\pi = 1$) or where prices are fully indexed to any variable ω_t with the property that its deterministic steady-state value equals the deterministic steady-state value of inflation (i.e., $\omega = \pi$), the variable s_t follows, up to first order, the univariate autoregressive process $\hat{s}_t = \alpha \hat{s}_{t-1}$. In these cases, the price dispersion measure s_t has no first-order real consequences for the stationary distribution of any endogenous variable of the model. This means that studies that restrict attention to linear approximations to the equilibrium conditions are justified to ignore the variable s_t if the model features no price dispersion in the deterministic steady state. But s_t matters up to first order when the deterministic steady state features movements in relative prices across goods varieties. More importantly, the price dispersion variable s_t must be taken into account if one is interested in higher-order approximations to the equilibrium conditions even if relative prices are stable in the deterministic steady state. Omitting s_t in higher-order expansions would amount to leaving out certain higher-order terms while including others. Finally, when prices are fully flexible, $\alpha = 0$, we have that $\tilde{p}_t = 1$ and thus $s_t = 1$. (Obviously, in a flexible-price equilibrium there is no price dispersion across varieties.)

As discussed above, equilibrium marginal costs and capital-labor ratios are identical across firms. Therefore, one can aggregate the firm's optimality conditions with respect to labor and capital, equations (26) and (27), as

$$\mathrm{mc}_t z_t F_h(u_t k_t, h_t^d) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{42}$$

and

$$\mathrm{mc}_t z_t F_k(u_t k_t, h_t^d) = r_t^k. \tag{43}$$

2.4.2 Market Clearing in the Labor Market

It follows from equation (33) that the aggregate demand for labor of type $j \in [0,1]$, which we denote by $h_t^j \equiv \int_0^1 h_{it}^j di$, is given by

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d, \tag{44}$$

where $h_t^d \equiv \int_0^1 h_{it} di$ denotes the aggregate demand for the composite labor input. Taking into account that at any point in time the nominal wage rate is identical across all labor markets at which wages are allowed to change optimally, we have that labor demand in each of those markets is

$$\tilde{h}_t = \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d.$$

Combining this expression with equation (44), describing the demand for labor of type $j \in [0,1]$, and with the time constraint (6), which must hold with equality, we can write

$$h_t = (1-\tilde{\alpha}) h_t^d \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s}\prod_{k=1}^{s}\pi_{t+k-s-1}^{\tilde{\chi}}}{W_t}\right)^{-\tilde{\eta}}$$

Let $\tilde{s}_t \equiv (1-\tilde{\alpha})\sum_{s=0}^{\infty}\tilde{\alpha}^s\left(\frac{\tilde{W}_{t-s}\prod_{k=1}^{s}\pi_{t+k-s-1}^{\tilde{\chi}}}{W_t}\right)^{-\tilde{\eta}}$. The variable $\tilde{s}_t$ measures the degree of wage dispersion across different types of labor. The above expression can be written as

$$h_t = \tilde{s}_t h_t^d. \tag{45}$$

The state variable $\tilde{s}_t$ evolves over time according to

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}}\left(\frac{\pi_t}{\pi_{t-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}}\tilde{s}_{t-1}. \tag{46}$$

We note that because all job varieties are ex-ante identical, any wage dispersion is inefficient. This is reflected in the fact that $\tilde{s}_t$ is bounded below by 1. The proof of this statement is identical to that offered earlier for s_t. To see this, note that $\tilde{s}_t$ can be written as $\tilde{s}_t = \int_0^1 \left(\frac{W_{it}}{W_t}\right)^{-\tilde{\eta}} di$. This inefficiency introduces a wedge that makes the number of hours supplied to the market, h_t, larger than the number of productive units of labor input, h_t^d. In an environment without long-run wage dispersion, the dead-weight loss created by wage dispersion is nil up to first order. Formally, a first-order approximation of the law of motion of $\tilde{s}_t$ yields a univariate autoregressive process of the form $\hat{\tilde{s}}_t = \tilde{\alpha}\hat{\tilde{s}}_{t-1}$, as long as there is no wage dispersion in the deterministic steady state. When wages are fully flexible, $\tilde{\alpha} = 0$, wage dispersion disappears, and thus $\tilde{s}_t$ equals 1.

It follows from our definition of the wage index given in equation (34) that in equilibrium the real wage rate must satisfy

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}. \tag{47}$$

Aggregating the expression for firm's profits given in equation (25) yields

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d - \nu(1-R_t^{-1}) w_t h_t^d. \tag{48}$$

In equilibrium, tax collection and real money holdings can be expressed as

$$m_t = m_t^h + \nu w_t h_t^d \tag{49}$$

and

$$\tau_t = \tau_t^k [r_t^k u_t - a(u_t) - q_t \delta] k_t + \tau_t^h h_t^d w_t + \tau_t^\phi \phi_t \tag{50}$$

2.5 Competitive Equilibrium

A stationary competitive equilibrium is a set of stationary processes u_t, c_t, h_t, i_t, k_{t+1}, v_t, m_t^h, m_t, a_t, λ_t, π_t, w_t, $\tilde{\mu}_t$, q_t, $\tilde{q}_t$, r_t^k, ϕ_t, f_t^1, f_t^2, $\tilde{w}_t$, h_t^d, y_t, mc_t, x_t^1, x_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, τ_t, τ_t^h, τ_t^k, τ_t^ϕ, R_t, z_t, g_t, n_t, satisfying (7), (8), (10), (12)-(22), (29)-(31), (37)-(43), and (45)-(50), given a monetary-fiscal regime (which adds 4 more restrictions), exogenous stochastic processes $\{\epsilon_t^g, \epsilon_t^z, \epsilon_t^n\}_{t=0}^\infty$, and initial conditions z_0, g_0, n_0, c_{-1}, w_{-1}, s_{-1}, $\tilde{s}_{-1}$, π_{-1}, i_{-1}, a_{-1}, and k_0.

2.6 The Ramsey Equilibrium

We assume that at $t = 0$, the benevolent government has been operating for an infinite number of periods. In choosing optimal policy, the government is assumed to honor commitments made in the past. This form of policy commitment has been referred to as 'optimal from the timeless perspective' (Woodford, 2003).

Formally, we define a Ramsey equilibrium as a set of stationary processes u_t, c_t, h_t, i_t, k_{t+1}, v_t, m_t^h, m_t, a_t, λ_t, π_t, w_t, $\tilde{\mu}_t$, q_t, $\tilde{q}_t$, r_t^k, ϕ_t, f_t^1, f_t^2, $\tilde{w}_t$, h_t^d, y_t, mc_t, x_t^1, x_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, τ_t,

τ_t^h, τ_t^k, τ_t^ϕ, R_t, z_t, g_t, and n_t, for $t \geq 0$ that maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t),$$

subject to the competitive equilibrium conditions (7), (8), (10), (12)-(22), (29)-(31), (37)-(43), and (45)-(50), and $R_t \geq 1$, for $t > -\infty$, given exogenous stochastic processes $\{\epsilon_t^g$, ϵ_t^z, $\epsilon_t^n\}_{t=0}^{\infty}$, values of the variables listed above dated $t < 0$, and values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$.

Technically, the difference between the usual Ramsey equilibrium concept and the one employed here is that here the structure of the optimality conditions associated with the Ramsey equilibrium is time invariant. By contrast, under the standard Ramsey equilibrium definition, the equilibrium conditions in the initial periods are different from those applying to later periods.

Our results concerning the business-cycle properties of Ramsey-optimal policy are comparable to those obtained in the existing literature under the standard definition of Ramsey optimality (e.g., Chari, Christiano, and Kehoe, 1995). The reason is that existing studies of business cycles under the standard Ramsey policy focus on the behavior of the economy in the stochastic steady state (i.e., they limit attention to the properties of equilibrium time series excluding the initial transition).

3 Calibration and Functional Forms

We use the following standard functional forms for utility and technology:

$$U = \frac{\left[(c_t - bc_{t-1})^{1-\phi_4} (1-h_t)^{\phi_4}\right]^{1-\phi_3} - 1}{1-\phi_3}$$

and

$$F(k,h) = k^\theta h^{1-\theta}.$$

Following Christiano, Eichenbaum, and Evans (2005) we assume an investment adjustment cost function of the form:

$$\mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2.$$

As in Schmitt-Grohé and Uribe (2004a) we assume that the transaction cost technology takes the form

$$\ell(v) = \phi_1 v + \phi_2/v - 2\sqrt{\phi_1\phi_2}. \tag{51}$$

The money demand function implied by this transaction technology is of the form

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{R_t - 1}{R_t}.$$

Note the existence of a satiation point for consumption-based money velocity, $\underline{v}$, equal to $\sqrt{\phi_2/\phi_1}$. The money demand has a unit elasticity with respect to consumption expenditures. This feature is a consequence of the assumption that transaction costs, $c\ell(c/m)$, are homogenous of degree one in consumption and real balances and is independent of the particular functional form assumed for $\ell(\cdot)$. Further, as the parameter ϕ_2 approaches zero, the transaction cost function $\ell(\cdot)$ becomes linear in velocity and the demand for money adopts the Baumol-Tobin square root form with respect to the opportunity cost of holding money, $(R-1)/R$. That is, the log-log elasticity of money demand with respect to the opportunity cost of holding money converges to 1/2, as ϕ_2 vanishes.

The costs of higher capacity utilization are parameterized as follows:

$$a(u) = \gamma_1(u-1) + \frac{\gamma_2}{2}(u-1)^2.$$

This is the parameterization estimated in Christiano, Eichenbaum, and Evans (2005).

The equilibrium conditions listed in section 2.5 contain 32 equations (without counting the 4 restrictions defining the monetary/fiscal regime) and 36 variables: 29 endogenous variables (u_t, c_t, h_t, i_t, k_{t+1}, v_t, m_t^h, m_t, a_t, λ_t, π_t, w_t, $\tilde{\mu}_t$, q_t, $\tilde{q}_t$, r_t^k, ϕ_t, f_t^1, f_t^2, $\tilde{w}_t$, h_t^d, y_t, mc_t, x_t^1, x_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, τ_t), 4 policy variables (τ_t^h, τ_t^k, τ_t^ϕ, and R_t), and 3 exogenous shocks (z_t, g_t, and n_t). In addition, the equilibrium conditions feature 27 parameters (ϕ_1, ϕ_2, ϕ_3, ϕ_4, γ_1, γ_2, θ, κ, b, β, δ, $\tilde{\eta}$, $\tilde{\alpha}$, η, α, χ, $\tilde{\chi}$, ψ, ν, ρ_z, σ_{ϵ^z}, ρ_g, σ_{ϵ^g}, , ρ_n, σ_{ϵ^n}, $\bar{g}$, $\bar{n}$). This means that in order to obtain values for the steady-state levels of variables and for the deep structural parameters, we need to impose 31 restrictions.

We take from Christiano, Eichenbaum and Evans (2005) the following 5 parameter values, which they estimate:

$$\eta = 6$$

$$\alpha = 0.6$$

$$\tilde{\alpha} = 0.64$$

$$b = 0.65$$

$$\kappa = 2.48$$

We draw from the estimates reported in Altig, et al. (2004) to assign values to the following parameter:

$$\epsilon_{m^h,R} \equiv \frac{1}{4}\frac{\partial \ln(m^h)}{\partial R} = -0.81$$

$$\frac{\gamma_2}{\gamma_1} = 2.02$$

The parameter $\epsilon_{m^h,R}$ denotes the annualized interest semielasticity of money demand. The assumed form for the transaction cost function implies that

$$\epsilon_{m^h,R} = -\frac{1}{8}\,\frac{1}{R\,(\phi_2\,R + R - 1)}.$$

We follow Christiano, Eichenbaum and Evans (2005) and impose the following 7 calibration (i.e., nonestimated) restrictions:

$$\tilde{\eta} = 21$$

$$\frac{m^h}{m} \equiv \mathrm{s}_{mh} = 0.44$$

$$\tilde{\chi} = 1$$

$$\phi = 0$$

$$u = 1$$

$$\phi_3 = 1$$

The last restriction imply that the period utility function is separable in consumption and leisure.

We set the capital share in value added at 25 percent, the discount rate at 4 percent per year, and the Frisch elasticity of labor supply with respect to the wage rate equal to 4 (which implies that households allocate about 20 percent of their time to remunerated work), that is,

$$\frac{r^k uk}{y} \equiv s_k = 0.25$$

$$\beta = 1.04^{-1/4}$$

$$h = 0.195$$

These values are commonly used in business-cycle studies.

Using postwar U.S. data, we measure the average money-to-output ratio as the ratio of M1 to GDP, and set it equal to 16.95 percent per year. We measure i_t as gross private investment and obtain a an average ratio of investment to GDP of 0.1275. We measure n_t as net transfer to private persons and obtain an average ratio of transfers to GDP of

0.078. Government spending is measured as public consumption expenditure and obtain an estimate its mean share in GDP to be 0.17. Finally, we measure inflation as the growth rate of the GDP deflator, and obtain an average annual rate of 4.2 percent. Thus,

$$\frac{m}{y} = s_m \equiv 0.1695 \times 4$$

$$\frac{i}{y} = s_i \equiv 0.1275$$

$$\frac{g}{y} \equiv s_g = 0.17$$

$$\frac{n}{y} \equiv s_n = 0.078$$

$$\pi = 1.042^{1/4}$$

In calibrating the model, we assume that the economy is in the deterministic steady state of a competitive equilibrium in which the capital and labor income tax rates are constant. We follow Mendoza, Razin, and Tesar (1994) in setting values for the income tax rates:

$$\tau^k = 0.407$$

$$\tau^h = 0.285$$

These tax rates correspond to the year 1988, which is the last year in their sample. Because tax rates appear to have a trend, we use the last observation rather than an average over their sample.

Note that because profits are restricted to be nil in the competitive equilibrium at which we calibrate the economy, the rate at which profits are taxed plays no role for the calibration.

We draw from Cogley and Sbordone (2004) and assume no indexation in product prices:

$$\chi = 0.$$

We estimate by OLS the processes for government purchases and transfers using data from 1947:I to 2004:III. In the case of transfers we eliminated the years 1947-1950 because they contain two outliers in which the government paid unusually high veteran benefits. To obtain the cyclical behavior of all variables involved in the estimation we use the HP filter with a parameter of 1600. We obtain

$$\rho_g = 0.87,$$

$$\sigma_{\epsilon^g} = 0.016,$$

$$\rho_n = 0.78,$$

and

$$\sigma_{\epsilon^n} = 0.022.$$

Given the multitude of distortions in our model, a simple Solow residual is a poor measure of the technology shock in our model. Thus, instead of relying on existing estimates for the stochastic process of Solow residuals, we calibrate the productivity shock process as follows. We require that the process is such, that is, ρ_z and σ_{ϵ^z} are such that the model matches the observed unconditional standard deviation and serial correlation of the cyclical component of output during the postwar era, 1.6 percent and 0.84, respectively. This exercise requires making assumptions about fiscal and monetary policy. We assume that income tax rates are constant at the levels given above (i.e, $\tau^k = 0.407$ and $\tau^h = 0.285$), and that fiscal policy is passive. Specifically, we assume that lump-sum taxes ensure the stationarity of government assets for any on- or off-equilibrium path of the price level. We assume that monetary policy takes the form of a simple Taylor-type rule whereby the current nominal interest rate is set as a function of contemporaneous inflation and output. We then pick the 4 parameters describing the technology process and the monetary policy rule so that the model matches the volatility and serial correlation of output and inflation observed in the postwar U.S. economy. The resulting parameter values for the technology shock process are $\rho_z = 0.8556$ and $\sigma_{\epsilon^z} = 0.0064$.

In computing Ramsey equilibria, we assume that the economy has been operating under the Ramsey policy for a long time and that the average ratio of government debt to GDP is 42 percent per year. That is, we impose that

$$\frac{(a-m)/R}{y} = 0.42 \times 4$$

This value corresponds to the average federal debt held by the public as a percent of GDP in the United States between 1984 and 2003.[4]

Table 1 gathers the values of the deep structural parameters of the model implied by our calibration strategy.

[4]The source is the Economic Report of the President, February 2004, table B79.

Table 1: Structural Parameters

Parameter	Value	Description
β	0.9902	Subjective discount factor (quarterly)
θ	0.25	Share of capital in value added
ψ	0.0594	Fixed cost
δ	0.0173	Depreciation rate (quarterly)
ν	0.5114	Fraction of wage bill subject to a CIA constraint
η	6	Price-elasticity of demand for a specific good variety
$\tilde{\eta}$	21	Wage-elasticity of demand for a specific labor variety
α	0.6	Fraction of firms not setting prices optimally each quarter
$\tilde{\alpha}$	0.64	Fraction of labor markets not setting wages optimally each quarter
b	0.65	Degree of habit persistence
ϕ_1	0.0267	Transaction cost parameter
ϕ_2	0.1284	Transaction cost parameter
ϕ_3	1	Preference parameter
ϕ_4	0.75	Preference parameter
κ	2.48	Parameter governing investment adjustment costs
$\gamma 1$	0.0339	Parameter of capacity-utilization cost function
$\gamma 2$	0.0685	Parameter of capacity-utilization cost function
χ	0	Degree of price indexation
$\tilde{\chi}$	1	Degree of wage indexation
$\bar{g}$	0.0505	Steady-state value of government consumption (quarterly)
$\bar{n}$	0.0232	Steady-state value of government transfers (quarterly)
ρ_z	0.8556	Serial correlation of the log of the technology shock
σ_{ϵ^z}	0.0064	Std. dev. of the innovation to log of technology
ρ_g	0.87	Serial correlation of the log of government spending
σ_{ϵ^g}	0.016	Std. dev. of the innovation to log of gov. consumption
ρ_n	0.78	Serial correlation of the log of government transfers
σ_{ϵ^n}	0.022	Std. dev. of the innovation to log of gov. transfers
b/y	1.68	Debt-to-GDP ratio (quarterly)

Table 2: Ramsey Steady States

Environment				Steady-State Outcome				
τ_t^ϕ	χ	$\bar{n}$	γ_2	π	R	τ^h	τ^k	profit share
τ_t^k				0.2	4.2	35.4	-6.3	0.6
τ_t^k	1			4.6	8.8	34.7	-6.6	0.6
τ_t^k	1	0		-3.8	0	24.1	-5.3	2.3
τ_t^k		0		-0.2	3.8	23.3	-5.2	2.3
1				0.3	4.3	38.2	-44.3	0.8
1			6850	0.3	4.3	37.8	-84.9	1.4
τ_t^k, τ_t^h				0.5	4.5	30.0	30.0	0.3

Note: The inflation rate, π, and the nominal interest rate, R, are expressed in percent per year. The labor income tax rate, τ^h, and the capital income tax rate, τ^k, are expressed in percent. Unless indicated otherwise, parameters take their baseline values, given in table 1.

4 Ramsey Steady States

Consider the long-run state of the Ramsey equilibrium in an economy without uncertainty. We refer to this state as the Ramsey steady state. Note that the Ramsey steady state is in general different from the allocation/policy that maximizes welfare in the steady state of a competitive equilibrium.

Table 2 displays the Ramsey steady-state values of inflation, the nominal interest rate, and labor and capital income tax rates under a number of environments of interests. The figures reported in the table correspond to the exact numerical solution to the steady-state of the Ramsey problem.

4.1 The Optimal Level of Inflation

Consider first the case in which profits are taxed at the same rate as income from capital ($\tau_t^\phi = \tau_t^k$ for all t). In this case, the Ramsey planner chooses to conduct monetary policy in such a way as to nearly stabilize the price level. The optimal inflation rate is 18 basis points per year (line 1 of table 2). It is worth noting that, although small, the steady-state inflation rate is positive. This finding is somewhat surprising, for a well-known result in the context of simpler versions of the new Keynesian model is that the Ramsey steady-state level of inflation is negative and lies between the one called for by the Friedman rule and the one corresponding to full price stabilization. In calibrated example economies, the optimal deflation rate is, however, small (see, for instance, Schmitt-Grohé and Uribe, 2004a; and

Khan, et al., 2003). In these simpler models the optimal inflation rate is determined by the tradeoff between minimizing the opportunity cost of holding money (which requires setting the inflation rate equal to minus the real interest rate) and minimizing price dispersion arising from nominal rigidities (which requires setting inflation at zero). Clearly, our finding of a positive inflation rate suggests that in the medium scale economy we study in this paper there must be an additional tradeoff that the Ramsey planner faces in setting the rate of inflation. To make the presence of the third tradeoff nitid, we consider the case of indexation of product prices to lagged inflation, $\chi = 1$ (line 2 of table 2). In this case, the long-run distortions stemming from nominal rigidities are nil. (Recall that in our calibration nominal wages are fully indexed, i.e., $\tilde{\chi} = 1$.) Therefore, in this case there is no tradeoff between the sticky-price and money-demand frictions. In the absence of any additional tradeoffs, one should expect the Friedman rule to be optimal in this case. However, line 2 of table 2 shows that under long-run price flexibility, the optimal rate of inflation is 4.6 percent per year, a value even further removed from the Friedman rule than the one that is optimal under no indexation in product markets (line 1 of the table).

The third tradeoff turns out to originate in the presence of government transfer payments to households, n_t. Line 3 of table 2 shows that under full indexation and in the absence of government transfers, the Friedman rule emerges as the optimal monetary policy. That is, the nominal interest rate is zero and the inflation rate is negative and equal to the rate of discount in absolute value. The reason why lump-sum government transfers induce positive inflation is that from the viewpoint of the Ramsey planner they represent pure rents accruing to households and as such can be taxed without creating a distortion. In the absence of a specific instrument to tax transfer income, the government chooses to tax this source of income indirectly when it is used for consumption. Specifically, in the model consumption purchases require money. As a result, a positive opportunity cost of holding money—i.e., a positive nominal interest rate—acts as a tax on consumption.[5]

Clearly, in the present model, if lump-sum transfers could be set optimally, they would be set at a negative value in a magnitude sufficient to finance government expenditures and output subsidies aimed at eliminating monopolistic distortions in product and factor markets. But in reality government transfers are positive and large. In the United States, they averaged 7 percent of GDP in the postwar era. Justifying this amount of government transfers as an optimal outcome lies beyond the scope of this paper. One obvious theoretical element that would introduce a rationale for positive government transfers would be the introduction of some form of heterogeneity across households.

[5] A formal analytical derivation of the result that the Friedman rule fails in the presence of government transfers is given in Schmitt-Grohé and Uribe (2005).

Whether set optimally or not, government transfers must be financed. Comparing lines 1 and 4 of table 2 it follows that the government must increase the labor income tax rate by 12 percentage points to finance transfer payments of 7 percent of GDP. Thus, the economy featuring transfers is significantly more distorted than the one without transfers. Because, in general, optimal stabilization policy will depend on the average level of distortion present in the economy, it is of importance for the purpose of this paper, to explicitly include transfers into the model. It is noteworthy that under the calibration shown in table 1 (particularly, under no indexation), allowing for transfers has virtually no effect on the steady-state Ramsey policy except for the level of the labor income tax rate. Specifically, comparing lines 1 and 4 of table 2 shows that removing transfers has virtually no bearing on the optimal rate of inflation and capital income taxation in the steady state.

We conclude that the tripodal tradeoff that determines the Ramsey long-run rate of inflation is resolved in favor of price stability. In this sense, the nominal price friction appears to dominate the money demand friction and the transfer-taxation motive in shaping optimal monetary policy in the long run.

4.2 Optimal Tax Rates

Consider first the economy where profit income are taxed at 100 percent ($\tau_t^{\phi} = 1$). In this case, shown in line 5 of table 2, the Ramsey plan calls for subsidizing capital at the rate of 44.3 percent in the deterministic steady state. It is well known from the work of Judd (2002) that in the presence of imperfect competition in product markets, the markup of prices over marginal costs introduces a distortion between the private and the social returns to capital that increases exponentially over the investment horizon. As a result, optimal policy calls for eliminating this distortion by setting negative capital income tax rates. To gain insight into the nature of the capital income subsidy, note that in steady state the private return to investment is given by $(1-\tau^k)(uF_k/\mu - \delta - a(u))$, where μ denotes the steady-state markup, uF_k denotes the marginal product of capital, δ denotes the depreciation rate, and $a(u)$ denotes the cost of utilizing capital at the rate u. The social return to capital is given by $uF_k - \delta - a(u)$. Equating the private and social returns to investment requires setting τ^k so that

$$(1-\tau^k)(uF_k/\mu - \delta - a(u)) = uF_k - \delta - a(u).$$

Because in the presence of market power in product markets, the markup is greater than unity ($\mu > 1$), it follows that τ^k must be negative. Using the fact that in the steady state

$1 = \beta[(1-\tau^k)(uF_k/\mu - \delta - a(u)) + 1]$, we can write the above expression as:

$$1 - \tau^k = \mu \left[\frac{(\beta^{-1} - 1)}{\beta^{-1} - 1 - (\mu - 1)(\delta + a(u))} \right]. \tag{52}$$

It is clear from this expression that if the depreciation rate is zero (δ=0), and capacity utilization is fixed at unity (so that $a(u) = 0$), then the optimal capital income tax rate is equal to the net markup in absolute value. The case of zero depreciation and constant capacity utilization is the one considered in Judd (2002). We find that the introduction of depreciation in combination with a depreciation allowance, which is clearly the case of greatest empirical interest, magnifies significantly the size of the optimal capital subsidy. For instance, in our economy the markup is 20 percent, the depreciation rate is 7 percent per year, and the discount factor is 4 percent per year. In the case of no depreciation and fixed capacity utilization, the formula in equation (52) implies a capital subsidy of 20 percent. However, with a conservative depreciation rate of 7 percent per year and fixed capacity utilization—which we induce by increasing γ_2 by a factor of 10^5—the optimal subsidy on capital income skyrockets to 85 percent (see line 6 of table 2). The reason for this tremendous rise in the size of the subsidy is that the government taxes the rate of return on capital net of depreciation, whereas the markup distorts the rate of return on capital gross of depreciation.

Allowing for variable capacity utilization (by setting γ_2 at its baseline value of 0.0685), reduces the capital subsidy from 85 percent (line 6 of table 2) to 44 percent (line 5 of table 2). The reason why the subsidy is smaller in this case is that $a(u)$ is negative, which results in a lower effective depreciation rate.[6]

An additional factor determining the size of the optimal subsidy on capital is the fiscal treatment of profits. The formula given in equation (52) applies when profits are taxed at a 100 percent rate. Consider instead the case in which profit income is taxed at the same rate as capital income ($\tau_t^\phi = \tau_t^k$), which is assumed in lines 1-4 of table 2. Because profits are pure rents, the Ramsey planner has an incentive to confiscate them. This creates a tension between setting τ^k equal to 100 percent, so as to fully tax profits, and setting τ^k at the negative value that equates the social and private returns to investment. This explains why when the Ramsey planner is constrained to tax profits and capital income at the same rate, the optimal subsidy to capital is 6.3 percent, a number much smaller than the 85 percent

[6] To see why the level of capacity utilization u_t is less than one (which is necessary for $(a(u_t))$ to be negative) in the Ramsey equilibrium, recall that in the competitive equilibrium used for the calibration of the function $a(\cdot)$, the tax rate on capital was set at 40.7 percent and u_t was set at unity. In the Ramsey equilibrium, τ^k is negative, which induces a larger level of capital. With a higher capital stock, its rate of return at full utilization falls, which induces capitalists to lower it degree of utilization. In the steady state shown in line 5 of table2, u equals 0.85.

implied by equation (52).

Line 7 of table 2 displays the case in which the Ramsey planner is constrained to follow an income tax policy. That is, fiscal policy stipulates $\tau_t^h = \tau_t^k = \tau_t^\phi$. Not surprisingly, the optimal income tax rate falls in between the values of the labor and capital income tax rates that are optimal when the fiscal authority is allowed to set these tax rates separately (line 5 of table 2). The optimal rate of inflation under an income tax is small, half a percent per annum, and not significantly different from the one that emerges when taxes can vary across income sources.[7]

5 Ramsey Dynamics Under Income Taxation

In this section, we study the business-cycle implications of Ramsey-optimal policy when tax rates are restricted to be identical across all sources of income. Specifically, we study the case in which,

$$\tau_t^h = \tau_t^k = \tau_t^\phi \equiv \tau_t^y,$$

for all t, where τ_t^y denotes the income tax rate.

We approximate the Ramsey equilibrium dynamics by solving a first-order approximation to the Ramsey equilibrium conditions. There is evidence that first-order approximations to the Ramsey equilibrium conditions deliver dynamics that are fairly close to those associated with the exact solution. For instance, in Schmitt-Grohé and Uribe (2004b) we compute the exact solution to the Ramsey equilibrium in a flexible-price dynamic economy with money, income taxes, and monopolistic competition in product markets. In Schmitt-Grohé and Uribe (2004a) we then compute the solution to the exact same economy using a first-order approximation to the Ramsey equilibrium conditions. We find that the exact solution is not significantly different from the one based on a first-order approximation.

It has also been shown in the context of environments with fewer distortions than the medium-scale macroeconomic model studied here that a first-order approximation to the Ramsey equilibrium conditions implies dynamics that are very close to the dynamics associated with a second-order approximation to the Ramsey system. Specifically, in Schmitt-Grohé and Uribe (2004a) we establish this result using a dynamic general equilibrium model with money, income taxes, sticky prices in product markets, and imperfect competition.[8]

[7]The reason why the inflation rate is higher than in the baseline case is that in this way the Ramsey planner can tax labor at a higher rate than capital, a point we discuss in detail below.

[8]More recently, Benigno and Woodford (2005) arrive at a similar conclusion in the context of optimal taxation in the standard RBC model. They show that the first- and second-order approximations of the Ramsey equilibrium conditions are similar to the approximation based on a minimum-weighted-residual method reported in Chari et al. (1995).

Table 3: Cyclical Implications of Optimal Policy Under Income Taxation

Variable	Steady state	Standard deviation	Serial correlation	Correlation with output
τ_t^y	30	1.1	0.62	-0.51
R_t	4.53	1.43	0.74	-0.11
π_t	0.51	1.1	0.55	0.11
y_t	0.3	1.96	0.97	1
c_t	0.21	1.16	0.98	0.89
i_t	0.04	7.87	0.98	0.95
h_t	0.19	1.34	0.75	0.59
w_t	1.17	0.94	0.93	0.80
a_t	0.72	4.44	0.99	0.31

Note: R_t and π_t are expressed in percent per year, and τ_t^y is expressed in percent. The steady-state values of y_t, c_t, i_t, w_t, and a_t are expressed in levels. The standard deviations, serial correlations, and correlations with output of these 5 variables correspond to percent deviations from their steady-state values.

Table 3 displays the standard deviation, serial correlation, and correlation with output of a number of macroeconomic variables of interest in the Ramsey equilibrium with income taxation. In computing these second moments, all structural parameters of the model take the values shown in table 1. Second moments are calculated using Monte Carlo simulations. We perform 1000 simulations of 200 quarters each. For each simulation, we compute second moments and then average these figures over the 1000 simulations.

An important result that emerges from table 3 is that under the optimal policy regime inflation is remarkably stable over the business cycle. This result is akin to the one derived in the context of models with a single distortion, namely sticky product prices and no fiscal considerations (Goodfriend and King, 1997 among many others). In the canonical Neo Keynesian model studied in Goodfriend and King, the optimality of price stability is a straightforward result. For in that environment, the single cause of inefficiencies is price dispersion due to exogenous impediments to the adjustment of nominal prices. By contrast, the medium-scale model studied here features, in addition to price stickiness, distortions that in isolation would call for a highly volatile inflation rate under the Ramsey plan.

First, the fact that the government does not have access to lump-sum taxation provides an incentive for the Ramsey planner to use unexpected variations in the inflation rate as a capital levy on private holdings of nominal assets to finance innovations in the fiscal deficit. In effect, Chari et al. (1995) show in the context of a flexible-price model that the optimal rate of inflation volatility is extremely high (above 10 percent per year). So in setting the optimal level of inflation volatility, the Ramsey planner faces a tradeoff between using inflation as a capital levy and minimizing the dispersion of nominal prices. For plausible calibrations, this tradeoff has been shown to be resolved overwhelmingly in favor of price stability. Schmitt-Grohé and Uribe (2004) show within a sticky-price model with distorting taxes that a miniscule amount of price stickiness suffices to induce the Ramsey planner to abandon the use of inflation as a fiscal instrument in favor of virtually complete price stability. Table 3 shows that this result survives in the much richer environment studied here, featuring a relatively large number of nominal and real rigidities.

Second, the fact that our model features sticky wages introduces an incentive for the Ramsey planner to adjust so as to bring about efficient real wage movements. As will be shown shortly, nominal wage stickiness in isolation calls for the Ramsey inflation rate to be highly volatile.

With the inflation rate not playing the role of absorber of fiscal shocks, the Ramsey planner must finance fiscal disturbances via deficits or changes in tax rates or both. Table 3 shows that in our model the role of shock absorber is picked up to a large extent by fiscal deficits (i.e., by adjustments in the level of public debt). Total government liabilities, a_t, are relatively volatile and display a near-unit-root behavior. The standard deviation of government liabilities is 4.4 percent per quarter and the serial correlation is 0.99 in our simulated sample paths. By contrast tax rates do not vary much over the business cycle. The Ramsey planner is able to implement tax smoothing by allowing the public liabilities to vary in response to fiscal shocks.

5.1 Nominal Rigidities and Optimal Policy

Table 4 presents the effects of changing the degree of wage or price stickiness on the behavior of policy variables. Panel A considers the case of no transfers ($n_t = 0$ for all t). This case is of interest because it removes the government's incentive to tax transfers through long-run inflation, making the economy more comparable to existing related studies. When product and factor prices are fully flexible ($\alpha = \tilde{\alpha} = 0$), the optimal policy features high inflation volatility (5.8 percentage points per quarter at an annual rate) and relatively stable tax rates, with a standard deviation of 0.1 percent. In this case, as discussed earlier, variations in inflation are used as a state-contingent tax on nominal government liabilities, allowing the Ramsey planner to smooth taxes. Public debt is stationary with a serial correlation of 0.84.

When prices are sticky but wages are flexible ($\alpha = 0.6$ and $\tilde{\alpha} = 0$), the optimal inflation volatility falls dramatically from its benchmark value of 1.1 percent at an annual rate to less than 0.1 percent. Because prices are costly to adjust, the Ramsey planner relinquishes

Table 4: Degree of Nominal Rigidity and Optimal Policy

A. No Transfers ($n_t = 0$)

α	$\tilde{\alpha}$		τ_t^y	R_t	π_t	w_t	a_t
		Mean	19.0	4.4	0.4	1.2	0.8
0	0	Std. dev.	0.1	0.2	5.8	1.4	2.5
		Ser. corr.	0.6	0.8	-0.1	0.8	0.84
		Mean	19.0	4.0	0.02	1.2	0.8
0.6	0	Std. dev.	0.4	0.7	0.1	1.4	6.3
		Ser. corr.	0.6	0.9	0.1	0.9	1
		Mean	19.0	4.4	0.4	1.2	0.8
0	0.64	Std. dev.	1.5	3.1	5.8	1.7	5.1
		Ser. corr	0.5	0.9	0.8	0.8	0.99
		Mean	19.0	4.0	0.02	1.2	0.8
0.6	0.64	Std. dev.	1.0	1.3	1.1	1	3.6
		Ser. corr.	0.6	0.7	0.6	0.9	0.99

B. Baseline Transfers

		Mean	27.5	21.2	16.6	1.2	0.7
0	0	Std. dev.	0.5	0.5	6.8	1.5	3.0
		Ser. corr.	0.4	0.9	-0.0	0.8	0.84
		Mean	30.0	4.5	0.5	1.2	0.7
0.6	0	Std. Dev.	0.6	0.9	0.2	1.3	7.0
		Ser. corr.	0.7	0.6	0.1	0.7	1
		Mean	27.5	21.2	16.6	1.2	0.7
0	0.64	Std. dev.	1.3	4.6	6.6	1.9	4.3
		Ser. corr.	0.5	0.9	0.83	0.8	0.99
		Mean	30	4.5	0.5	1.2	0.7
0.6	0.64	Std. dev.	1.1	1.4	1.1	0.9	4.4
		Ser. corr.	0.6	0.7	0.6	0.9	0.99

Note: See note to table 3.

the use of surprise inflation as a fiscal shock absorber. Instead, he uses variations in fiscal deficits and some small adjustments in the income tax rate to guarantee fiscal solvency. This practice results in a drastic increase in the serial correlation in government assets, which become a (near) random-walk process. These effects of price stickiness on optimal monetary and fiscal policy are known to emerge in the context of models without capital and fewer nominal and real frictions (e.g., Schmitt-Grohé and Uribe, 2004a).

In the benchmark case, where both prices and wages are sticky ($\alpha = 0.6$ and $\tilde{\alpha} = 0.64$), inflation is more volatile than under product price stickiness alone. As stressed by Erceg et al. (2000) in the context of a much simpler model without a fiscal sector or capital, the reason for the increased volatility of inflation in the case of both price and wage stickiness relative to the case of price stickiness alone, is that the central bank faces a tradeoff between minimizing relative product price dispersion and minimizing relative wage dispersion. Quantitatively, however, this tradeoff appears to be resolved in favor of minimizing product price dispersion rather than wage dispersion. In effect, under price stickiness alone, the volatility of inflation is 0.09 percent, whereas under wage stickiness alone it is 5.8 percent. When both nominal rigidities are present, the optimal inflation volatility falls in between these two values, but, at 1.1 percent, is much closer to the lower one. Interestingly, this result obtains even if one assumes that nominal wages are not indexed to past inflation ($\tilde{\chi} = 0$). In this case, the optimal inflation volatility is 0.9 percent, which is even lower than under full wage indexation (see table 5 and the discussion around it). We note that indexation to past consumer price inflation, being an arbitrary scheme, may not necessarily be welfare improving in our model.

Panel B of table 4 considers the case of positive transfers. All of the results obtained under the assumption of no transfers carry over to the economy with transfers, except for the fact that the mean rate of inflation increases dramatically when product prices are flexible. As discussed earlier in section 4.1, the reason why the Ramsey planner chooses to inflate when all prices are flexible, is that inflation is an indirect tax on transfer payments.

We close this section with a short digression. One may wonder why in the case of fully flexible product and factor prices and no transfers ($\alpha = \tilde{\alpha} = n_t = 0$), the Friedman rule fails to be Ramsey optimal. The reason is that in this case a positive nominal interest rate allows the Ramsey planner to effectively tax labor at a rate higher than capital. The planner engineers this differential tax treatment by exploiting the fact that firms are subject to a cash-in-advance constraint on the wage bill. The reason why it is optimal for the planner to tax labor at a higher rate than capital is clear from our analysis of the Ramsey steady state in section 4. When tax rates on capital and labor income can be chosen independently, the Ramsey planner selects to subsidize capital and to tax labor. Under the income-tax regime studied here, the planner is unable to set different tax rates across sources of income. But

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Table 5: Indexation and Optimal Policy

χ	$\tilde{\chi}$		τ_t^y	R_t	π_t	w_t	a_t
		Mean	30	4.1	0.11	1.2	0.72
0	0	Std. dev.	0.66	1.2	0.94	1	4.9
		Ser. corr.	0.56	0.6	0.44	0.96	0.99
		Mean	30	4.1	0.13	1.2	0.72
1	0	Std. dev.	0.66	1	1.1	1.1	5
		Ser. corr.	0.51	0.58	0.77	0.96	0.99
		Mean	30	4.5	0.51	1.2	0.72
0	1	Std. dev.	1.1	1.4	1.1	0.95	4.3
		Ser. corr.	0.62	0.74	0.55	0.93	0.99
		Mean	28	21	17	1.1	0.74
1	1	Std. dev.	1	2.7	2.9	1.2	4
		Ser. corr.	0.47	0.88	0.94	0.96	1

Note: See note to table 3.

he does so indirectly by levying an inflation tax on labor. The inflation bias introduced by the combination of an income tax and a cash-in-advance constraint on wages is large, above 4 percent per year. If one were to lift the cash-in-advance constraint on wage payment by setting the parameter ν equal to zero, the Friedman rule would reemerge as the Ramsey outcome.

5.2 Indexation and Optimal Policy

An important policy implication of our analysis of optimal fiscal and monetary policy in a medium scale model under income taxation is the desirability of price stability. Because our benchmark calibration assumes full indexation in factor prices but no indexation in product prices, one may worry that our central policy result may be driven too much by the assumed indexation scheme. But this turns out not to be the case.

Consider a symmetric indexation specification in which neither factor nor good prices are indexed ($\chi = \tilde{\chi} = 0$). This case is shown in line 1 of table 5 In the non-indexed economy the Ramsey plan calls for even more emphasis on price stability than in the environment with factor price indexation. The mean and standard deviation of inflation both fall from 0.51 and 1.1, respectively, in the economy with wage indexation to 0.11 and 0.94 in the economy without any type of indexation. The reason why the average inflation rate is lower in the absence of indexation is that removing wage indexation creates an additional source of long-run inefficiency stemming from inflation, namely, wage dispersion. The reason why inflation volatility also falls when one removes wage indexation is less clear. We simply note, as we did before, that the indexation scheme assumed here, namely indexing to past price inflation, being arbitrary, may or may not be welfare improving in the short run.

Consider now the case that prices are fully indexed but wages are not ($\chi = 1$ and $\tilde{\chi} = 0$). If our main result, namely the optimality of inflation stabilization, was driven by our indexation assumption, then the indexation scheme considered now would stack the deck against short-run price stability. Line 2 of table table:wage-vs-price-indexation shows that even when prices are indexed and wages are not, the Ramsey plan calls for the same low level of inflation volatility as under the reverse indexation scheme considered in the benchmark economy (line 3 of table table:wage-vs-price-indexation). The reason is that if the planner were to move prices around over the business cycle so as to minimize the distortions introduced by nominal wage stickiness, then such price movements still would lead to important inefficiencies in the product market because prices although indexed are still sticky. Indexation removes the distortions associated with nominal rigidities only in the long run, not necessarily in the short run.

The fact that indexation removes the long-run inefficiencies associated with nominal product and factor price dispersion due to price stickiness is illustrated in line 4 of table table:wage-vs-price-indexation, displaying the case of indexation in both product and factor markets. The Ramsey-optimal mean inflation rate is in this case 17 percent per year. This large number is driven by two fiscal policy factors identified earlier in this paper: high inflation allows the Ramsey planner to tax transfers indirectly and at the same time provides an opportunity to tax labor income at a higher rate than capital income.

6 Optimized Policy Rules

Ramsey outcomes are mute on the issue of what policy regimes can implement them. The information on policy one can extract from the solution to the Ramsey problem is limited to the equilibrium behavior of policy variables such as tax rates, the nominal interest rate, etc. But this information is in general of little use for central banks or fiscal authorities seeking to implement the Ramsey equilibrium. Specifically, the equilibrium process of policy variables in the Ramsey equilibrium is a function of all of the states of the Ramsey equilibrium. These state include all of the exogenous driving forces and all of the endogenous predetermined variables. Among this second set of variables are past values of the Lagrange multipliers associated with the constraints of the Ramsey problem. Even if the policymaker could observe the state of all of these variables, using the equilibrium process of the policy variables to define a policy regime would not guarantee the Ramsey outcome as the competitive equilibrium. The problem is that such a policy regime could give rise to multiple equilibria.

In this section, we do not attempt to resolve the issue of what policy implements the Ramsey equilibrium in the medium-scale model under study. Rather, we focus on finding parameterizations of monetary and fiscal rules that satisfy the following 3 conditions: (a) They are simple, in the sense that they involve only a few observable macroeconomic variables. (b) They guarantee (local) uniqueness of the rational expectations equilibrium. (c) They minimize some distance (to be specified shortly) between the competitive equilibrium they induce and the Ramsey equilibrium. We refer to rules that satisfy criteria (a) and (b) as implementable. We refer to implementable rules that satisfy criterion (c) as optimized rules.

We define the distance between the competitive equilibrium induced by an implementable rule and the Ramsey equilibrium as follows. Let $IR^R_{T,S,Y}$ denote the impulse response function associated with the Ramsey equilibrium of length T quarters, for shocks in the set S, and variables in the set Y. Similarly, let $IR^{CE}_{T,S,Y}$ denote the impulse responses associated with the competitive equilibrium induced by a particular policy rule. Let $x \equiv \text{vec}(IR^R_{T,S,Y} - IR^{CE}_{T,S,Y})$. Then, we define the distance between the Ramsey equilibrium and the competitive equilibrium associated with a particular implementable rule as $x'x$.

In the present analysis, we take as reference the Ramsey equilibrium under the restriction of an income tax. We set the length of the impulse response function at 20 quarters ($T = 20$). The set of shocks is given by the three shocks that drive business cycles in the model presented above, productivity, government consumption, and government transfers shocks. That is, $S = \{z_t, g_t, n_t\}$. Finally, we limit attention to two endogenous variables, consumption and hours. That is, we let $Y = \{c_t, h_t\}$. Although we focus on this small set of endogenous variables, we will explore the fit of other endogenous variables of interest not included in the set Y.[9]

The family of rules that we will consider here consists of an interest-rule and a tax-rate rule. In the interest-rate rule, the nominal interest rate depends linearly on its own lag, the rates of price and wage inflation, and the log deviation of output from its steady state value. The tax-rate rule features the tax rate depending linearly on its own lag and log deviations of government liabilities and output from their respective steady-state values. Formally, the interest-rate and tax-rate rules are given by

$$\ln(R_t/R^*) = \alpha_\pi \ln(\pi_t/\pi^*) + \alpha_W \ln(\pi^W_t/\pi^*) + \alpha_y \ln(y_t/y^*) + \alpha_R \ln(R_{t-1}/R^*)$$

[9]Ideally, we would like to include in the set Y all the endogenous variables that appear in a second-order expansion of utility after all first-order terms have been substituted out as in the methodology pioneered by Woodford (2003).

and

$$\tau_t^y - \tau^{y*} = \beta_a \ln(a_t/a^*) + \beta_y \ln(y_t/y^*) + \beta_\tau \ln(\tau_{t-1}^y - \tau^{y*}).$$

The target values $R^*, \pi^*, y^*, \tau^{y*}$, and a^* are assumed to be the Ramsey steady-state values of their associated endogenous variables, given in the second column of table 3. The variable $\pi_t^W \equiv W_t/W_{t-1}$ denotes wage inflation. It follows that in our search for the optimized policy rule, we pick 7 parameters so as to minimize the Euclidean norm of the vector x containing 120 elements. We set the initial impulse equal to 1 standard deviation of the innovation in the corresponding shock. That is, for impulse responses associated with shocks z_t, g_t, and n_t, the initial impulse is given by $\sqrt{\frac{\sigma^2_{\epsilon^z}}{(1-\rho_z^2)}}$, $\sqrt{\frac{\sigma^2_{\epsilon^g}}{(1-\rho_g^2)}}$, and $\sqrt{\frac{\sigma^2_{\epsilon^n}}{(1-\rho_n^2)}}$, respectively.

The optimized rule is given by

$$\ln(R_t/R^*) = -1.4 \ln(\pi_t/\pi^*) + 1.68 \ln(\pi_t^W/\pi^*) - 0.077 \ln(y_t/y^*) + 0.42 \ln(R_{t-1}/R^*)$$

and

$$\tau_t^y - \tau^{y*} = -0.26 \ln(a_t/a^*) + 0.18 \ln(y_t/y^*) + 0.29 \ln(\tau_{t-1}^y - \tau^{y*})$$

The optimized interest-rate rule turns out to be active with a negative coefficient on product price inflation and a positive coefficient on wage inflation. The coefficient of output is close to zero and the optimal rule features a mild degree of interest rate smoothing.

The optimized tax-rate rule is active in the sense that an increase in government liabilities is met by a decrease in the income tax rate and an increase in the income tax base calls for tightening fiscal policy. Taken together these two response coefficients will force tax revenues to increase at a time when the financing need of the government is lower. The tax-rate rule also displays a mild degree of tax rate inertia, in the sense that the coefficient on lagged tax rates is positive but significantly less than unity.

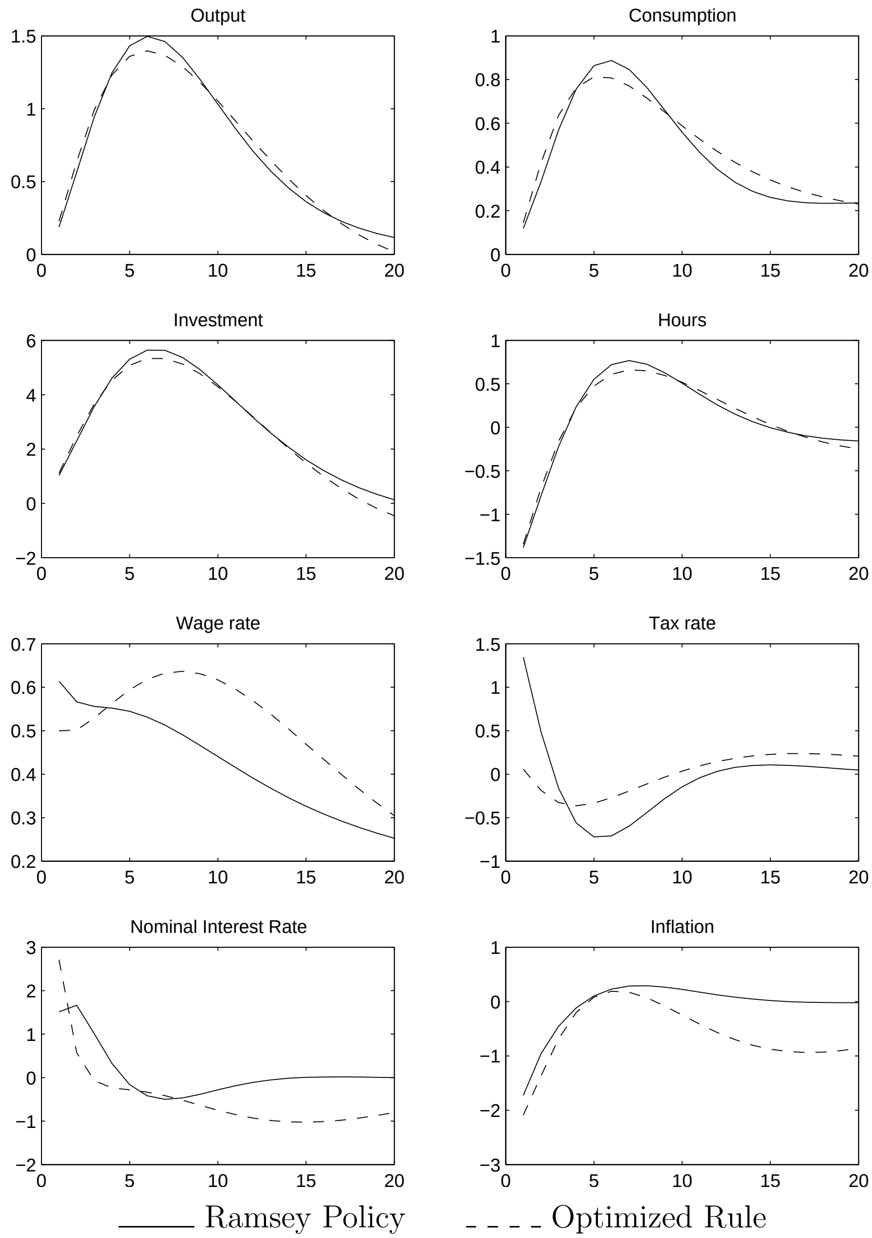
TO BE ADDED: Second-order-accurate welfare difference between the Ramsey equilibrium and the optimized rule.

6.1 Ramsey and Optimized Impulse Responses

To provide a sense of how close the dynamics induced by the Ramsey policy and the optimized rule are, in this section we study theoretical impulse responses to the three shocks driving business cycles in our model economy. Figure 1, displays impulse response functions to a one-standard-deviation increase in productivity ($z_0 = 1.2$ percent). Solid lines correspond to the Ramsey equilibrium, and broken lines correspond to the optimized policy rules.

Remarkably, in response to an increase in productivity, hours worked fall more than one for one. The reason for this sharp decline in labor effort is the presence of significant costs of

Figure 1: Impulse Response To A Productivity Shock



Note: The size of the initial innovation to the technology shock is one standard deviation, $\ln(z_1) = 7\%$. The nominal interest rate and the inflation rate are expressed in percent per year, the tax rate is expressed in percentage points, and the remaining variables are expressed in percentage deviations from their respective steady-state values.

adjustment in investment and consumption. Notice that neither consumption nor investment move much on impact. As a result, the increase in productivity must be accompanied by an increase in leisure large enough to ensure that output remains little changed on impact. The contraction in hours following a positive productivity shock is in line with recent econometric studies using data from the U. S. economy (see, for example, Galí, 2004 NBER Macro Annual).

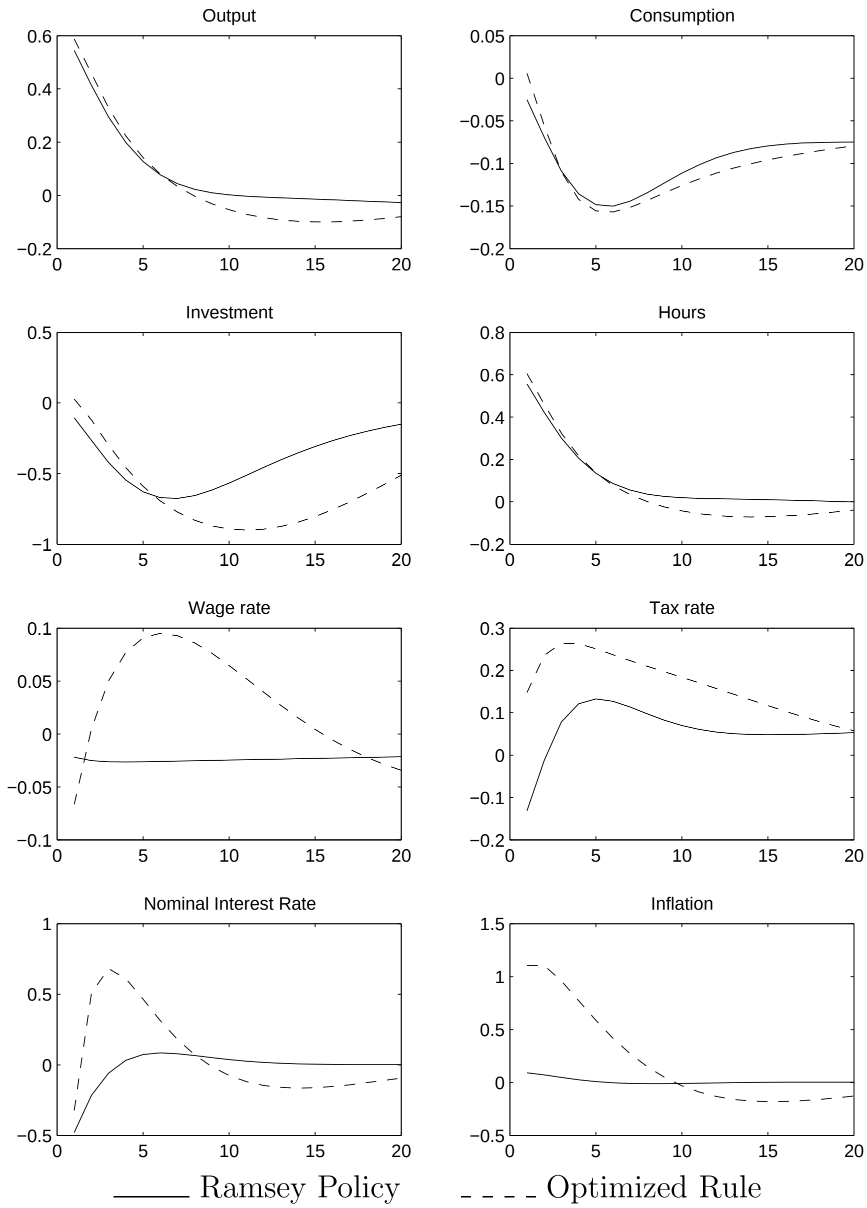
The equilibrium dynamics induced by the optimized policy rules mimic those associated with the Ramsey economy not just for consumption and hours, which would be somewhat less surprising because these two variables enter in the distance measure used to estimate the optimized rule, but also along other dimensions, particularly investment, output, and to a lesser extend inflation and the nominal interest rate. In response to a positive productivity shock, the monetary authority reacts by sharply increasing the nominal interest rate. This reaction is more vigorous under the simple optimized rule. As a result of the monetary contraction, inflation falls significantly. In the ensuing periods, inflation accelerates quickly, displaying overshooting in the Ramsey regime. It is remarkable that the income tax behaves very different under each regime. In the Ramsey economy, the slow initial response of output is accompanied by a sharp increase in taxes on impact followed by a vertical decline to a level below the steady-state value. By contrast, under the optimized rule regime, the tax rate is virtually flat. That is, the optimized rule induces more tax smoothing than is Ramsey optimal.

Figures 2, and 3, display impulse responses to a government spending shock and a government transfer shock, respectively. In both cases, the size of the initial impulse equals one standard deviation of the shock 3.2 percent for the government spending shock, and 3.5 for the government transfer shock). The equilibrium dynamics under the optimized policy rule appears to mimic those associated with the Ramsey policy not as closely as in the case of a productivity shock. This is understandable, however, if one takes into account that these two shocks explain only a small fraction of aggregate fluctuation. In effect productivity shocks alone explain 97 percent of variations in consumption and 88 percent of variations in hours under the Ramsey policy. The optimization estimation procedure therefore naturally assigns a smaller weight on fitting the dynamics induced by g_t and n_t.

7 Capital and Labor Taxation

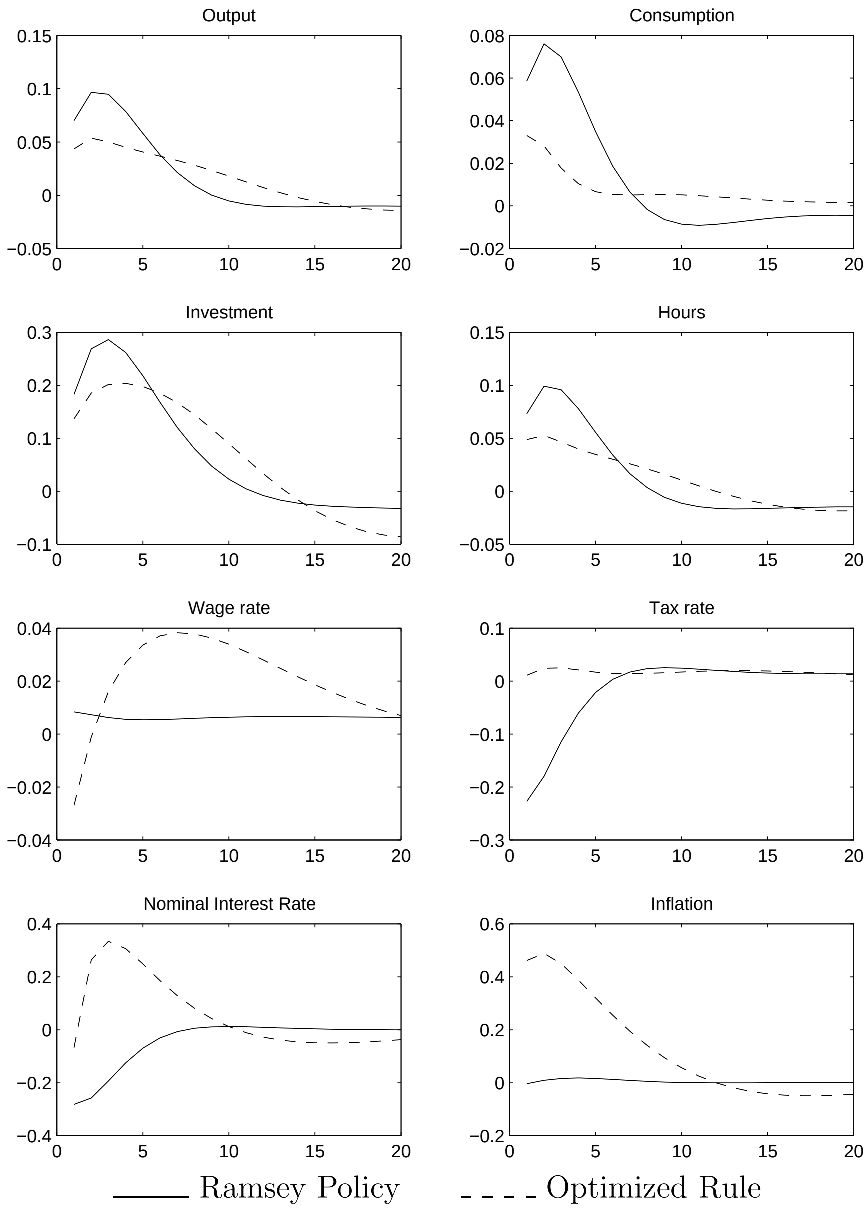
In this section, we characterize dynamic Ramsey policy under the assumption that the fiscal authority has access to three tax instruments, taxes on capital income (τ_t^k), taxes on labor income (τ_t^h), and taxes on pure profits (τ_t^ϕ). Clearly, the optimal tax rate on profits is 100

Figure 2: Impulse Response To A Government Spending Shock



Note: The size of the initial innovation to the government spending shock is one standard deviation, $\ln(g_1/\bar{g}) = 1.6\%$. The nominal interest rate and the inflation rate are expressed in percent per year, the tax rate is expressed in percentage points, and the remaining variables are expressed in percentage deviations from their respective steady-state values.

Figure 3: Impulse Response To A Government Transfer Shock



Note: The size of the initial innovation to the government spending shock is one standard deviation, $\ln(n_1/\bar{n}) = 2.2\%$. The nominal interest rate and the inflation rate are expressed in percent per year, the tax rate is expressed in percentage points, and the remaining variables are expressed in percentage deviations from their respective steady-state values.

percent. We thus set $\tau_t^{\phi} = 1$ for all t for the remainder of the section.

We analyzed the Ramsey steady state of this economy earlier in section 4. As shown on line 5 of table 2, in the Ramsey steady state the labor income tax rate is 38.2 percent and the capital subsidy is 44.3 percent. For the calibration shown in table 1, we find that the standard deviation of the capital income tax rate under the Ramsey policy is 148 percent. The natural reaction to this number is that in this economy the constraint that capital subsidies and taxes should be less than 100 percent will be frequently violated and in this regard the optimal policy makes little economic sense.[10] Qualitatively, however, the intuition for why the volatility of the capital income tax is high is clear. Because capital is a predetermined state variable, unexpected variations in the capital income tax rate act as a nondistorting levy, which the fiscal authority uses to finance innovations in the government budget. The (population) serial correlation of capital tax rates is very close to zero at -0.07. When capital income tax rates can play the role of a fiscal shock absorber, government liabilities no longer display the near random walk behavior as in the case of an income tax. In fact, the (population) serial correlation of a_t now is only 0.6.

To put the number we obtain for the optimal volatility of τ_t^k into perspective, we use as a point of reference two simpler but related economies. First, Chari et. al. (1995), study optimal taxation in a standard real-business-cycle model with exogenous long-run growth and report a standard deviation of the capital income tax rate of 40 percentage points for the stochastic steady state of the Ramsey equilibrium.[11] Recently, Benigno and Woodford (2005) using a different numerical technique replicate this finding. As a test of our numerical procedure, we also study this economy and are able to reproduce the numbers reported in Benigno and Woodford.[12]

The second economy we study as a point of reference is a stationary version of the RBC model of Chari et al. (1995). We find that if one assumes no long-run growth in the Chari et al. economy, the standard deviation of capital income taxes shoots up to about 60 percent (assuming that the steady state level of government assets in the steady state is the same as in the economy with growth). This result illustrates that relatively minor modifications in the economic environment can lead to drastic changes in the optimal volatility of capital income tax rates. Still, these values are not as high as the ones we find in our much more complex model economy. In what follows we complete the reconciliation of our finding with

[10]Our computational strategy does not allow us to consider the case that tax rates are bounded above and below explicitly. But even if one were to use an alternative computational method, one should find that Ramsey capital income tax rates vary significantly over the business cycle.

[11]Chari et. al. consider an annual calibration (that is somewhat different from the one considered here) with business cycles driven by government purchases and technology shocks.

[12]The matlab code for this case will also be available at the authors websites shortly.

those available in the existing literature.

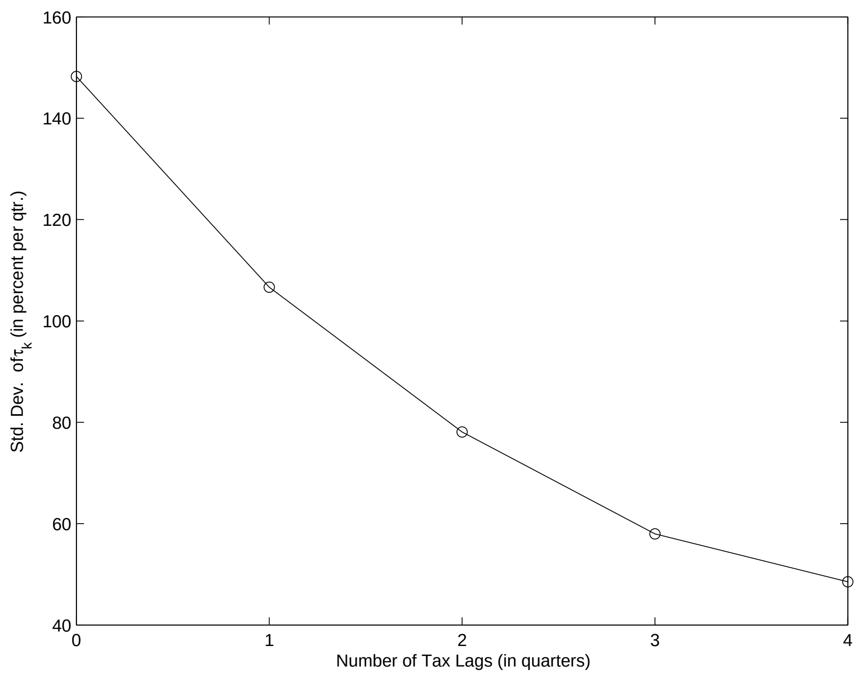
7.1 Time To Tax

Two modifications to our medium-scale macroeconomic model allow us to drive the optimal volatility of capital tax rates down to a level that is comparable to the one that obtains in the standard real business cycle model without growth. First, the model studied by Chari et al. features no impediments to adjusting the level of investment over the business cycle, whereas our model economy incorporates significant investment adjustment costs. Lowering the investment adjustment cost parameter κ by a factor of 1,000 reduces the optimal capital income tax volatility from 148 percent to 66 percent which is close to the volatility of the RBC model without growth. If in addition we assume that tax rates are set one period in advance, then the optimal capital tax volatility falls to 20 percent. The reason why adjustment costs induce a higher optimal volatility of the capital income tax is that investment adjustment costs make capital more akin to a fixed factor of production thereby making movements in the capital tax rate less distorting.

Second, the time unit in the Chari et al. model is one year. By contrast, the time unit in our model is one quarter. Our choice of a time unit is guided by the fact that we study optimal monetary policy as well as optimal fiscal policy. It is unrealistic to assume that the government adjusts monetary policy only once a year. For example, in the United States the FOMC meets every 8 weeks. At the same time, it is equally unrealistic to assume that tax rates change every quarter. One possible way to resolve this conflict is to continue to assume that the time unit is one quarter and to impose that tax rates are determined several quarters in advance, that is, that there are tax lags.

Figure 4 depicts the standard deviation of the capital income tax as a function of the number of tax lags. It shows the results for the economy calibrated using the parameter values shown in table 1. The graph illustrates that the optimal volatility of the capital income tax rate falls steadily with the number of tax lags. Under the assumption that tax rates are determined four quarters in advance, the optimal volatility of capital taxes is driven down to about 49 percentage points. This level of volatility is lower than the values obtained in a non-growing RBC model. (If one constructs a time series of annual tax rates from the arithmetic mean of the quarterly tax rates, then the optimal tax rate volatility at the annual frequency is just 17 percent.)

Figure 4: Time to Tax and Capital Tax Rate Volatility



Note: The standard deviation of the capital income tax rate is expressed in percentage points.

7.2 Capital Tax Volatility and Cost of Varying Capacity Utilization

Another difference between the simple RBC model of Chari et al. (1995) and the model studied here, is that our model economy incorporates variable capacity utilization. One may think that the presence of variable capacity utilization could induce lower capital income tax volatility. For in this case the effective stock of capital is no longer predetermined. As a result, one would expect that variations in the capital income tax rate should be more distorting and hence used less. It turns out, however, that the volatility of the capital income tax rate is not significantly affected when the cost of varying the intensity of capacity utilization falls (in our model, when γ_2 is reduced). For example, when we hold γ_1 constant and reduce γ_2 by a factor of 2, the optimal capital tax volatility increases from 149 percent to 153 percent. [Discussion to be added]

8 Conclusion

[To be added]

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