

# STRUCTURAL ANALYSIS OF US AND EA BUSINESS CYCLES

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## Abstract

Using Bayesian techniques, we estimate a two-country dynamic stochastic general equilibrium (DSGE) model for the United States (US) and the euro area (EA). The main features of the new open economy macroeconomics (NOEM) are embodied in our framework: in particular, imperfect exchange rate pass-through and incomplete financial markets internationally. Each country model incorporates the wide range of nominal and real frictions found in the closed-economy literature: staggered price and wage settings, variable capital utilization and fixed costs in production. The model is estimated on 18 quarterly time series and allows for a relatively large number of structural shocks. In its benchmark version, the model is able to compete with BVAR models in terms of marginal density. Such US-EA DSGE model is well suited for monetary policy analysis at the international level, forecasting exercises and risk analysis.

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# 1 Introduction

This paper advances the New Open Economy Macroeconomics (NOEM) literature in estimating a two-country Dynamic Stochastic General Equilibrium (DSGE) model using Bayesian techniques while incorporating most of the relevant frictions and shocks identified in the New Keynesian closed economy literature.

On the one hand, Bergin (2004) estimated a small scale two-country DSGE for the US and G7 using maximum likelihood and incorporating the main features of the NOEM framework. On the other hand, Smets and Wouters (2003a, 2003b, 2003c) showed that closed economy DSGE embodying a large range of frictions and shocks can be successfully estimated with Bayesian methods. Such models turn out to perform relatively well in terms of forecasts compared with standard and Bayesian VAR models.

Therefore, we intend to improve on both approaches by estimating a relatively large DSGE for the US and the euro area, well suited for forecasting and policy analysis. It seems appropriate to step directly towards a two-country framework as a small open economy specification may not be appropriate for the US or the euro area.

The model shares many features common in NOEM models. Exchange rate pass-through is incomplete due to some nominal rigidity in the buyer's currency. The specification is flexible enough to let the data discriminate between the polar cases of local-currency-pricing (LCP) and producer-currency-pricing (PCP). Financial markets are incomplete internationally and a risk premium on external borrowing is related to net foreign assets. Finally, even under flexible prices and wages, purchasing power parity does not hold due to home bias in aggregate domestic demand.

As in Christiano, Eichenbaum and Evans (2001), we introduced a number of nominal and real frictions such as sticky prices, sticky wages, variable capital utilization costs and habit persistence. A large set of structural shocks enters the model. We follow Smets and Wouters (SW) (2003a, 2003b, 2003c) in the estimation strategy.

The model is fitted on 18 data series: GDP, consumption, investment, labor, real wage, GDP inflation, CPI inflation, short term interest rate for the US and the euro area, the Eurodollar exchange rate and the US current account as a share of GDP. The dataset is quarterly from 1973:1 to 2003:3. Compared with SW, 4 new variables are introduced. Therefore, the open economy dimension also requires additional disturbances. We add a shock on the uncovered interest rate parity condition (UIP) which is often done in the open economy literature, a shock on the relative home bias and two shocks on the distribution sector markups (affecting the CPI equations).

The paper is organized as follows. In section 2, the theoretical model is derived. Section 3 contains a short description of data used, a discussion of parameter calibration and prior distributions, and then reports our estimation results. Section 4 focuses on propagation of shocks and variance decomposition.

## 2 The Model

The world economy is composed by two symmetric countries: Home and Foreign. In each country, there is a continuum of “single-good-firms” indexed on  $[0,1]$ , producing differentiated goods that are imperfect substitute. The number of households is proportional to the number of firms. Consumers receive utility from consumption and disutility from labor. They are identical to each other in the sense that they share the same intertemporal elasticity of substitution and the same elasticity of labor supply with respect to the real wage. But, in each country, the consumption baskets aggregating products from both countries have biased preferences towards locally produced goods. Households own the capital and act as monopolistic suppliers of labor services. Their behavior consists in an intertemporal smoothing of consumption, a supply of rental services from capital and a staggered wage setting. Financial markets are complete domestically but incomplete internationally.

Intermediate firms are monopolistic competitors, produce differentiated products and set prices on a staggered basis *à la* Calvo (1983). Exporter prices are sticky in the producer currency for a fraction of firms and in the buyer currency for the rest.

Not only can the economies be affected by various “efficient” shocks like technological or demand shocks, but it is also possible to introduce inefficient shocks that lead to a short run inflation/output gap tradeoff for the conduct of monetary policy. In our model, we might rationalize those shocks as markup fluctuations in the labor market, as markup fluctuations in the goods markets and financial frictions affecting investment choice.

### 2.1 Consumer’s program

At time  $t$ , the utility function of a generic domestic consumer  $b$  belonging to country H is

$$U_t^h = E_t \sum_{s \geq t} \beta^{s-t} \left( U \left( C_{t+s}^h, C_{t+s-1}^h, L_{t+s}^h, E_t^B, E_t^L \right) \right) \quad (1)$$

Households obtain utility from consumption of an aggregate index  $C_t^h$  while receiving disutility from labor  $L_t^h$ . Utility also incorporates a consumption preference shock  $E_t^B$  and a labor supply shock  $E_t^L$ .

Financial markets are incomplete internationally. As assumed generally in the literature (see for example Benigno P. (2001) or Bergin P. (2004)), Home households can trade two nominal risk-less bonds denominated in the domestic and foreign currency. But foreign residents can only use bonds denominated in the foreign currency. A risk premium as a function of real holdings of the foreign assets in the entire economy, is introduced on international financing of Home consumption expenditures.

Each household  $b$  maximizes its utility function under the following budgetary constraint:

$$\frac{B_{H,t}^h}{P_t(1+i_t)} + \frac{S_t B_{F,t}^h}{P_t(1+i_t^*) \Psi \left( \frac{S_t B_{F,t}^h}{P_t} \right)} + C_t^h = \frac{B_{H,t-1}^h}{P_t} + \frac{S_t B_{F,t-1}^h}{P_t} + \frac{W_t^h L_t^h + A_t^h}{P_t} + \frac{R_t^k C U_t^h K_t^h - \Phi(C U_t^h) K_t^h}{P_t} \quad (2)$$

where  $W_t^h$  is the wage,  $A_t^h$  is a stream of income coming from state contingent securities,  $S_t$  is the nominal exchange rate,  $R_t^k C U_t^h K_t^h - \Phi(C U_t^h) K_t^h$  represents the return on the real capital stock minus the cost associated with variations in the degree of capital utilisation. As in CEE (2001), the income from renting out capital services depends on the level of capital augmented for its utilisation rate and the cost of capacity utilisation is zero when capacity are fully used ( $\Phi(1)=0$ ). Finally  $B_{H,t}^h$  and  $B_{F,t}^h$  are the individuals holding of domestic and foreign bonds denominated in local currency. The risk premium function  $\Psi(\bullet)$  is differentiable, decreasing and verifies  $\Psi(0)=1$ .

The first order conditions corresponding to the quantity of contingent bonds are

$$U_{C,t} = (1+i_t) \beta E_t \left( U_{C,t+1} \frac{P_t}{P_{t+1}} \right) \quad (3)$$

$$U_{C,t} = (1+i_t^*) E_t^{\Delta S} \Psi \left( \frac{S_t B_{F,t}^h}{P_t} \right) \beta E_t \left( U_{C,t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right) \quad (4)$$

The previous equations imply an arbitrage condition on bond prices which corresponds to a modified uncovered interest rate parity:

$$\frac{1+i_t}{(1+i_t^*) E_t^{\Delta S} \Psi \left( \frac{S_t B_{F,t}^h}{P_t} \right)} = \frac{E_t \left( U_{C,t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right)}{E_t \left( U_{C,t+1} \frac{P_t}{P_{t+1}} \right)} \quad (5)$$

$E_t^{\Delta S}$  is a unitary-mean disturbance affecting the time-varying risk premium. There are different ways of rationalizing such a term (see Bergin (2004)) which remains a common device in macro models.

### 2.1.2 Labour supply and wage setting

Moreover, as in Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labor service. For the sake of simplicity, we assume that he sells his services to a perfectly competitive firm which transforms it into an aggregate labor input using the following technology

$$L_t = \left[ \int_0^1 L_t^h \frac{\varepsilon_W - 1}{\varepsilon_W} dh \right]^{\frac{\varepsilon_W}{\varepsilon_W - 1}} \quad (6)$$

The household faces a labor demand curve with constant elasticity of substitution:

$$L_t^h = \left( \frac{W_t^h}{W_t} \right)^{-\varepsilon_W} L_t \quad (7)$$

where  $W_t = \left( \int_0^1 W_t^h \frac{1-\varepsilon_W}{\varepsilon_W} dh \right)^{\frac{1}{1-\varepsilon_W}}$  is the aggregate wage rate.

Households set its wage on a staggered basis. Each period, any household faces a constant probability  $1-\alpha_W$  of changing its wage. In such a case, the wage is set to  $\tilde{w}_t$ , taking into account that it will not be re-optimized in the near future. Otherwise, wages are adjusted following an indexation rule on CPI inflation.

$$W_t^h = \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_W} W_{t-1}^h \quad (8)$$

The first order condition for the re-optimized wage is given by

$$E_t \sum_{s=0}^{\infty} (\beta \alpha_W)^s \left[ \frac{\tilde{w}_t}{P_t} \frac{P_t}{P_{t+s}} \left( \frac{P_{t-1+s}}{P_{t-1}} \right)^{\gamma_W} \frac{\varepsilon_W - 1}{\varepsilon_W} L_{t+s}^h U_{C,t+s} - L_{t+s}^h U_{L,t+s} \right] = 0 \quad (9)$$

When wages are perfectly flexible, this relation collapses to

$$\frac{\varepsilon_W}{\varepsilon_W - 1} U_{L,t}^h = U_{C,t}^h \frac{W_t^h}{P_t}$$

The real wage is equal to a constant markup over the marginal rate of substitution between consumption and labor.

The dynamics of the aggregate wage index is given by

$$W_t^{1-\varepsilon_w} = \alpha_w \left[ \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}^h \right]^{1-\varepsilon_w} + (1-\alpha_w) (\tilde{w}_t)^{1-\varepsilon_w} \quad (10)$$

### 2.1.3 Investment decisions

In each country, the capital is owned by households and rented out to the intermediate firms at a rental rate  $R_t^k$ . Households choose the capital stock, investment and the capacity utilisation rate in order to maximize their intertemporal utility function subject to the intertemporal budget constraint and the capital accumulation equation given by:

$$K_{t+1} = (1-\delta)K_t + \left[ 1 + E_t^I - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad (11)$$

where  $\delta$  is the depreciation rate and  $S(\cdot)$  the cost adjustment function.

First-order conditions result in the following equations for the real value of capital, investment and the capacity utilisation rate:

$$Q_t = E_t \left[ \beta \frac{U_{C,t+1}}{U_{C,t}} \left( Q_{t+1} (1-\delta) + R_{t+1}^k CU_{t+1} - \Phi(CU_{t+1}) \right) \right] E_t^Q \quad (12)$$

$$Q_t \left[ 1 + E_t^I + \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) \right] = \beta E_t Q_{t+1} \frac{U_{C,t+1}}{U_{C,t}} \left( \frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) + 1 \quad (13)$$

$$R_t^k = \Phi'(CU_t) \quad (14)$$

## 2.2 Optimal risk sharing

It is worth examining the case of complete asset market structure because our definition of the flexible price equilibrium will assume that financial markets are also complete internationally. In that case, Households in both countries are allowed to trade in the contingent one-period nominal bonds denominated in the home currency. This leads to the following risk sharing condition<sup>1</sup>:

$$\frac{U_{C,t}^*}{U_{C,t}} = \kappa \frac{S_t P_t^*}{P_t} \quad (15)$$

where  $\kappa$  is a constant depending on initial conditions (here normalized to 1). Equation (15) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among state-contingent securities.

When markets are complete, it is no use evaluating the current account path in order to determine the relative consumption dynamics. Consumption levels in both countries differ only to the extent that the real exchange rate deviates from purchasing power parity. In our model, those deviations are allowed by two assumptions. The first one is the preference bias for locally produced goods, implying that real exchange rate depends on terms of trade. The second one is the possibility that prices might not be denominated in the producer currency, which generates failures of the law of one price.

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<sup>1</sup> A full derivation of this result can be found in Chari et al (2000).

## 2.3 Final goods sector

Final producers for local sales and imports are in perfect competition and aggregate a continuum of differentiated intermediate products from home and foreign intermediate sector.

$Y_H$  and  $Y_F$  are sub-indexes of the continuum of differentiated goods produced respectively in country H and F.  $\xi$  is the elasticity of substitution between bundles  $Y_H$  and  $Y_F$ . The elementary differentiated goods are imperfect substitutes with elasticity of substitution denoted  $\varepsilon$ .

$$Y_H = \left[ \int_0^1 Y(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad Y_F = \left[ \int_0^1 Y(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad Y_H^* = \left[ \int_0^1 Y^*(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad Y_F^* = \left[ \int_0^1 Y^*(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

For a domestic product  $h$ , we denote  $p(h)$  its price on the local market and  $p^*(h)$  its price on the foreign import market.

The domestic demand-based price indexes associated are defined as

$$P_H = \left[ \int_0^1 p(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \quad P_F = \left[ \int_0^1 p(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}, \quad P_H^* = \left[ \int_0^1 p^*(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \quad P_F^* = \left[ \int_0^1 p^*(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}.$$

And domestic demand is allocated across the differentiated goods as follows

$$\begin{cases} \forall h \in [0,1] & Y(h) = \left( \frac{p(h)}{P_H} \right)^{-\varepsilon} Y_H & Y^*(h) = \left( \frac{p^*(h)}{P_H^*} \right)^{-\varepsilon} Y_H^* \\ \forall f \in [0,1] & Y(f) = \left( \frac{p(f)}{P_F} \right)^{-\varepsilon} Y_F & Y^*(f) = \left( \frac{p^*(f)}{P_F^*} \right)^{-\varepsilon} Y_F^* \end{cases} \quad (16)$$

## 2.4 Intermediate firms

On the supply side, goods are produced with a Cobb-Douglas technology as follows:

$$\begin{cases} \forall h \in [0,1], & Y_t(h) = E_t^A (CU_t(h) K_t(h))^\alpha (L_t(h))^{1-\alpha} - \Omega \\ \forall f \in [0,1], & Y_t^*(f) = E_t^{A*} (CU_t^*(f) K_t^*(f))^\alpha (L_t^*(f))^{1-\alpha} - \Omega \end{cases} \quad (17)$$

where  $E_t^A$  and  $E_t^{A*}$  are exogenous technology parameters.

Each firm sells its products in the local market and in the foreign market. We denote  $Y_H(h)$  and  $Y_H^*(h)$  (respectively  $Y_F^*(f)$  and  $Y_F(f)$ ) the local and foreign sales of domestic producer  $h$  (respectively foreign producer  $f$ ) and we define  $L_H(h)$  and  $L_H^*(h)$  (respectively  $L_F^*(f)$  and  $L_F(f)$ ) the corresponding labor demand.

Firms are monopolistic competitors and produce differentiated products. For local sales, firms set prices on a staggered basis *à la* Calvo (1983). In each period, a firm  $h$  (resp.  $f$ ) faces a constant probability,  $1 - \alpha_H$  (resp.  $1 - \alpha_F^*$ ), of being able to reoptimize its nominal price. This probability is independent across firms and time in a same country. The average duration of a rigidity period is  $\frac{1}{1 - \alpha_H}$  (resp.  $\frac{1}{1 - \alpha_F^*}$ ). If a firm cannot reoptimize its price, the price evolves according to the following simple rule:

$$p_t(h) = \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\gamma_H} p_{t-1}(h) \quad (18)$$

Therefore, the firm  $b$  chooses  $\tilde{p}_t(h)$  to maximize its intertemporal profit

$$E_t \sum_{j=0}^{\infty} \alpha_H^j \Xi_{t,t+j} Y_{H,t+j}(h) \left[ (1-\tau) \tilde{p}_t(h) \left( \frac{P_{H,t-1+j}}{P_{H,t-1}} \right)^{\gamma_H} - MC_{t+j} P_{H,t+j} \right] \quad (19)$$

where  $\Xi_{t,t+j} = \beta^j \frac{U_{C,t+j} P_t}{U_{C,t} P_{t+j}}$  is the marginal value of one unit of money to the household,  $MC_{t+j}$  is the real marginal cost.  $\tau$  is a tax on firm's revenue. Due to our assumptions on the labor market and the rental rate of capital, the real marginal cost is identical across producers.

$$MC_t = \frac{W_t^{(1-\alpha)} R_t^{k\alpha}}{E_t^A \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}} \quad (20)$$

In our model, all firms that can reoptimize their price at time  $t$  choose the same level (see Woodford (1999b) for example).

The first order condition associated with the firm's choice of  $\tilde{p}_t(h)$  is

$$E_t \sum_{j=0}^{\infty} \alpha_H^j \Xi_{t,t+j} Y_{H,t+j}(h) P_{H,t+j} \left[ (1-\tau) \frac{\tilde{p}_t(h)}{P_{H,t+j}} \left( \frac{P_{H,t-1+j}}{P_{H,t-1}} \right)^{\gamma_H} - \frac{\varepsilon}{\varepsilon-1} MC_{t+j} \right] = 0. \quad (21)$$

When the probability of being able to change prices tends towards unity, (22) implies that the firm sets its price equal to a constant markup,  $\frac{\varepsilon}{(\varepsilon-1)(1-\tau)}$ , over marginal cost as in the flexible-price model.

Otherwise the firm imposes this markup to the weighted-average of marginal costs over time.

Only a fraction  $(1-\alpha_H)$  of producers in country  $H$  can reoptimize its price, each period. So the aggregate producer-price-index has the following dynamic:

$$P_{H,t}^{1-\varepsilon} = \alpha_H \left( \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\gamma_H} P_{H,t-1} \right)^{1-\varepsilon} + (1-\alpha_H) \tilde{p}_t^{1-\varepsilon}(h) \quad (22)$$

Equations analogous to (21) and (22) hold for foreign producers.

$$E_t \sum_{j=0}^{\infty} (\alpha_F^*)^j \Xi_{t,t+j}^* Y_{F,t+j}^*(f) P_{F,t+j}^* \left[ (1-\tau) \frac{\tilde{p}_t^*(f)}{P_{F,t+j}^*} \left( \frac{P_{F,t-1+j}^*}{P_{F,t-1}^*} \right)^{\gamma_F^*} - \frac{\varepsilon}{\varepsilon-1} MC_{t+j}^* \right] = 0 \quad (23)$$

$$P_{F,t}^{*1-\varepsilon} = \alpha_F^* \left( \left( \frac{P_{F,t-1}^*}{P_{F,t-2}^*} \right)^{\gamma_F^*} P_{F,t-1}^* \right)^{1-\varepsilon} + (1-\alpha_F^*) \tilde{p}_t^{*1-\varepsilon}(f) \quad (24)$$

Concerning exports, we assume that, in country  $H$ , a fraction  $\eta$  (respectively  $\eta^*$  in country  $F$ ) of exporters exhibit producer-currency-pricing (PCP) while the remaining firms exhibit local-currency-pricing (LCP). Consequently, aggregate export prices are given by

$$P_H^* = \left[ \int_0^{\eta} \frac{1}{S} p(h)^{1-\varepsilon} dh + \int_{\eta}^1 p^*(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \text{ and } P_F = \left[ \int_0^{\eta^*} S p^*(f)^{1-\varepsilon} df + \int_{\eta^*}^1 p(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \quad (25)$$

LCP exporters also set its prices  $\hat{p}^*(h)$  and  $\hat{p}(f)$  on a staggered basis and features of nominal rigidities are the same as for the local producers.

Let us define the aggregate LCP export price indexes:

$$\tilde{P}_H^* = \left[ \frac{1}{1-\eta} \int_{\eta} p^*(h)^{1-\varepsilon} dh \right]^{\frac{1}{1-\varepsilon}}, \text{ and } \tilde{P}_F = \left[ \frac{1}{1-\eta^*} \int_{\eta^*} p(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \quad (26)$$

LCP export prices verify the following first order conditions

$$E_t \sum_{j=0}^{\infty} (\alpha_F^*)^j \Xi_{t,t+j} Y_{H,t+j}^* (h) P_{H,t+j} \left[ (1-\tau) \frac{\hat{p}_t^*(h)}{P_{H,t+j}^*} \frac{S_{t+j} P_{H,t+j}^*}{P_{H,t+j}} \left( \frac{\tilde{P}_{H,t-1+j}^*}{\tilde{P}_{H,t-1}^*} \right)^{\gamma_F^*} - \frac{\varepsilon}{\varepsilon-1} MC_{t+j} \right] = 0 \quad (27)$$

$$E_t \sum_{j=0}^{\infty} \alpha_H^j \Xi_{t,t+j}^* Y_{F,t+j} (f) P_{F,t+j}^* \left[ (1-\tau^*) \frac{p_t(f)}{P_{F,t+j}} \frac{P_{F,t+j}}{S_{t+j} P_{F,t+j}^*} \left( \frac{\tilde{P}_{F,t-1+j}}{\tilde{P}_{F,t-1}} \right)^{\gamma_H} - \frac{\varepsilon}{\varepsilon-1} MC_{t+j}^* \right] = 0 \quad (28)$$

$MC_{t+j}$  is the real marginal cost deflated by interior-production-price-index.

As seen before, the dynamics of price indexes are:

$$\tilde{P}_{H,t}^{*1-\varepsilon} = \alpha_H^* \left( \left( \frac{\tilde{P}_{H,t-1}^*}{\tilde{P}_{H,t-2}^*} \right)^{\gamma_H^*} \tilde{P}_{H,t-1}^* \right)^{1-\varepsilon} + (1-\alpha_H^*) p_t^*(h)^{1-\varepsilon} \quad (29)$$

$$\tilde{P}_{F,t}^{1-\varepsilon} = \alpha_F \left( \left( \frac{\tilde{P}_{F,t-1}}{\tilde{P}_{F,t-2}} \right)^{\gamma_F} \tilde{P}_{F,t-1} \right)^{1-\varepsilon} + (1-\alpha_F) p_t(f)^{1-\varepsilon} \quad (30)$$

Capital labour ratio is equalized across firms and linked to the relative cost of factors:

$$\frac{W_t L_t}{R_t^k z_t K_t} = \frac{1-\alpha}{\alpha} \text{ and } \frac{W_t^* L_t^*}{R_t^{k*} z_t^* K_t^*} = \frac{1-\alpha}{\alpha} \quad (31)$$

## 2.5 Distribution sector

Two segments compose the distribution sector. At the end, a competitive segment aggregates differentiated consumer baskets with the following technology:

$$Y = \left[ \int_0^1 Y_j^{\frac{\varepsilon_D-1}{\varepsilon_D}} dj \right]^{\frac{\varepsilon_D}{\varepsilon_D-1}}, \quad Y^* = \left[ \int_0^1 Y_j^{*\frac{\varepsilon_D-1}{\varepsilon_D}} dj \right]^{\frac{\varepsilon_D}{\varepsilon_D-1}}$$

At the source, a continuum of companies in monopolistic competition mixes local production with imports. There is a home bias in the aggregation, which pins down the degree of openness at steady state. The distributor technology is given by

$$\forall i \in [0,1], \quad Y_i = \left[ n_t^{\frac{1}{\xi}} Y_{i,H}^{\frac{\xi-1}{\xi}} + (1-n_t)^{\frac{1}{\xi}} Y_{i,F}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \quad Y_i^* = \left[ (1-n_t^*)^{\frac{1}{\xi}} Y_{i,H}^{*\frac{\xi-1}{\xi}} + n_t^{*\frac{1}{\xi}} Y_{i,F}^{*\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \quad (32)$$

Degrees of home bias are subject to shocks. But as only difference of openness rates enters linearized aggregate equations, home bias shocks are given by  $n_t = n(1 + e_t^{\Delta n}/2)$  and  $n_t^* = n(1 - e_t^{\Delta n}/2)$



Here again, we assume a staggered price setting with an associated probabilities  $\alpha_D$  and  $\alpha_D^*$  and re-optimized price denoted  $\tilde{P}_{i,t}$  and  $\tilde{P}_{i,t}^*$ . First-order conditions are given by

$$E_t \sum_{j=0}^{\infty} \alpha_D^j \Xi_{t,t+j} Y_{i,t+j} P_{t+j} \left[ \frac{\tilde{P}_{i,t}}{P_{t+j}} \left( \frac{P_{t-1+j}}{P_{t-1}} \right)^{\gamma_D} - \frac{\varepsilon_D}{\varepsilon_D - 1} MC_{D,t+j} \right] = 0. \quad (33)$$

$$E_t \sum_{j=0}^{\infty} (\alpha_D^*)^j \Xi_{t,t+j}^* Y_{i,t+j}^* P_{t+j}^* \left[ \frac{\tilde{P}_{i,t}^*}{P_{t+j}^*} \left( \frac{P_{i,t-1+j}^*}{P_{i,t-1}^*} \right)^{\gamma_D^*} - \frac{\varepsilon_D}{\varepsilon_D - 1} MC_{D,t+j}^* \right] = 0 \quad (34)$$

where

$$MC_D = \left[ n_t \left( \frac{P_H}{P} \right)^{1-\xi} + (1-n_t) \left( \frac{P_F}{P} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}}, \quad MC_D^* = \left[ n_t^* \left( \frac{P_F^*}{P^*} \right)^{1-\xi} + (1-n_t^*) \left( \frac{P_H^*}{P^*} \right)^{1-\xi} \right]^{\frac{1}{1-\xi}} \quad (35)$$

Aggregate consumer price dynamics are

$$P_t^{1-\varepsilon_D} = \alpha_D \left( \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_D} P_{t-1} \right)^{1-\varepsilon_D} + (1-\alpha_D) \tilde{P}_{i,t}^{1-\varepsilon_D} \quad (36)$$

$$P_t^{*1-\varepsilon_D} = \alpha_D^* \left( \left( \frac{P_{t-1}^*}{P_{t-2}^*} \right)^{\gamma_D^*} P_{t-1}^* \right)^{1-\varepsilon_D^*} + (1-\alpha_D^*) \tilde{P}_{i,t}^{*1-\varepsilon_D^*} \quad (37)$$

Note that cost minimization implies

$$\begin{cases} \frac{Y_H}{Y_F} = \frac{n_t}{(1-n_t)} \left( \frac{P_H}{P_F} \right)^{-\xi} \\ \frac{Y_F^*}{Y_H^*} = \frac{n_t^*}{(1-n_t^*)} \left( \frac{P_F^*}{P_H^*} \right)^{-\xi} \end{cases} \quad (38)$$

## 2.5 Market clearing conditions

Aggregate productions are obtained using the CES aggregator  $\left[ \int_0^1 \bullet^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$  and labor demands are given by the following relations:

$$\begin{cases} Y_{H,t} = \frac{E_t^A (CU_t K_{H,t})^\alpha (L_{H,t})^{1-\alpha} - \Omega}{D_{H,t}}, \quad Y_{H,t}^* = \frac{E_t^A (CU_t K_{H,t}^*)^\alpha (L_{H,t}^*)^{1-\alpha} - \Omega}{D_{H,t}^*} \\ Y_{F,t} = \frac{E_t^{A*} (CU_t^* K_{F,t})^\alpha (L_{F,t})^{1-\alpha} - \Omega}{D_{F,t}}, \quad Y_{F,t}^* = \frac{E_t^{A*} (CU_t^* K_{F,t}^*)^\alpha (L_{F,t}^*)^{1-\alpha} - \Omega}{D_{F,t}^*} \end{cases} \quad (39)$$

$$\text{where } D_{H,t} = \int_0^1 \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\varepsilon} dh, \quad D_{H,t}^* = \int_0^1 \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} dh, \quad D_{F,t} = \int_0^1 \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\varepsilon} df, \quad D_{F,t}^* = \int_0^1 \left( \frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\varepsilon} df.$$

Total output and labor supplies are denoted  $Z$  and  $L$  in country  $H$ ,  $Z^*$  and  $L^*$  in country  $F$ .

Market clearing conditions on the labour market lead to

$$L_t = \frac{L_{H,t} + L_{H,t}^*}{D_{W,t}} \text{ and } L_t^* = \frac{L_{F,t} + L_{F,t}^*}{D_{W,t}^*} \quad (40)$$

$$\text{where } D_{W,t} = \int_0^1 \left( \frac{W_t^h}{W_t} \right)^{-\varepsilon_w} dh \text{ and } D_{W,t}^* = \int_0^1 \left( \frac{W_t^f}{W_t^*} \right)^{-\varepsilon_w} df$$

Market clearing conditions on goods market are given by:

$$\begin{cases} Z = \left( \frac{P_H}{P} \right)^{-\xi} n_t Y + \left( \frac{P_H^*}{P^*} \right)^{-\xi} (1 - n_t^*) Y^* \\ Z^* = \left( \frac{P_F}{P} \right)^{-\xi} (1 - n_t) Y + \left( \frac{P_F^*}{P^*} \right)^{-\xi} n_t^* Y^* \end{cases} \quad (41)$$

Domestic demands are given by

$$\begin{cases} Y_t / D_{Y,t} = C_t + I_t + E_t^G + \Phi(z_t) K_t \\ Y_t^* / D_{Y,t}^* = C_t^* + I_t^* + E_t^{G*} + \Phi(z_t^*) K_t^* \end{cases} \quad (42)$$

$$\text{where } D_{Y,t} = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon_D} di \text{ and } D_{Y,t}^* = \int_0^1 \left( \frac{P_{i,t}^*}{P_t^*} \right)^{-\varepsilon_D} di$$

Turning to bond markets, we assume that bonds denominated in currency  $H$  are in zero-net supply within that country. Moreover, demand for bonds denominated in currency  $F$  emanating from agents in country  $H$  is given by

$$\frac{S_t B_{F,t} / P_t}{(1 + i_t^*) \Psi \left( \frac{S_t B_{F,t}^h}{P_t} \right)} = \frac{S_t B_{F,t-1}}{P_t} + \left( \frac{P_{H,t}}{P_t} \right)^{1-\xi} n_t Y_t + \left( \frac{P_{H,t}^*}{P_t^*} \right)^{1-\xi} (1 - n_t^*) Y_t^* - C_t \quad (43)$$

Several relative price indicators are worth noting.

$$RER_{H,t} = \frac{S_t P_{H,t}^*}{P_{H,t}} \text{ and } RER_{F,t} = \frac{P_{F,t}}{S_t P_{F,t}^*} \text{ represent the aggregate relative margins on exports for producers}$$

in country  $H$  and  $F$  respectively. If there is some form of international price discrimination, those ratios figure the relative profitability of foreign sales compared with the local ones.

$T = \frac{P_F}{P_H}$  and  $T^* = \frac{P_F^*}{P_H^*}$  denote the interior terms of trade and measure the competitiveness of local producers against import competitors.

$$RER_t = \frac{S_t P_t^*}{P_t} \text{ is the real exchange rate measured with the consumer price indexes.}$$

Specifying the interest rate rules followed by the monetary authorities finally closes the model. We use Taylor type rules as in Smets and Wouters (2003a, 2003b, 2003c). In an open economy framework, the choice of the price deflator in the reaction function remains an issue. In the benchmark model, we intended to limit the departure from the closed-economy results and assumed that monetary authorities target domestic objectives: the domestic output gap and GDP inflation rate.

## 3 Bayesian estimation of the linearized Model

### 3.1 The linearized model

The two-country model is linearized around a symmetric steady state. Considering the US and the euro area, this constraint may not be so restrictive. In the steady state, all macroeconomic variables are equalized across country and there is no detention of foreign assets.

The function utility is given by

$$U_t^h = \left[ \frac{1}{1-\sigma_c} (C_t - h C_{t-1})^{1-\sigma_c} - \frac{E_t^L}{1+\sigma_L} L_t^{h1+\sigma_L} \right] E_t^B \quad (44)$$

The details of the linearized sticky-prices model are presented in Appendix A. The flexible-price equilibrium is given by the same set of equations without nominal rigidities on prices and wages (i.e. law of one price holds), with complete financial markets and considering only efficient shocks. The linearized model is written with lower cases.

The monetary authorities follow generalized Taylor rules which incorporate deviations of lagged inflation and the lagged output gap defined as the difference between actual and flexible-price output.

The exogenous can be divided in three categories:

- Efficient shocks: shocks on technology, investment, labour supply, public expenditures, consumption preferences and relative home bias.
- Inefficient shocks: shocks on goods market markups, labour market markups, Tobin's Q and UIP.
- Policy shocks: shocks on short term interest rates and inflation target.

Efficient shocks follow AR(1) processes whereas inefficient shocks and Taylor rules residuals are white noises.

The benchmark model abstracts from nominal rigidities in the distribution sectors. Accordingly, CPI inflation rates equations are given by

$$\pi_t = n\pi_{H,t} + (1-n)\pi_{F,t} + \varepsilon_t^{CPI} \quad \text{and} \quad \pi_t^* = n\pi_{F,t}^* + (1-n)\pi_{H,t}^* + \varepsilon_t^{CPI*}$$

### 3.2 Estimation methodology

Compared with the closed-economy version of the model that has been estimated in the US and the euro area separately by Smets and Wouters (2003a,2003b,2003c), the two-country framework embodies four additional variables in the estimation and four additional shocks closely related to the new variables: the exchange rate together with the UIP shock, the current account with the relative home bias shock, CPI inflation rates with distribution markup shocks.

Introducing two price deflators per country is necessary in order to describe the imperfect exchange pass-through. We will come back to this point in the sensitivity analysis below. The current account has been incorporated in the estimation to improve the inference of the financial frictions.

## Data

Data for the US will be used for the country H. For each country we consider 8 key macro-economic quarterly time series from 1973q1 to 2003q3: output, consumption, investment, hours worked, real wages, GDP deflator inflation rate, CPI inflation rate and 3 month short-term interest rate. US series come from BEA and BLS. Euro area data are taken from Fagan et al (2001) and Eurostat. Concerning the euro area, employment numbers replace hours. Consequently, as in Smets and Wouters (2003a), hours are linked to the number of people employed  $e_t^*$  with the following dynamics:

$$e_t^* = \beta E_t e_{t+1}^* + \frac{(1 - \beta \lambda_e)(1 - \lambda_e)}{\lambda_e} (l_t^* - e_t^*)$$

The exchange rate is the euro/dollar exchange rate. Due to statistical problems in computing long series of bilateral current account and current account for the euro area, we used the US current account as a share of US GDP. Aggregate real variables are expressed per capita by dividing with working age population.

All the data are detrended before the estimation.

The SDGE is fitted on 18 macro variables,  $z_t, z_t^*, c_t, c_t^*, i_t, i_t^*, l_t, e_t, w_t, w_t^*, \pi_t, \pi_t^*, \pi_t^{GDP}, \pi_t^{GDP*}, r_t, r_t^*, \Delta s_t, ca_t$ , assuming 24 exogenous disturbances,  $e_t^A, e_t^{A*}, e_t^B, e_t^{B*}, e_t^G, e_t^{G*}, e_t^I, e_t^{I*}, e_t^L, e_t^{L*}, e_t^{\Delta n}, e_t^{\Delta s}, \varepsilon_t^P, \varepsilon_t^{P*}, \varepsilon_t^W, \varepsilon_t^{W*}, \varepsilon_t^Q, \varepsilon_t^{Q*}, \varepsilon_t^R, \varepsilon_t^{R*}, \varepsilon_t^{CPI}, \varepsilon_t^{CPI*}, \varepsilon_t^R, \varepsilon_t^{R*}$ .

## Calibrated parameters

Some parameters are fixed prior to the estimation. This concerns generally parameters driving the steady state values of the state variables to which the econometric model including detrended data is quasi uninformative. Those parameters are assumed to be the same for the US and the euro area. The discount factor  $\beta$  is calibrated to 0.99, which implies annual steady state real interest rates of 4%. The depreciation rate  $\delta$  is equal to 0.0025 per quarter. Given labor income share in total output of 70%, we set  $\alpha$  to 0.3. Consequently, the steady state shares of consumption and investment in total output are respectively 0.6 and 0.2. External trade is balanced in the steady state. Finally, the labour market markup in wage setting is not identified and is equal to 0.5 as in Smets and Wouters (2003,2004).

## Econometric Methods

All the results reported here are obtained with Dynare, a matlab toolbox aimed at simulating and estimating DSGE models. The estimation strategy may be decomposed in three steps. First the linearized version of the rational expectation model is solved, so that the world dynamics are described in a state-space representation (non linear in the deep parameters). Second, the posterior kernel of the model (i.e. the log-prior densities plus the log-likelihood of the model obtained by running a Kalman filter) is evaluated and maximized. Third, once the posterior mode is found, we get the entire posterior distribution by implementing a Metropolis-Hastings Monte-Carlo.

### Prior distribution of parameters

The priors are assumed to be the same across countries.

The standard errors of the innovations are assumed to follow uniform distributions on the compact  $[0, 6]$ .

In SDGE models, data are often very informative about the structural disturbances so those very loose priors seem well suited. The distribution of the persistence parameters in the efficient and policy shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1.

Concerning the parameters of the Taylor rules, we follow Smets and Wouters (2003a, 2003b, 2003c): the long run coefficient on inflation and output gap are described by a Normal distribution with mean 1.5 and 0.125, and standard errors 0.1 and 0.05 respectively. The persistence parameter follows a normal around 0.75 with a standard error of 0.1. The prior on the short run reaction coefficients to inflation and output gap changes reflect the assumptions of a gradual adjustment towards the long run.

Concerning preference parameters, the intertemporal elasticity of substitution is set at 1 with standard error of 0.375. The habit parameter is centered on 0.7 with standard deviation of 0.1 and the elasticity of labor supply has mean 2 and standard error of 0.75. Adjustment cost parameter for investment follows a  $N(4, 2)$  and the capacity utilization elasticity is set at 0.2 with a standard error of 0.1. Finally the share of fixed costs in the production function is assumed to be distributed around 0.25.

Concerning the Calvo probabilities of price and wage settings, we assume a beta distribution around 0.75. The degree of indexation to past inflation is centered on 0.5.

Regarding the open economy parameters, the intratemporal elasticity of substitution follows a gamma distribution with mean 2 and standard error 0.75. Coefficients of contemporaneous exchange rate pass-through into trade prices are normally distributed around 0.5 with standard deviation of 0.3. Finally the semi elasticity of risk premium with respect to net foreign assets has a gamma distribution centered on 0.005 with standard error of 0.002.

Finally, the steady state value of the openness ratio is also estimated. As a rest of the world is not included in our framework, we tried to “let the data speak” about the effective openness ratio in this reduced form model of the international linkages between the US and the euro area. The point is to see if the estimated openness ratio is closer to the bilateral openness, around 2%, than to the overall openness, around 13%. The prior for this parameter is a beta distribution centered on 0.85 with standard deviation 0.05.

### 3.3 Parameter estimates: comparison with the closed economy model

The posterior distributions of the parameters are reported in Appendix B. The first set of charts show the likelihood function in the vicinity of posterior modes and in the direction of each parameter which enables us to check the curvature of the objective function and the point estimates. Table 1 and 2 gather posterior modes and standard errors corresponding to the Hessian. Point estimates are satisfying as far as convergence and “significance” are concerned.

Tables 3 and 4 present the mean, 90% percent Highest Probability Density interval of the posterior distribution of the parameters obtained through the Metropolis-Hasting sampling algorithm. The latter is based on two chains of 500000 draws. Convergence diagnostics suggest that the convergence of the algorithm is satisfying.

For individual countries, results seem to be in line with the work of Smets and Wouters (2003a, 2003b, 2003c).

Table 8 and 9 report the posterior modes of the closed economy model. In this specification, both countries are closed to international trade in goods and financial assets. The model then collapses to a pooling of closed economies. The exchange rate and the current account are not defined anymore in this framework and have been excluded. Consequently, model comparison based on marginal density cannot be performed directly from these results.

Between the closed and the open economy models, point estimates are relatively similar. The main differences appear on the wage setting parameters and on intertemporal elasticity of substitution. In the open economy framework, CPI inflation enters the wage setting and the Euler equation instead of GDP inflation. Consequently, nominal rigidity and inertia in the wage settings seem to be reduced whereas the intertemporal elasticity of substitution (proportional to  $1/\sigma_c$ ) is lower.

Tables 5, 6 and 7 (respectively 10 and 11 for the closed economy model) summarize the results for the unconditional variance decomposition of the forecast errors for fitted variables. On output, it seems that compared with the closed economy model, the introduction of open economy shocks lowers the relative contribution of demand shocks in the short term and, to a lesser extent, the contribution of monetary policy shocks. Concerning consumption and investment, UIP and Home bias shocks contribute mainly on asymptotic variance and have an impact on the overall decomposition.

Regarding asymmetries, parameters driving the behavioral equations for the US and the euro area are relatively close. The main differences appear on the shock processes and in the price setting. Concerning the price setting, it seems that prices are less sticky in the US than in the euro area but the weight on past inflation in the Phillips curve is higher. We estimated a version of the model assuming that most of the US and euro area structural parameters are identical but asymmetries remain on price setting, exchange rate pass-through and exogenous disturbance processes. Those constraints seem to be well accepted by the data and the marginal density is not lower with homogenous behaviors. This point deserves however more elaborated investigation before drawing strong conclusions.

### 3.4 Open economy features

#### Intratemporal elasticity of substitution

The intratemporal elasticity of substitution is around 1 in the benchmark model, the highest probability density interval going approximately from 0.8 to 1.5. This parameter is crucial for a wide range of international economics issues. NOEM literature frequently used unitary assumptions in order to improved the tractability of theoretical developments. However, empirical studies on international trade, generally obtained with disaggregated data, find much higher estimates whereas trade equations on national accounts trade volumes often deliver price elasticity significantly below one. Given that we did not introduce trade data in the estimation and that US - EA competition on bilateral and third markets is embodied in a reduced form framework, a low substitution between US and EA products is a plausible outcome. This estimate is similar to the unitary elasticity found by Bergin (2004) in a US-G7 framework.

#### Imperfect pass-through

The extent to which nominal exchange rate fluctuations pass through core prices and the way to incorporate such features in theoretical models are topical issues in international economics. In this paper, imperfect pass-through is achieved through a combination of nominal rigidities and/or currency denomination of exports. Should all prices be flexible, firms would have no incentives to discriminate international markets and the law of one price would hold.

In our benchmark setting, we estimate the share between producer-currency-pricing firms and local-currency-pricing firms. For the US, 80% of firms are PCP with a distribution between 70% to 99%. In the euro area, the share of PCP firms is much lower and is centered on 60% with a distribution between 40% and 80%. Therefore, the estimated immediate exchange rate pass-through on import prices is relatively high and the US exporting firms seem to be more “price maker” than the European firms.

Introducing an additional staggered price setting between trade prices and consumer prices deteriorates noticeably the performance of the model (in terms of marginal density, see Table 27). This friction should be reassessed with different modeling of the exchange rate pass-through on trade prices and eventually trying to fit trade price data.

Finally, parameter posteriors related to exchange rate pass-through depend crucially on the price deflators introduced in the estimation procedure. If GDP inflation rates are removed, immediate pass-through is negligible which support the LCP assumption. Conversely, if CPI inflation rates are removed, immediate pass-through is close to complete as in the PCP case. This result put into perspective the finding of Bergin (2004) who, using CPI inflation, concluded that LCP was the appropriate specification.

### **International financial markets friction**

The model also gives some information about the risk premium of the UIP linked to net foreign assets. Even if the posterior distribution is close to the prior one, the “distance” between both distributions (and in particular the mean shift) suggests that this parameter is relatively well identified. Our estimate implies that a 20% decrease in US net foreign assets increases the risk premium by 12 basis point (the distribution ranging from 6 bp to 20 bp). Bergin (2004) finds a result of 0.00384 which is in our posterior distribution.

### **Steady state openness ratio**

Our model tries to estimate a reduced form of the US and EA interactions in the world economy. A rest of the world sector is not introduced. Consequently, the relevant value of the steady state openness ratio can be higher than the bilateral openness ratio in order to take into account third markets effects. In the benchmark version, the openness ratio is estimated and the result points to value quite close to the bilateral openness ratio.

Alternatively, we estimated a model keeping the steady state openness ratio fixed at 12% which is close to the openness of the US and the euro area over the recent years. Overall, this restriction deteriorates considerably the performance of the model as the likelihood function decreases strongly (see Table 27). The intratemporal elasticity of substitution is then substantially lower at 0.6. The exchange rate pass-through is also estimated to be lower and asymmetries between the US and the euro area are more pronounced as only 37% of EA firms are PCP against 76% in the US.

### **Taylor rules**

The open economy framework allows introducing different inflation targets in the reaction function of monetary authorities. In the benchmark version, monetary reaction functions include model based output gap and GDP inflation rate. Coefficients on the level of inflation in the Taylor rules prove not to be identified. Prior and posterior distributions for those parameters match quite perfectly indicating that our sample is uninformative about them. Similar results are found in SW (2003a, 2003b).

In theoretical papers (see for example Darracq (2003)), it can be shown that under PCP, monetary policy should adjust output gap to producer prices whereas, under LCP, targets should be consumption gaps and CPI. Without developing further this topic, we only estimated the model replacing GDP inflation rates by CPI inflation rates in the Taylor rules. This alternative specification does not modify substantially the results as parameter estimates are very close to the benchmark case but the marginal density is lower. Replacing the model based output gap by the model based consumption gap in the Taylor rule worsens further the performance of the model (see Table 27).

## **3.5 Empirical performance of the model**

Following Smets and Wouters (2003a, 2003b, 2003c) the empirical performance of the above models may be evaluated by comparing their (log) marginal densities to the (log) marginal density of a BVAR model. The idea here is to compare the structural models to a less constrained statistical model. Logged marginal densities of structural models and BVAR models are respectively reported in Table 27. The priors of our BVAR(p) models are defined as in Sims (2002). The best BVAR model (with lag order two) is far beaten by all the structural models.

## 4 Interdependence of US and euro area business cycle

### 4.1 Propagation of shocks

Figures 47 to 90 present the impulse response functions of the benchmark model with parameters at the mode. The main features of the IRFs depend on the typology of shocks.

#### Efficient supply shocks

Productivity and labor supply shocks raise the natural output of the domestic economy, creating a slack in resource use, and call for real depreciation in order for demand to absorb the excess supply. Monetary policy accommodates those shocks by decreasing interest rates. Exchange rate overshoots, depreciating on impact and then gradually appreciating. Current account increases slightly as the relative price effect overcomes the income effect.

Spill-over to the foreign economy of those shocks is a priori ambiguous with conflicting relative price and income effects. Spill-over is negative on activity following a US shock and positive following euro area shock. However, the size of the international transmission is limited.

Note that, due to a high degree of home bias, the “Marshall-Lerner” condition holds even if the intratemporal elasticity of substitution is close to one. However, as trade volumes react immediately to relative prices, the current account does not exhibit J-curve profile after a relative price shock. Adding adjustment costs on trade flows would circumvent this drawback. However, as our model is not estimated on trade data, not incorporating this additional friction should not alter significantly the performance of the model.

#### Demand shocks

Preference and public spending shocks increase the output gap and requires real appreciation so that lower external demand counterbalances excess domestic demand. Indeed monetary policy appears to lean against these shocks by increasing interest rate. Exchange rate overshoots, appreciating on impact and then gradually depreciating. Current account records a deficit given that both relative output and relative price effects worsen the external position.

Ex post demand multipliers on economic activity between the US and the euro area are close to 0.05 – 0.1 in the first three years.

#### Markup shocks

Markup shocks on the products and labor markets induce a tradeoff for monetary policy by pushing output gap and inflation in opposite directions. Considering the product market markup shock, domestic monetary policy (described by a Taylor rule) is slightly restrictive on impact but then becomes accommodative after one year. Nominal exchange rate depreciates slowly for two years and the current account decreases slightly as the impact of the cost-push shock on competitiveness dominates in the short run. Under those markup shocks, international correlation of output is negative and relatively small.

#### Monetary policy shocks

The open economy framework adds the exchange rate channel to the monetary policy transmission mechanism. Interest rate increases trigger an instantaneous appreciation of the exchange rate. Both external and domestic demand decreases. The net impact on current account is negative.

Overall, ex post 100 basis points hike in interest rate decreases domestic output by 0.8 and foreign output by 0.08 during the first year. Exchange rate appreciates by around 2%.



## Open economy shocks

The UIP and the home bias shocks have both strong impact on exchange rate and current account but imply correlation of opposite signs.

The UIP shock leads to a strong appreciation of the nominal exchange rate on impact and a decrease in domestic interest rate. Over the first quarters, domestic output contract and foreign output expands by a similar amount. Net impact on activity is driven by external demand in short run. Consequently current account records a relatively high deficit.

The home bias shock is equivalent to an asymmetric world demand shock. Domestic output expands, domestic interest rate rises, the exchange rate appreciates strongly and the current account increases. Macro variables in the foreign country mirror these developments on the negative side.

Mixing UIP and home bias shocks can generate “exchange rate disconnect” as they have similar impact on exchange rate but opposite impact on current account.

## 4.2 Variance decomposition

As in the closed economy case, unexpected short run output fluctuations are explained by domestic demand shocks in the near term and domestic supply shocks in the long run. In the US, long term contribution of investment shocks and monetary shocks are higher than in the euro area where productivity shocks matter much more. Open economy shocks contribute to 15% of forecast errors during the first year but this contribution decreases dramatically over longer period.

Over the medium term, half of inflation rate forecast errors come from markup domestic fluctuations. Domestic supply factors and monetary policy (and UIP shock for the CPI inflation) explain the remaining part.

As already mentioned in the last section, international spill-overs are quite small. Therefore, the contributions of domestic shocks on the foreign economy are limited. The highest spill-over concerns the US investment shock on euro area output.

Concerning the nominal exchange rate change, the UIP shock contributes up to 60% and the home bias shock explains 25%. The remaining marginal contributors are monetary policy and efficient supply shocks. We should not conclude from this decomposition that relative prices are marginally driven by supply shocks on the long run. Indeed performing this decomposition for the real exchange rate increases substantially the contribution of “fundamental” shocks.

Turning now to the current account, the home bias shock explains 60% of the forecast errors and the UIP shock provides most of the remaining part. Those two shocks are key to explain the current account and the nominal exchange rate in our framework. The importance of the risk premium shock on the current account is in line with the finding of Bergin (2004). However, the very high share of the home bias shock is quite disappointing. However, taking the US current account to proxy the bilateral net foreign assets accumulation between the US and the euro area may lead to such results. Sensitivity analysis using bilateral net trade or euro area net trade will be performed.

### 4.3 Correlation of structural shocks

Our benchmark model assumes independent structural shocks. This assumption resulted in very low contribution of foreign shocks to domestic fluctuations.

#### Common versus asymmetric shocks

Therefore, we estimate a version of the model where common components are introduced on all type of shocks. The relevant common factors appear for productivity, preference, investment, CPI markup and monetary policy shocks. This model improves on the benchmark model as far as marginal density is concerned (see Table 27).

Tables 14 to 16 report the dynamic variance decomposition. Concerning output, common factors contribute up to 15% on all horizon for both the US and euro area. Common shock on CPI markups only explains CPI inflation rates and may be linked to oil prices and third country export prices.

#### Correlation of efficient shocks with the UIP shock

As the risk premium shock to the UIP relation has less structural foundations, we introduce the possibility of non-zero correlation between the UIP shock and the efficient shocks. Results suggest that allowing less restrictive covariance matrix of structural innovations improves the performance of the model.

In addition, signs of correlation are interesting: a UIP shock leading to an appreciation of the dollar vis-à-vis the euro, is associated with a positive technological shock in the US. Conversely, it is associated with a negative preference in the US. The UIP shock is also positively correlated to public spending and labor supply shocks in the US and negatively in the euro area.

## 4 Conclusion

This paper aims at estimating a fully-fledged DSGE for the US and the euro area. Using Bayesian techniques, we show that a two-country model incorporating the relevant shocks and frictions identified in both the closed and open economy literature performs relatively well compared with BVAR models.

The estimated model is likely to provide a valuable input for calibration exercise in the theoretical literature. As far as the US and the euro area are concerned, the intratemporal elasticity of substitution is found to be close to one. The model gives an estimated share of firms pricing in local currency and points out to asymmetries between the US and the euro area. It also provides a semi-elasticity of UIP risk premium with respect to net foreign assets.

Concerning the US - EA business cycles, open economy shocks seem to play a significant role but the estimated spill - over of domestic shock to the foreign economy is relatively small. When the structural shocks are assumed to be independent internationally, the contribution of domestic disturbances to the foreign country macroeconomic volatility is negligible. Including common components improves the model and indicates that interdependence between the US and the euro area is strengthened by correlation among structural shocks.

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## Appendix A

This section presents the linearized equations. As emphasized earlier, we consider a symmetric steady state where macroeconomic aggregates and relative prices are equalized across countries. In particular, external accounts are balanced (i.e.  $B_F = B_F^* = 0$ ).

We denote  $X^R = (X - X^*)/2$  and  $X^W = (X + X^*)/2$ .

**Euler equations:**

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1} - \frac{1-h}{\sigma_c(1+h)} (r_t - E_t \pi_{t+1}) + \frac{1-h}{\sigma_c(1+h)} e_t^B \quad (45)$$

$$c_t^* = \frac{h^*}{1+h^*} c_{t-1}^* + \frac{1}{1+h^*} E_t c_{t+1}^* + \frac{\sigma_c^* - 1}{\sigma_c^*(1+\lambda_w^*)(1+h^*)} (l_t^* - E_t l_{t+1}^*) + \frac{1-h^*}{\sigma_c(1+h^*)} e_t^{B*} \quad (46)$$

**Wage setting:**

$$w_t = \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta} E_t w_{t+1} + \frac{\beta}{1+\beta} E_t (\pi_{t+1} - \bar{\pi}_t) - \frac{1+\beta\gamma_w}{1+\beta} (\pi_t - \bar{\pi}_t) + \frac{\gamma_w}{1+\beta} (\pi_{t-1} - \bar{\pi}_t) \\ - \frac{1}{1+\beta} \left[ \frac{(1-\beta\alpha_w)(1-\alpha_w)}{1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w}} \right] \alpha_w \left[ w_t - \sigma_L l_t - \frac{\sigma_c}{1-h} (c_t - h c_{t-1}) + e_t^L \right] + \varepsilon_t^W \quad (47)$$

$$w_t^* = \frac{1}{1+\beta} w_{t-1}^* + \frac{\beta}{1+\beta} E_t w_{t+1}^* + \frac{\beta}{1+\beta} E_t (\pi_{t+1}^* - \bar{\pi}_t^*) - \frac{1+\beta\gamma_w}{1+\beta} (\pi_t^* - \bar{\pi}_t^*) + \frac{\gamma_w}{1+\beta} (\pi_{t-1}^* - \bar{\pi}_t^*) \\ - \frac{1}{1+\beta} \left[ \frac{(1-\beta\alpha_w^*)(1-\alpha_w^*)}{1+\frac{(1+\lambda_w^*)\sigma_L^*}{\lambda_w^*}} \right] \alpha_w^* \left[ w_t^* - \sigma_L^* l_t^* - \frac{\sigma_c^*}{1-h^*} (c_t^* - h^* c_{t-1}^*) + e_t^{L*} \right] + \varepsilon_t^{W*} \quad (48)$$

**Investment equations:**

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1/\varphi}{(1+\beta)} (q_t + e_t^I) \quad (49)$$

$$i_t^* = \frac{1}{1+\beta} i_{t-1}^* + \frac{\beta}{1+\beta} E_t i_{t+1}^* + \frac{1/\varphi^*}{(1+\beta)} (q_t^* + e_t^{I*}) \quad (50)$$

where  $\varphi = \bar{S}''$  and  $\varphi^* = \bar{S}^{*''}$ .

**Real price of capital equations:**

$$q_t = -\left(r_t - E_t \pi_{t+1}\right) + \beta(1-\delta) E_t q_{t+1} + [1-\beta(1-\delta)] E_t r_{t+1}^k + \varepsilon_t^Q \quad (51)$$

$$q_t^* = -\left(r_t^* - E_t \pi_{t+1}^*\right) + \beta(1-\delta) E_t q_{t+1}^* + [1-\beta(1-\delta)] E_t r_{t+1}^{k*} + \varepsilon_t^{Q*} \quad (52)$$

**Capital accumulation equations:**

$$k_t = (1 - \delta)k_{t-1} + \delta i_{t-1} + \delta e_t^I \quad (53)$$

$$k_t^* = (1 - \delta)k_{t-1}^* + \delta i_{t-1}^* + \delta e_t^{I*} \quad (54)$$

**Aggregate production functions:**

$$z_t = \omega \alpha \left( k_{t-1} + 1/\phi r_t^k \right) + \omega(1 - \alpha)l_t + \omega e_t^A \quad (55)$$

$$z_t^* = \omega \alpha \left( k_{t-1}^* + 1/\phi^* r_t^{k*} \right) + \omega(1 - \alpha)l_t^* + \omega e_t^{A*} \quad (56)$$

where  $\phi = \frac{\Phi''(1)}{\Phi'(1)}$  is the elasticity of the capital utilisation cost function and  $\omega$  one plus the share of the fixed cost in production.

**Labour demands:**

$$l_t - k_{t-1} = - \left[ w_t - (1 + 1/\phi) r_t^k \right] \quad (57)$$

$$l_t^* - k_{t-1}^* = - \left[ w_t^* - (1 + 1/\phi^*) r_t^{k*} \right] \quad (58)$$

**Real marginal cost (deflated by the interior producer price index):**

$$mc_t = \alpha r_t^k + (1 - \alpha) w_t + (1 - n) t_t - e_t^A + \varepsilon_t^P \quad (59)$$

$$mc_t^* = \alpha r_t^{k*} + (1 - \alpha) w_t^* - (1 - n) t_t^* - e_t^{A*} + \varepsilon_t^{P*} \quad (60)$$

**Price setting for local sales:**

$$\pi_{H,t} = \frac{\beta}{1 + \beta \gamma_H} E_t \pi_{H,t+1} + \frac{\gamma_H}{1 + \beta \gamma_H} \pi_{H,t-1} + \lambda_H (mc_t + \varepsilon_t^P) \quad (61)$$

$$\pi_{F,t}^* = \frac{\beta}{1 + \beta \gamma_F^*} E_t \pi_{F,t+1}^* + \frac{\gamma_F^*}{1 + \beta \gamma_F^*} \pi_{F,t-1}^* + \lambda_F^* (mc_t^* + \varepsilon_t^{P*}) \quad (62)$$

**Export price equation for country H:**

$$\pi_{H,t}^* = (1 - \eta) \tilde{\pi}_{H,t} + \eta (\pi_{H,t} - \Delta s_t) \quad (63)$$

$$\tilde{\pi}_{H,t} = \frac{\beta}{1 + \beta \gamma_H^*} E_t \tilde{\pi}_{H,t+1} + \frac{\gamma_H^*}{1 + \beta \gamma_H^*} \tilde{\pi}_{H,t-1} + \lambda_H^* (mc_t - rer_{H,t} + \varepsilon_t^P) \quad (64)$$

**Export price equation for country F:**

$$\pi_{F,t} = (1 - \eta^*) \tilde{\pi}_{F,t} + \eta^* (\pi_{F,t}^* + \Delta s_t) \quad (65)$$

$$\tilde{\pi}_{F,t} = \frac{\beta}{1 + \beta \gamma_F} E_t \tilde{\pi}_{F,t+1} + \frac{\gamma_F}{1 + \beta \gamma_F} \tilde{\pi}_{F,t-1} + \lambda_F (mc_t^* - rer_{F,t} + \varepsilon_t^{P*}) \quad (66)$$

### CPI inflation rates

$$\pi_t = \frac{\beta}{1 + \beta\gamma_D} E_t \pi_{t+1} + \frac{\gamma_D}{1 + \beta\gamma_D} \pi_{t-1} + \lambda_D \left( n t_{H,t} + (1-n) t_{F,t} + \varepsilon_t^{CPI} \right) \quad (67)$$

$$\pi_t^* = \frac{\beta}{1 + \beta\gamma_D^*} E_t \pi_{t+1}^* + \frac{\gamma_D^*}{1 + \beta\gamma_D^*} \pi_{t-1}^* + \lambda_D^* \left( n t_{F,t}^* + (1-n) t_{H,t}^* + \varepsilon_t^{CPI*} \right) \quad (68)$$

### Market clearing conditions on goods markets:

$$z_t = n \left[ c_y c_t + i_y i_t + \varepsilon_t^G \right] + (1-n) \left[ c_y c_t^* + i_y i_t^* + \varepsilon_t^{G*} \right] + \xi \left( n t_{H,t} + (1-n) t_{H,t}^* \right) + n e_t^{\Delta n} \quad (69)$$

$$z_t^* = (1-n) \left[ c_y c_t + i_y i_t + \varepsilon_t^G \right] + n \left[ c_y c_t^* + i_y i_t^* + \varepsilon_t^{G*} \right] - \xi \left( n t_{F,t}^* + (1-n) t_{F,t} \right) - n e_t^{\Delta n} \quad (70)$$

### Modified uncovered interest rate parity

$$E_t \Delta s_{t+1} = i_t - i_t^* + \chi b_t + e_t^{\Delta s} \quad (71)$$

### Net foreign assets dynamics

$$\beta b_t = b_{t-1} - 2(1-n) \left[ c_y c_t^R + i_y i_t^R + e_t^{GR} - rer_t/2 \right] + (\xi - 1) \left( n t_{H,t} + (1-n) t_{H,t}^* \right) + n e_t^{\Delta n} \quad (72)$$

### Definition of relative prices:

$$t_t = t_{t-1} + \pi_{F,t} - \pi_{H,t} \quad (73)$$

$$t_t^* = t_{t-1}^* + \pi_{F,t}^* - \pi_{H,t}^* \quad (74)$$

$$t_{H,t} = t_{H,t-1} + \pi_{H,t} - \pi_t \quad (75)$$

$$t_{F,t} = t_{F,t-1} + \pi_{F,t} - \pi_t \quad (76)$$

$$t_{H,t}^* = t_{H,t-1}^* + \pi_{H,t}^* - \pi_t^* \quad (77)$$

$$t_{F,t}^* = t_{F,t-1}^* + \pi_{F,t}^* - \pi_t^* \quad (78)$$

$$rer_{H,t} = rer_{H,t-1} + \Delta s_t + \pi_{H,t}^* - \pi_{H,t} \quad (79)$$

$$rer_{F,t} = rer_{F,t-1} + \pi_{F,t} - \Delta s_t - \pi_{F,t}^* \quad (80)$$

$$rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \quad (81)$$

**Monetary policy rules:**

$$r_t = \bar{\pi}_{t-1} + \rho(r_{t-1} - \bar{\pi}_{t-1}) + (1 - \rho) \left[ r_\pi(\pi_{t-1}^{GDP} - \bar{\pi}_{t-1}) + r_Y(z_{t-1} - \bar{z}_{t-1}) \right] \\ + r_{\Delta\pi} \left[ (\pi_t^{GDP} - \bar{\pi}_t) - (\pi_{t-1}^{GDP} - \bar{\pi}_{t-1}) \right] + r_{\Delta y} \left[ (z_t - \bar{z}_t) - (z_{t-1} - \bar{z}_{t-1}) \right] + \varepsilon_t^R \quad (82)$$

$$r_t^* = \bar{\pi}_{t-1}^* + \rho^*(r_{t-1}^* - \bar{\pi}_{t-1}^*) + (1 - \rho^*) \left[ r_\pi^*(\pi_{t-1}^{GDP*} - \bar{\pi}_{t-1}^*) + r_{\Delta Y}^*(z_{t-1}^* - \bar{z}_{t-1}^*) \right] \\ + r_{\Delta\pi}^* \left[ (\pi_t^{GDP*} - \bar{\pi}_t^*) - (\pi_{t-1}^{GDP*} - \bar{\pi}_{t-1}^*) \right] + r_{\Delta y}^* \left[ (z_t^* - \bar{z}_t^*) - (z_{t-1}^* - \bar{z}_{t-1}^*) \right] + \varepsilon_t^{R*} \quad (83)$$

**Preferences and technology shocks:**

$$e_t^A = \rho_A e_{t-1}^A + \varepsilon_t^A, \quad e_t^{A*} = \rho_A^* e_{t-1}^{A*} + \varepsilon_t^{A*}$$

$$e_t^B = \rho_B e_{t-1}^B + \varepsilon_t^B, \quad e_t^{B*} = \rho_B^* e_{t-1}^{B*} + \varepsilon_t^{B*}$$

$$e_t^G = \rho_G e_{t-1}^G + \varepsilon_t^G, \quad e_t^{G*} = \rho_G^* e_{t-1}^{G*} + \varepsilon_t^{G*}$$

$$e_t^I = \rho_I e_{t-1}^I + \varepsilon_t^I, \quad e_t^{I*} = \rho_I^* e_{t-1}^{I*} + \varepsilon_t^{I*}$$

$$e_t^L = \rho_L e_{t-1}^L + \varepsilon_t^L, \quad e_t^{L*} = \rho_L^* e_{t-1}^{L*} + \varepsilon_t^{L*}$$

$$e_t^{\Delta n} = \rho_{\Delta n} e_{t-1}^{\Delta n} + \varepsilon_t^{\Delta n}$$

**Inefficient shocks:**

$$\varepsilon_t^P, \varepsilon_t^{P*}, \varepsilon_t^W, \varepsilon_t^{W*}, \varepsilon_t^Q, \varepsilon_t^{Q*}, \varepsilon_t^R, \varepsilon_t^{R*}, \varepsilon_t^{CPI}, \varepsilon_t^{CPI*}.$$

**Monetary policy shocks:**

$$\bar{\pi}_t = \rho_{\bar{\pi}} \bar{\pi}_{t-1} + \varepsilon_t^{\bar{\pi}}, \quad \bar{\pi}_t^* = \rho_{\bar{\pi}}^* \bar{\pi}_{t-1}^* + \varepsilon_t^{\bar{\pi}*}$$

$$\varepsilon_t^R, \varepsilon_t^{R*}$$

**UIP shock:**

$$e_t^{\Delta s} = \rho_{\Delta s} e_{t-1}^{\Delta s} + \varepsilon_t^{\Delta s}$$