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***Aggregating Phillips Curves***  
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# Aggregating Phillips Curves\*

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## Abstract

The New Keynesian Phillips Curve is at the center of two raging empirical debates. First, how can purely forward looking pricing account for the observed persistence in aggregate inflation. Second, price-setting responds to movements in marginal costs, which should therefore be the driving force to observed inflation dynamics. This is not always the case in typical estimations. In this paper, we show how heterogeneity in pricing behavior is relevant to both questions. We detail the conditions under which imposing homogeneity results in overestimating a backward-looking component in (aggregate) inflation, and, potentially underestimating the importance of (aggregate) marginal costs for (aggregate) inflation. We provide intuition for the direction of these biases, and verify them in sectoral French data. Our analytics identify the sources of heterogeneity most likely to result in aggregation bias under the two standard (homogeneous) estimators used to test the New Keynesian Phillips curve, the Generalized Method of Moments and the Maximum Likelihood approach. Our econometrics results provide a simple blueprint which, if disaggregated data are available, makes it possible to identify and correct for heterogeneity biases, and to predict their magnitudes depending on the source and extent of industry-level specificities.

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# 1 Introduction

Since it burst onto the scene of mainstream monetary economics, the New Keynesian Phillips Curve has been the focus of two important empirical debates. First, to what extent purely forward-looking pricing behavior can be reconciled with observed inflation persistence. Second, to what extent properly measured marginal costs affect inflation dynamics. Both issues are crucial. If inflation is purely forward looking, its persistence arises only from that of shocks to marginal costs (provided they matter), and perfectly anticipated changes in inflation are costless. Second, that shocks to marginal costs affect inflation is the basis of the forward-looking pricing rule profit that maximizing firms are assumed to follow. And the magnitude of the estimated relation relates directly to the extent of nominal rigidities. Both issues have recently been hotly debated, and for good reason.<sup>1</sup>

In this paper, we show that heterogeneity in the pricing behavior of firms matters for both empirical questions. If pricing is heterogeneous, any estimation that ignores the issue is flawed. We show that the direction and magnitude of the bias are not the same for marginal costs or for expected inflation, and that both may also vary with the estimator used. We derive analytical expressions for both biases, which are helpful to garner intuition on their direction and magnitude. We use simulations to assess the sensitivity of our conclusions, which we then confirm in sectoral quarterly French data on prices and marginal costs, in two ways. First, we use them to calibrate our analytical expressions for the biases. Second, we compare Phillips Curve estimates arising from standard homogeneous approaches to what is obtained when heterogeneity is allowed.

Inasmuch as it stresses a source of mis-specification, the paper informs the empirical debate surrounding the New Keynesian Phillips Curve in a general sense. Our contribution has three further implications. First, we discuss how the bias created by heterogeneity changes in magnitude depending on whether Generalized Method of Moments (GMM) or Maximum Likelihood (ML) techniques are implemented. Thus, we provide a (partial) explanation for the discrepancy in results the literature has uncovered. Second, we model heterogeneity as arising from two potential sources: the extent (and duration) of nominal rigidities and the extent of backward indexation. If only the former source of heterogeneity were present in our data, heterogeneity would only plague the aggregate Phillips Curve via marginal costs, not via lead or lagged inflation. This can be tested.

Third, our approach underlines the importance of disaggregated information to improve the structural modeling of aggregate inflation. This is related to the flurry of recent empirical evidence on disaggregate price dynamics, pioneered by [Bils and Klenow \(2005\)](#) and the series of country specific studies implemented by the European Central Bank summarized in [Altissimo et al \(2006\)](#). A conclusion drawn from this vast body of evidence seems to be that price dynamics are heterogeneous and inflation persistence could be an artefact of aggregation. More specifically, macroeconomic estimates have been widely criticized on the ground that the average duration of sticky prices is too large to make economic sense and, in particular,

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<sup>1</sup>A non exhaustive list of issues includes the model's ability to capture inflation persistence ([Fuhrer \(1997\)](#), [Fuhrer and Moore \(1995\)](#)), the plausibility of its implied dynamics ([Mankiw and Reis \(2002\)](#)), and the validity of the empirical approach. For instance, [Guay and Pelgrin \(2005\)](#), [Rudd and Whelan \(2003, 2005\)](#), [Nason and Smith \(2004\)](#) or [Lindé \(2005\)](#) cast doubt on the validity of GMM estimates. [Dufour, Khalaf, and Kichian \(2006\)](#) and [Mavroeidis \(2004\)](#) stress sensitivity to the choice of an instrument set. [Jondeau and Le Bihan \(2003\)](#) and [Kurmman \(2006\)](#) argue Maximum Likelihood estimators ought to be preferred. See the special issue of the *Journal of Monetary Economics* (2005).

is inconsistent with the results observed in microeconomic data.<sup>2</sup>

Our paper is closely related to Zaffaroni (2004) and Altissimo, Mojon, and Zaffaroni (2004). Both papers are also concerned with inflation dynamics, and apply insights on the effects of cross-sectional aggregation of heterogeneous processes that were first introduced by Robinson (1978) and Granger (1980).<sup>3</sup> Unlike them however, here we ask from a structural model what heterogeneity will do empirically. This makes it possible for us to evaluate the effect heterogeneous pricing may have on the validity of a structural model of inflation, and correct it accordingly.

Carvalho (2006) derives a generalized New Keynesian Phillips Curve in the presence of heterogeneity in the frequency of price adjustments across industries. In his calibrated model, monetary shocks have larger and more persistent effects than under homogeneity, and mimicking the data requires only shorter, more plausible, nominal rigidities. Our approach is complementary. Rather than introducing heterogeneity in a calibrated general equilibrium model, we implement the adequate econometrics to account for heterogeneity in the data. We bring the data closer to the theoretically standard homogeneous case, rather than sophisticating the theory away from the representative firm case.<sup>4</sup>

Since our data contain information on marginal costs at the industry level, we are able to aggregate theory-implied Phillips curves involving marginal cost rather than output gap, which simplifies considerably the derivations. We are able to identify alternative sources of sectoral heterogeneity, and test for their relevance. Our contribution details how, armed with sector-level data on prices and marginal costs, it is possible to back out unbiased aggregate estimates of the New Keynesian Phillips Curve, that account for possible heterogeneity in pricing behavior.

Our results suggest industry differences in the extent of nominal rigidities engender large upward biases in the aggregate, particularly on the extent of measured backward looking behavior. On the other hand, the dispersion in the extent of backward indexation has substantially less impact on aggregates. At least in French data, there is little heterogeneity bias in the estimated importance of marginal costs in driving aggregate inflation. On the other hand, heterogeneity more than doubles the importance of lagged inflation. This does not predicate what would happen in a dataset where the sources of heterogeneity were different, or with less granular information: French data display relatively little dispersion in industry-specific nominal rigidities. But our analytics will help pinpoint and address heterogeneity biases in any data.

The rest of the paper is organized as follows. In Section 2, we briefly review how to derive an expression for a sectoral Phillips Curve allowing for nominal rigidities and backward looking indexation that are both sector specific. We aggregate sectors up and obtain the now standard New Keynesian inflation dynamics, amended for heterogeneity. We then analyze a simple two sector model and explain

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<sup>2</sup>See Chari, Kehoe, and McGrattan (2000) or Dhyne et al.(2005) for an analysis of the issue as it pertains to the Euro zone.

<sup>3</sup>For more recent discussions of the effects of aggregation under heterogeneity, see Pesaran, Pierse, and Kumar (1989), Pesaran, Pierse, and Lee (1994) or Pesaran and Smith (1995). Imbs et al. (2005) applied the insights to the real exchange rate.

<sup>4</sup>Using scanner data, Midrigan (2006) shows the cross-sectional distribution of (non zero) price changes has fat tails. He argues the high moments properties of the heterogeneity in price adjustments are crucial when aggregating microeconomic rigidities in menu-costs models of macroeconomic fluctuations.

how heterogeneity matters qualitatively. In Section 3, we present the expressions that render homogeneous estimators problematic when pricing is sector specific. We provide an intuition for biases, whose magnitude and direction depend on parameter values, but also on the estimator implemented. Simulations results are discussed, which illustrate how heterogeneity matters. In Section 4, we describe the econometric methods used in the paper to deal with heterogeneity. In Section 5, we introduce our data. and discuss discrepancies between estimates implied by homogeneous and heterogeneous estimators. Section 6 concludes.

## 2 Aggregating Sectoral Phillips Curves

We first derive an expression for a sectoral Phillips Curve, where a sector is characterized by the extent of nominal rigidities and indexation to past inflation.<sup>5</sup> We then aggregate up to the country level, assuming away any cross-sectoral influences as for instance ones implied by input-output relations - just as most aggregate Phillips curves assume away international linkages. Price dynamics in each sector are assumed to respond only to the dynamics of marginal costs there.<sup>6</sup> The derivation follows directly from Woodford (2003) and Christiano, Eichenbaum, and Evans (2005).<sup>7</sup>

### 2.1 The Model

We briefly derive the New Keynesian Phillips curve for a sector  $j$ , where technology shocks, price rigidity and the extent of backward indexation are all specific to  $j$ .<sup>8</sup> Each firm  $i$  in sector  $j$  uses labor  $H_{ij,t}$  to produce a differentiated good according to the production

$$Y_{ij,t} = A_{j,t} f(H_{ij,t})$$

where  $A_{j,t}$  denotes (sector specific) labor productivity and  $f(\cdot)$  is an increasing and concave function. Assuming perfect labor mobility within sectors, cost minimization implies

$$S_{ij,t} = \frac{W_{j,t}}{P_{ij,t} A_{j,t}} \Phi(Y_{ij,t}/A_{j,t})$$

where  $S_{ij,t}$  denotes the real marginal cost for firm  $i$  in sector  $j$ ,  $\Phi(\cdot) = 1/f' [f^{-1}(\cdot)]$ , and  $W_{j,t}$  is the sectoral nominal wage.

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<sup>5</sup>Backward-looking pricing could equally be introduced using the formulation of Galí and Gertler (1999) or Galí, Gertler and Lopez-Salido (2001), without loss of generality.

<sup>6</sup>We later allow for industry shocks to be correlated across sectors. That is not quite the same as constructing an explicit model of technological linkages between sectors. Going that route while preserving the level of generality we endeavor would simply be intractable, both in theory and in empirical applications. Justiniano, Kumhof and Ravenna (2006) propose a model of specifically vertical input-output relations between industries. They calibrate their model to show appropriate linkages can account for the discrepancy between price sluggishness in the aggregate and rapid adjustment at the microeconomic level. Dupor (1999) focuses on the persistence in real quantities, and shows that, in general, input-output linkages are incapable of driving a wedge between sectoral and aggregate real output dynamics.

<sup>7</sup>We are far from the first ones to take interest in heterogeneous pricing in monetary models. Erceg and Levin (2002) characterize a sector on the demand side, focusing on differences between durable and non-durables goods. Aoki (2001), Benigno (2004) and Huang and Liu (2004) analyze the implications of sectoral heterogeneity for the design of monetary policy. Dixon and Kara (2005) study the impact of heterogeneity in the context of Taylor staggered wage setting. Bouakez, Cardia, and Ruge-Murcia (2005) construct and estimate a model with heterogenous production sectors, and show substantial heterogeneity across sectors in the degree of sectoral sensitivity to monetary policy shocks. Álvarez, Burriel, and Hernando (2005) analyze the impact of heterogeneity under a variety of different assumptions on price-setting behavior.

<sup>8</sup>We also assume sector specific price elasticity of demand, which turns out to be innocuous for price dynamics.

Monopolistic competition in each sector implies that demand faced by firm  $i$  writes

$$Y_{ij,t} = \left( \frac{P_{ij,t}}{P_{j,t}} \right)^{-\eta_j} Y_{j,t}$$

where  $\eta_j > 1$  denotes the (sector specific) elasticity of substitution across varieties. We assume price setting decisions follow a modified version of the Calvo's (1983) staggering mechanism. In addition to the baseline mechanism, we allow for the possibility that firms that do not optimally set their prices may nonetheless adjust them to keep up with the previous period increase in the general price level.<sup>9</sup> In each period, a firm faces a constant probability  $1 - \alpha_j$  of being able to re-optimize its price and chooses the price  $P_{ij,t}$  that maximizes the expected discounted sum of profits

$$E_t \sum_{k=0}^{\infty} \alpha_j^k \Delta_{t,t+k} P_{j,t+k} \left[ \left( \frac{P_{ij,t} \Psi_{t,t+k}}{P_{j,t+k}} \right)^{1-\eta_j} - \left( \frac{P_{ij,t} \Psi_{t,t+k}}{P_{j,t+k}} \right)^{-\eta_j} S_{ij,t+k} \right] Y_{j,t+k}$$

where

$$\Psi_{t,t+k} = \begin{cases} \prod_{\nu=0}^{k-1} (\pi_{ij,t+\nu})^{\xi_j} (\bar{\Pi}_j)^{1-\xi_j} & k > 0 \\ 1 & k = 0 \end{cases}$$

and  $\bar{\Pi}_j$  denotes the (sector specific) exogenous trend inflation, which we later account for through detrending. The coefficient  $\xi_j \in [0, 1]$  indicates the degree of indexation to past prices in sector  $j$ , during the periods in which firms are not allowed to re-optimize.  $\Psi_{t,t+k}$  is a correcting term accounting for the fact that, if firm  $i$  does not re-optimize its price, it updates it according to the rule  $P_{ij,t} = (\bar{\Pi}_j)^{1-\xi_j} (\pi_{ij,t-1})^{\xi_j} P_{ij,t-1}$ . When  $\xi_j = 0$ , firms mechanically impute trend inflation when setting future prices; when  $\xi_j = 1$ , realized inflation rates between  $t+v-1$  and  $t+v$  are used to choose prices in  $t+v+1$ .  $\Delta_{t,t+k} = \beta^k \mathcal{U}_C(C_{t+k}) / \mathcal{U}_C(C_t)$  is the discount factor between dates  $t$  and  $t+k$  of the representative household, where  $\mathcal{U}_C(C_t)$  is the marginal utility of consumption at date  $t$ .  $\Delta_{t,t+k}$  is constant across sectors.

The optimal price is given by

$$P_{ij,t}^* = \frac{\eta_j}{\eta_j - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha_j^k \Delta_{t,t+k} \left( \frac{P_{ij,t}^* \Psi_{t,t+k}}{P_{j,t+k}} \right)^{-1-\eta_j} \Psi_{t,t+k} Y_{j,t+k} S_{ij,t+k}}{E_t \sum_{k=0}^{\infty} \alpha_j^k \Delta_{t,t+k} \left( \frac{P_{ij,t}^* \Psi_{t,t+k}}{P_{j,t+k}} \right)^{-1-\eta_j} \frac{\Psi_{t,t+k}^2}{P_{j,t+k}} Y_{j,t+k}} \quad (1)$$

Under fully flexible prices,  $\alpha_j = 0$  and optimal pricing implies  $P_{ij,t}^* / P_{j,t} = \mu_j S_{ij,t}$  where  $\mu_j \equiv \eta_j / (\eta_j - 1)$  is the optimal markup. As there is no firm-specific shock in this economy, all firms that are allowed to re-optimize their price at date  $t$  select the same optimal price  $P_{ij,t}^* = P_{j,t}^*$ . The equilibrium is symmetric across firms in each sector. Staggered price setting under partial indexation implies the price index in sector  $j$  follows

$$P_{j,t} = \left[ \alpha_j \left[ (\Pi_{j,t-1})^{\xi_j} (\bar{\Pi}_j)^{1-\xi_j} P_{j,t-1} \right]^{1-\eta_j} + (1 - \alpha_j) (P_{j,t}^*)^{1-\eta_j} \right]^{\frac{1}{1-\eta_j}} \quad (2)$$

Log-linearizing the expressions for optimal pricing in equation (1), for aggregate prices in equation (2) and the definition of marginal costs yields

$$\pi_{j,t} = \frac{\xi_j}{1 + \beta \xi_j} \pi_{j,t-1} + \frac{\beta}{1 + \beta \xi_j} E_t \pi_{j,t+1} + \frac{(1 - \beta \alpha_j)(1 - \alpha_j)}{(1 + \beta \xi_j) \alpha_j} s_{j,t}$$

<sup>9</sup>See Sbordone (2003) or Christiano, Eichenbaum, and Evans (2005) for discussions.

where lower case variables denote log-deviations from the steady state.

To economize on notation, define  $\lambda_j^b = \xi_j/(1 + \beta\xi_j)$ ,  $\lambda_j^f = \beta/(1 + \beta\xi_j)$  and  $\theta_j = \frac{(1 - \beta\alpha_j)(1 - \alpha_j)}{(1 + \beta\xi_j)\alpha_j}$  to rewrite the Phillips Curve in its well-known hybrid form

$$\pi_{j,t} = \lambda_j^b \pi_{j,t-1} + \lambda_j^f E_t \pi_{j,t+1} + \theta_j s_{j,t} + \varepsilon_{j,t} \quad (3)$$

where we introduced an error term  $\varepsilon_{j,t}$ , which may include sectoral or aggregate shocks. In the case where the only source of sectoral heterogeneity stems from the extent of nominal rigidities  $\alpha_j$ , we have  $\xi_j = \xi$ ,  $\lambda_j^b = \lambda^b$ , and  $\lambda_j^f = \lambda^f$  for all  $j$ . Only the coefficient on marginal costs  $\theta_j$  will then be heterogeneous. This will also be true in the absence of any backward-looking indexing when  $\xi = 0$ . The Phillips Curve becomes then purely forward looking, and only the coefficient on marginal costs is sector-specific.

The industry level Phillips curve in equation (3) does not include any reference to an aggregate variable, nor indeed any relative prices. At face value, this may seem a contradiction relative to the findings in Aoki (2001), Benigno (2004) or Ghironi, Carlstrom and Fuerst (2006). But all these authors use versions of the New Keynesian Phillips curve that refer to the output gap as a measure of economic activity. In contrast, here we can refer directly to marginal costs, which, under relatively benign assumptions on the labor market, we actually observe in our data. Woodford (2003) shows that a sector-level New Keynesian Phillips curve ceases to refer to any aggregate variables, or to relative sectoral prices, when it is written in terms of marginal costs.<sup>10</sup> The intuition is clarified on page 669 of Woodford (2003), where sectoral marginal costs are shown to embed directly relative prices in general equilibrium. Ghironi et al (2006) show this to be a key ingredient of determinacy when aggregating heterogeneous Phillips curves. This result considerably simplifies the theoretical impact of aggregation, and our econometric approach in addressing heterogeneity.

## 2.2 Aggregation

We model heterogeneity as sector-specific deviations from a common mean. In particular, we assume

$$\begin{aligned} \xi_j &= \xi + \tilde{\xi}_j \\ \alpha_j &= \alpha + \tilde{\alpha}_j \end{aligned}$$

where  $\tilde{\xi}_j$  and  $\tilde{\alpha}_j$  have zero means and constant variances and covariances.<sup>11</sup> Let  $w_j$  denote the weight of sector  $j$  in the aggregate economy. Straightforward aggregation of equation (3) gives

$$\pi_t = \sum_{j=1}^J w_j \lambda_j^b \pi_{j,t-1} + \sum_{j=1}^J w_j \lambda_j^f E_t \pi_{j,t+1} + \sum_{j=1}^J w_j \theta_j s_{j,t} + \sum_{j=1}^J w_j \varepsilon_{j,t} \quad (4)$$

Our purpose in this paper is to evaluate the validity of the *standard* Phillips curve at the country level in the presence of heterogeneity at a lower level of aggregation. We seek to characterize the econometric properties of the residuals in a version of equation (4) that simplifies into

$$\pi_t = \lambda^b \pi_{t-1} + \lambda^f E_t \pi_{t+1} + \theta s_t + \bar{\varepsilon}_t \quad (5)$$

<sup>10</sup>This is developed in the Appendix B.7 to Chapter 3, and in particular in equation B.33 on page 668.

<sup>11</sup>Whether heterogeneity is random or deterministic will matter for the estimation procedure. Since this is an empirical question, we leave the discussion for later.

with  $\lambda^b = \frac{\xi}{1+\beta\xi}$ ,  $\lambda^f = \frac{\beta}{1+\beta\xi} = \beta(1 - \beta\lambda^b)$  and  $\theta = \frac{(1-\beta\alpha)(1-\alpha)}{(1+\beta\xi)\alpha}$ .

This simplification implies a specific structure of heterogeneity: we assume that linear heterogeneity at the level of the structural parameters  $\xi_j$  and  $\alpha_j$  translates into linear heterogeneity in the reduced form Phillips curve. In other words, we impose  $\lambda_j^b = \lambda^b + \tilde{\lambda}_j^b$ ,  $\lambda_j^f = \lambda^f + \tilde{\lambda}_j^f$  and  $\theta_j = \theta + \tilde{\theta}_j$ .<sup>12</sup> This is obviously not the case in general, but ours is not a paper proposing an alternative structural form to account for aggregate inflation dynamics under sector-level heterogeneity. Rather it is one that seeks to evaluate the effects of (a specific form of) heterogeneity on the empirical validity of the *standard* model. Exploration of the alternative, theoretical, route has just begun with papers by Carvalho (2006) or Justiniano et al (2006).<sup>13</sup>

Estimates of  $\lambda^b$ ,  $\lambda^f$  and  $\theta$  in equation (5) are the object of an enormous literature. Our key assumption is all three estimates differ linearly from their average (aggregate) values at the sectoral level because of different realizations of  $\tilde{\xi}_j$  and  $\tilde{\alpha}_j$ . Under this assumption, the residuals in equation (5) are given by

$$\bar{\varepsilon}_t = \sum_{j=1}^J w_j \varepsilon_{j,t} + \sum_{j=1}^J w_j \tilde{\lambda}_j^b \pi_{j,t-1} + \sum_{j=1}^J w_j \tilde{\lambda}_j^f E_t \pi_{j,t+1} + \sum_{j=1}^J w_j \tilde{\theta}_j s_{j,t} \quad (6)$$

where

$$\begin{aligned} \tilde{\lambda}_j^b &= \frac{\xi_j}{1+\beta\xi_j} - \frac{\xi}{1+\beta\xi} = \frac{\tilde{\xi}_j}{(1+\beta\xi)(1+\beta\xi_j)} \\ \tilde{\lambda}_j^f &= \frac{\beta}{1+\beta\xi_j} - \frac{\beta}{1+\beta\xi} = \frac{-\beta^2\tilde{\xi}_j}{(1+\beta\xi)(1+\beta\xi_j)} = -\beta^2\tilde{\lambda}_j^b \\ \tilde{\theta}_j &= \frac{(1-\beta\alpha_j)(1-\alpha_j)}{(1+\beta\xi_j)\alpha_j} - \frac{(1-\beta\alpha)(1-\alpha)}{(1+\beta\xi)\alpha} = -\frac{(1-\beta\alpha\alpha_j)(1+\beta\xi)\tilde{\alpha}_j + \beta(1-\alpha)\alpha_j\tilde{\xi}_j}{(1+\beta\xi_j)(1+\beta\xi)\alpha_j} \end{aligned}$$

As in Pesaran and Smith (1995) ignoring heterogeneity in equation (5) results in a residual that is inevitably correlated with the dependent variables. Instrumenting will not alleviate the pathology since good instruments are correlated with the dependent variables, and therefore will mechanically be so as well with the residuals. The result is well known in theory, and a few applications have by now been developed in macroeconomics.<sup>14</sup> The issue is particularly pressing in the present case, and not only because modeling inflation dynamics is important in and of itself. First, in a multivariate setting, heterogeneity biases may have different signs and different magnitudes on different co-variates. In the next section, we show the biases may indeed have different signs on  $\lambda^b$ ,  $\lambda^f$  and  $\theta$ . We then implement simulations exercises strongly suggestive that they also have different magnitudes. Second, equation (5) involves an expected term, which complicates substantially the approach, especially when it comes to instrumenting these expectations. In

<sup>12</sup>We also assume information is perfectly common across sectors, so that pricing decisions are taken across the whole economy on the basis of exactly the same data.

<sup>13</sup>Equation (5) is hideously non linear when heterogeneity is introduced in the most general way. Econometric methods that can account for heterogeneity under such non linearities simply do not exist, not least because heterogeneity ( $\alpha_j$  and  $\xi_j$ ) now enters the very coefficients to be estimated. The model in Carvalho (2006) is solved in the aggregate under general heterogeneity at the industry level. But it is a model - not an econometric correction of the data.

<sup>14</sup>Imbs, Mumtaz, Ravn and Rey (2005) show heterogeneity biases the estimated persistence of the real exchange rate. Canova (2006) reviews the relevance of the issue across a wide range of empirical applications in macroeconomics.



section 3, we discuss the usual ways in which the expectational term is accounted for, and relate them with the issue of parameter heterogeneity.

### 2.3 A Two-Sector Example

We illustrate the potential magnitude of heterogeneity biases in estimates of the New Keynesian Phillips Curve in the context of simulations based on a simple two-sector economy. For simplicity, we impose additional structure on the model and in particular assume marginal costs are driven by an autoregressive process of order one.<sup>15</sup> In particular, in each sector  $j$  we assume

$$\begin{aligned}\pi_{j,t} &= \lambda_j^b \pi_{j,t-1} + \lambda_j^f E_t \pi_{j,t+1} + \theta_j s_{j,t} + \varepsilon_{j,t} \\ s_{j,t} &= \rho s_{j,t-1} + u_{j,t}\end{aligned}$$

We have allowed for heterogeneity in  $\lambda_j^b$ ,  $\lambda_j^f$  and  $\theta_j$ , which the previous section showed is akin to assuming heterogeneous values for  $\xi_j$  and  $\alpha_j$ . Straightforward algebra yields a reduced form expression for sectoral inflation

$$\pi_{j,t} = \xi_j \pi_{j,t-1} + \psi_j s_{j,t} + \eta_{j,t}$$

with

$$\psi_j = \frac{1 - \beta\alpha_j}{1 - \beta\rho} \frac{1 - \alpha_j}{\alpha_j} \quad \text{and} \quad \eta_{j,t} = (1 + \beta\xi_j) \varepsilon_{j,t}$$

We use simulations to evaluate the relative impact of dispersion in the sectoral values of  $\xi_j$  and  $\alpha_j$  on the aggregate structural parameters  $\xi$  and  $\alpha$ , and the implied dynamics of aggregate inflation.

We first impose homogeneity with parameters in both sectors taking the same initial values. We choose  $\xi_j = 0.5$ ,  $\alpha_j = 0.7$ ,  $\rho = 0.9$  and  $V = \sigma_{\eta_j}^2 / \sigma_{u_j}^2 = 1$ , where  $\sigma_{\eta_j}^2$  ( $\sigma_{u_j}^2$ ) denotes the variance in  $\eta_{j,t} = (1 + \beta\xi_j) \varepsilon_{j,t}$  ( $u_{j,t}$ ). We only need to parametrize the ratio of volatilities, as we only seek to simulate the second moments of aggregate inflation, and in particular its persistence.<sup>16</sup> The subjective discount factor is set at  $\beta = 0.99$ . We introduce sector-level heterogeneity by assuming a Normal distribution from which values of  $\xi_j$  and  $\alpha_j$  are drawn, centered around their initial values, but with non zero variance. The extent of simulated heterogeneity increases with this variance, i.e. with the range from which  $\xi_j$  and  $\alpha_j$  are drawn.<sup>17</sup> Armed with sector-specific (and heterogeneous) structural parameters, we simulate inflation series according to reduced form Phillips curve (and the assumed process for marginal costs) in each sector. We then aggregate them up, using equal weights, obtain a series for aggregate inflation and aggregate marginal costs, and use them to estimate the values of  $\alpha$  and  $\xi$  with either ML or GMM. We iterate the procedure 100,000 times and report results pertaining to both aggregate estimators.<sup>18</sup>

Figures 1 and 2 report the simulated values of  $\xi$  and  $\alpha$  for values of  $\xi_j$  drawn from  $[0.25, 0.75]$ , and values of  $\alpha_j$  drawn from  $[0.45, 0.95]$ , respectively. Several results are worth mentioning. First, both Figures

<sup>15</sup>This is discussed and motivated in more details in section 3.2, where we introduce the Maximum Likelihood estimator. Appendix 1 presents a detailed derivation of the Phillips curve in reduced form under the present assumptions.

<sup>16</sup>These parameter values correspond to the unbiased estimates obtained from the French data used in this paper. The results are robust to alternative initial values. We discuss how we tackle the heterogeneity bias in Sections 4 and 5.

<sup>17</sup>We also considered heterogeneity on the autoregressive parameter  $\rho_j$  and the ratio  $\sigma_{\eta_j}^2 / \sigma_{u_j}^2$ . These played little role in the aggregate and the corresponding results are not reported for the sake of brevity. They are available upon request.

<sup>18</sup>We also experimented with asymmetric sectors, with no sizable differences.

confirm the existence of a positive bias in the aggregate estimates of  $\alpha$  and  $\xi$ . On both plots, the highest values of the aggregate structural parameters are obtained when the cross-sectoral dispersion of  $\xi_j$  and  $\alpha_j$  is maximal. Second, both Figures suggest that the aggregate parameter that is most affected by an heterogeneity bias is  $\xi$ , rather than  $\alpha$ . Figure 1 shows that dispersion in  $\xi_j$  affects substantially more the estimates of  $\xi$ ; Figure 2 shows that dispersion in  $\alpha_j$  creates barely any bias on  $\alpha$ , but continues to imply a substantial one on  $\xi$ . Third, the dispersion in  $\alpha_j$  creates a larger bias on  $\xi$  than the dispersion in  $\xi_j$ .

In words, our simulations are suggestive of an asymmetry in the manner heterogeneity biases affect estimates of the New Keynesian Phillips curve. First, it is the extent of indexation  $\xi$  that reacts most to modeled heterogeneity in price-setting behavior. Second, a given dispersion in industry specificities in the extent of nominal rigidities, measured by  $\alpha_j$  has a substantially larger effect on the bias on  $\xi$  than a comparable dispersion in  $\xi_j$ . Put differently, heterogeneity is most likely to bias estimates of  $\xi$  upwards, i.e. underestimate the extent of forward-looking behavior ( $\lambda^f$ ) and underestimate the importance of nominal rigidities ( $\theta$ ). This will happen for little dispersion in  $\alpha_j$ , but requires substantial dispersion in  $\xi_j$ . We now turn to an intuition for these asymmetries.

### 3 The Biases

In this section, we describe the biases that plague aggregate estimates of the New Keynesian Phillips Curve in the presence of unaccounted heterogeneity. We discuss the biases affecting both the coefficient on marginal costs and the coefficients on inflation. The expressions and intuitions depend on what (homogeneous) estimator is implemented on aggregate data. At this juncture, we separate our discussion in two distinct sections. We first discuss biases implied by GMM, when future expected inflation is instrumented in a standard way. The nature of the inconsistencies will depend on the correlation between the chosen instruments and the residuals. Second, we analyze heterogeneity biases when the Phillips Curve is estimated using ML estimators, and marginal costs are assumed to follow an autoregressive process.<sup>19</sup> There, no instrumentation is necessary, but the possibility of a bias subsists. Throughout, we seek to characterize the size and magnitude of the heterogeneity biases on the three dependent variables. Since our analytical results are obtained at the cost of several simplifying assumptions, we close the section with simulation exercises that confirm our conjectures.

#### 3.1 Generalized Method of Moments

The Generalized Method of Moments (GMM) estimator builds on orthogonality conditions imposed on an amended version of equation (5) where expected inflation is replaced by its effectively observed value. In particular, identification requires that the instrument set  $Z_t$  be uncorrelated with the residuals  $v_{t+1} = \bar{\varepsilon}_t - \lambda^f (\pi_{t+1} - E_t \pi_{t+1})$  in

$$\pi_t = \lambda^b \pi_{t-1} + \lambda^f \pi_{t+1} + \theta s_t + v_{t+1}$$

The moment conditions write

$$E \left[ (\pi_t - \lambda^b \pi_{t-1} - \lambda^f \pi_{t+1} - \theta s_t) Z_t \right] = 0$$

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<sup>19</sup>Recent work has suggested this approach might be more accurate. See Kurmann (2006).

$Z_t$  is a vector of instruments dated  $t$  or earlier.

The choice of  $Z_t$  is an important point of debate. Galí and Gertler (1999) use four lags each of inflation, the labor income share (which measures marginal costs), the output gap, the long-short interest rate spread, wage inflation and commodity price inflation. Galí, Gertler, and Lopez-Salido (2001) choose a smaller number of lags for instruments other than inflation, in order to minimize the potential estimation bias that arises in small samples. Their instrument set reduces to four lags of inflation, and two lags each of the output gap, wage inflation and the labor income share. Heterogeneity is an issue for GMM estimators because the residual in equation (5) correlates with the dependent variables, and in particular with  $s_t$  and  $\pi_{t-1}$  as shown in equation (6).<sup>20</sup> In other words,  $s_t$  and  $\pi_{t-1}$  are *always* prominent in the instrument sets proposed in the literature, and they are indeed the ones at the core of an heterogeneity bias under GMM. Including further lags of inflation, of marginal costs or alternative instruments will not alter the intuition. In addition, since there are two structural parameters to estimate, two moment conditions are sufficient for identification. In what follows, we focus on the special case  $Z_t = \{s_t, \pi_{t-1}\}$ .<sup>21</sup>

The GMM estimator of  $(\lambda^b, \theta)'$  is given by the two moment conditions

$$\begin{aligned} E \left[ \pi_{t-1} \left( \pi_t - \lambda^b \pi_{t-1} - \lambda^f \pi_{t+1} - \theta s_t \right) \right] &= 0 \\ E \left[ s_t \left( \pi_t - \lambda^b \pi_{t-1} - \lambda^f \pi_{t+1} - \theta s_t \right) \right] &= 0 \end{aligned}$$

From equation (5), we know  $\lambda^f = \beta(1 - \beta\lambda^b)$ , which confirms two moment conditions are enough to achieve identification. Substituting,

$$\begin{aligned} E \left[ \pi_{t-1} \left( \pi_t - \beta\pi_{t+1} - \lambda^b (\pi_{t-1} - \beta^2\pi_{t+1}) - \theta s_t \right) \right] &= 0 \\ E \left[ s_t \left( \pi_t - \beta\pi_{t+1} - \lambda^b (\pi_{t-1} - \beta^2\pi_{t+1}) - \theta s_t \right) \right] &= 0 \end{aligned}$$

Under heterogeneity however, the residual  $\bar{\varepsilon}_t$  is not orthogonal to the instruments and the estimates are inconsistent. Simple algebraic manipulation implies expressions for the probability limits of the heterogeneity biases on all the structural estimates. In particular, in probability limits, the asymptotic bias on each estimator is given by

$$\begin{aligned} \Lambda \text{plim}(\lambda_{GMM}^b - \lambda^b) &= E(s_t^2) E(\pi_{t-1}\bar{\varepsilon}_t) - E(s_t\pi_{t-1}) E(s_t\bar{\varepsilon}_t) \\ \Lambda \text{plim}(\theta_{GMM} - \theta) &= E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) E(s_t\bar{\varepsilon}_t) - E(s_t(\pi_{t-1} - \beta^2\pi_{t+1})) E(\pi_{t-1}\bar{\varepsilon}_t) \end{aligned} \quad (7)$$

where we made use of the fact that  $\tilde{\lambda}_j^f = -\beta^2\tilde{\lambda}_j^b$  in computing

$$\bar{\varepsilon}_t = \sum_{j=1}^J w_j \varepsilon_{j,t} + \sum_{j=1}^J w_j \tilde{\lambda}_j^b (\pi_{j,t-1} - \beta^2\pi_{j,t+1}) + \sum_{j=1}^J w_j \tilde{\theta}_j s_{j,t}$$

and

$$\Lambda = E(s_t^2) E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) - E(s_t\pi_{t-1}) E(s_t(\pi_{t-1} - \beta^2\pi_{t+1}))$$

<sup>20</sup>The aggregate error term  $\bar{\varepsilon}_t$  actually involves sectoral values of lagged inflation and marginal costs, and so the bias also depends on the correlation between aggregate instruments and their sectoral components.

<sup>21</sup>In fact, this is actually the optimal set of instruments. See Nason and Smith (2004).

The bias on the degree of indexation  $\xi$  is given by

$$\Lambda (\xi_{GMM} - \xi) = (1 + \beta\hat{\xi}) (1 + \beta\xi) (\lambda_{GMM}^b - \lambda^b)$$

so that the biases on  $\xi_{GMM}$  and  $\lambda_{GMM}^b$  have the same sign. We observe that  $\Lambda$  is positive for large enough values of  $\beta$  and as soon as  $E(s_t\pi_{t-1})$  and  $E(s_t\pi_{t+1})$  are non negative. Both conditions are typically satisfied in macroeconomic data, and in particular in the French case we observe.<sup>22</sup> The signs of the asymptotic biases on the structural parameters are therefore given by the right-hand side of equations (7). We now turn to their characterization.

In order that we can sign the biases, we maintain four simplifying assumptions. (H1) Heterogeneity is deterministic. (H2) The weights of all sectors in the economy are exogenous and uncorrelated with the magnitude of sector-specific estimates of the Phillips curve. (H3) Sector-specific shocks are independent. (H4) Marginal costs follow a (potentially sector-specific) autoregressive process of order one. Our estimations later relax all four hypotheses; our purpose now is to obtain tractable expressions for all biases, at the cost of relatively benign assumptions. Under these assumptions, the sign of the bias affecting estimates for  $\lambda^b$  and  $\theta$  are given in Proposition 1.

**Proposition 1** *Assume that H1, H2, H3 and H4 all hold. Then*

- *The asymptotic bias of  $(\lambda_{GMM}^b - \lambda^b)$  can be decomposed into the sum of three terms:*

$$\begin{aligned} A_1 &= \frac{1}{\Lambda} E(s_t^2) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(\pi_{t-1}(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) \\ A_2 &= -\frac{1}{\Lambda} E(s_t\pi_{t-1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(s_t(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) \\ A_3 &= \frac{1}{\Lambda} E(s_t^2) \sum_{j=1}^J w_j \tilde{\theta}_j E(\pi_{t-1}s_{j,t}) - \frac{1}{\Lambda} E(s_t\pi_{t-1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(s_t s_{j,t}) \end{aligned}$$

where, as  $\beta \rightarrow 1$  and  $J \rightarrow \infty$ ,

$$A_1 > 0, \quad A_2 > 0, \quad A_3 > 0$$

so that  $\lambda_{GMM}^b$  and  $\xi_{GMM}$  tend to over-estimate their true values.

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<sup>22</sup>On average,  $E(s_t\pi_{t-1})$  equals 1.18 and  $E(s_t\pi_{t+1})$  equals 21.10 in our data.

- The asymptotic bias of  $(\theta_{GMM} - \theta)$  can be decomposed into the sum of three terms

$$\begin{aligned}
B_1 &= \frac{1}{\Lambda} E(\pi_{t-1}^2 - \beta^2 \pi_{t-1} \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(s_t (\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\
B_2 &= -\frac{1}{\Lambda} E(s_t \pi_{t-1} - \beta^2 s_t \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(\pi_{t-1} (\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\
B_3 &= \frac{1}{\Lambda} E(\pi_{t-1}^2 - \beta^2 \pi_{t-1} \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(s_t s_{j,t}) \\
&\quad - \frac{1}{\Lambda} E(s_t \pi_{t-1} - \beta^2 s_t \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(\pi_{t-1} s_{j,t})
\end{aligned}$$

where, as  $\beta \rightarrow 1$  and  $J \rightarrow \infty$ ,

$$B_1 < 0, B_2 > 0, B_3 > 0.$$

□

Proof: See Appendix 2.

From our simple simulations, we know that a quantitatively important source of bias is the dispersion in  $\alpha_j$ , and the end effect falls mostly on aggregate estimates of  $\xi$ . Translated into analytical terms, this suggests  $\xi_{GMM} - \xi$  should be larger than  $\theta_{GMM} - \theta$ , which is likely given that the former is always positive whereas the latter has ambiguous sign. What is more, the importance of heterogeneity in  $\alpha_j$  in our simulations suggests  $A_3$  and  $B_3$  should be the main sources of the biases. Indeed, in both cases, the biases decompose into correlation terms involving  $\tilde{\lambda}_j^b$  on the one hand ( $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$ ), and terms involving  $\tilde{\theta}_j$  on the other ( $A_3$  and  $B_3$ ). By definition, the heterogeneity in  $\alpha_j$  only affects  $\tilde{\theta}_j$ .

Our simulations therefore suggest  $A_3$  (and to a lesser extent  $B_3$ ) should be the main source of the heterogeneity bias. In order to garner an intuition in economic terms, it is useful to think of heterogeneity across sectors in the terminology introduced by Angeloni et al (2005). High values of  $A_1$  require that industries with high intrinsic inflation persistence (high realizations of  $\tilde{\lambda}_j^b$ ) also have large inflation variance, but low inflation persistence. Given an industry-level Phillips curve, this requires either that extrinsic inflation be particularly high (realizations of  $\tilde{\theta}_j$  be high), or  $E s_{j,t} \pi_{j,t}$  be large, or both. It is otherwise impossible to achieve simultaneously high inflation variance, low inflation persistence and high  $\tilde{\lambda}_j^b$ .

High values of  $A_2$  in turn require that industries with high intrinsic inflation also be ones with high values of  $E s_{j,t} \pi_{j,t+1}$  but low values of  $E s_{j,t} \pi_{j,t-1}$ . As such, this requires rather convoluted inflation dynamics. In addition, we note a potential contradiction in that large values for  $A_1$  require for the cross-correlogram between marginal costs and inflation to reach a peak for contemporaneous observations. But large values of  $A_2$  require low values for leads and lags of inflation. This complicates further the requirement on the cross-dynamics of  $s_{j,t}$  and  $\pi_{j,t}$ .

Large values for  $A_1$  and  $A_2$  may therefore only occur under relatively constrained sets of requirements, with restrictions on both the extents of intrinsic and extrinsic inflation at the industry level. In contrast,  $A_3$  will tend to take high values naturally. Consider the first term in  $A_3$ . Industries with high extrinsic

inflation will be ones with large realizations of  $\tilde{\theta}_j$ ; but this means by definition that inflation and marginal cost tend to be highly correlated, i.e.  $E(\pi_{j,t-1}s_{j,t})$  tends to take high values. For a given set of weights  $w_j$ , this acts to increase the weighted average  $\sum_{j=1}^J w_j \tilde{\theta}_j E(\pi_{t-1}s_{j,t})$ . On the other hand, the last term of  $A_3$  is likely to be negligible as the variance of the marginal cost is not under our assumptions at all related to  $\tilde{\theta}_j$ .

The exact same reasoning applies to  $\theta_{GMM} - \theta$ . In particular,  $B_3$  will also tend to be the dominant (and positive) component of the bias.<sup>23</sup> But since  $B_1$  is negative, the overall heterogeneity bias on  $\theta$  is likely to be smaller than  $\xi_{GMM} - \xi$ . Finally, note that if  $(\theta_{GMM} - \theta)$  and  $(\xi_{GMM} - \xi)$  happen to have the same sign, the GMM bias on  $\alpha$  must be of the opposite sign since  $\theta = (1 - \beta\alpha)(1 - \alpha) / [(1 + \beta\xi)\alpha]$  decreases in  $\alpha$ . If they have opposite signs, on the other hand, the bias on the GMM estimate of  $\alpha$  may be of either sign.

### 3.2 Maximum Likelihood

The GMM estimator requires an instrument set for expected inflation. An alternative, introduced for instance by Fuhrer and Moore (1995), Kurmann (2005), and Sbordone (2001, 2003) assumes a data generating process for marginal costs and implements a Maximum Likelihood estimator (ML) to back out the model's estimated coefficients. Under the additional hypothesis, it becomes possible to solve future expected inflation out of the Phillips curve, and obtain a model that can be brought to the data directly. We now consider the role of heterogeneity under this alternative estimation approach.<sup>24</sup>

For simplicity, we will assume marginal costs are generated by an autoregressive process of order one. The derivations that follow become substantially more complicated under more sophisticated autoregressive models, but there is no fundamental reason why the intuition we develop should be altered. In addition, the assumption does not differ from the existing literature.<sup>25</sup> The full model of inflation then rests on the following system

$$\begin{aligned}\pi_{j,t} &= \lambda_j^b \pi_{j,t-1} + \lambda_j^f E_t \pi_{j,t+1} + \theta_j s_{j,t} + \varepsilon_{j,t} \\ s_{j,t} &= \rho_j s_{j,t-1} + u_{j,t}\end{aligned}\tag{9}$$

where  $u_{j,t}$  denotes an independent and identically distributed shock to real marginal costs in sector  $j$ ,  $|\rho_j| < 1$ ,  $\sigma_{\varepsilon_j}^2 = E(\varepsilon_{j,t}^2)$  and  $\sigma_{u_j}^2 = E(u_{j,t}^2)$ . We have assumed marginal costs have similar autoregressive properties across sectors, but the persistence coefficient may differ. Appendix 1 shows the dynamics of inflation rewrite

$$\pi_{j,t} = \xi_j \pi_{j,t-1} + \psi_j s_{j,t} + \eta_{j,t}\tag{10}$$

with

$$\psi_j = \frac{1 - \beta\alpha_j}{1 - \beta\rho_j} \frac{1 - \alpha_j}{\alpha_j} = \frac{1 + \beta\xi_j}{1 - \beta\rho_j} \theta_j \quad \text{and} \quad \eta_{j,t} = (1 + \beta\xi_j) \varepsilon_{j,t}.$$

<sup>23</sup>To see this, note that  $E(s_t \pi_{t-1} - \beta^2 s_t \pi_{t+1}) < 0$ . See Appendix 2 for proofs.

<sup>24</sup>Our signing of the heterogeneity bias under GMM actually required that we assume a functional form for the dynamics of marginal costs. But the expressions for the heterogeneity bias themselves did not require the assumption.

<sup>25</sup>See for instance Kurmann (2005), Sbordone (2001, 2003) or Mavroeidis (2005).

As before, imposing homogeneity on an aggregated Phillips curve will force heterogeneity into the residual, and thus result in inconsistency in parameter estimates. We continue to assume that aggregation preserves the linearity property in heterogeneity; in particular, we assume the aggregate Phillips curve is true on average whenever  $\xi_j = \xi + \tilde{\xi}_j$ ,  $\alpha_j = \alpha + \tilde{\alpha}_j$  and  $\rho_j = \rho + \tilde{\rho}_j$ . Then we have

$$\begin{aligned}\pi_t &= \xi \pi_{t-1} + \psi s_t + \bar{\eta}_t \\ s_t &= \rho s_{t-1} + \bar{u}_t\end{aligned}\tag{11}$$

where  $\psi = \frac{1-\beta\alpha}{1-\beta\rho} \frac{1-\alpha}{\alpha}$  is the average value of the coefficient because of our maintained assumption on the nature of heterogeneity. In particular, we have  $\psi_j = \psi + \tilde{\psi}_j$  and

$$\tilde{\psi}_j = \frac{-(1-\beta\alpha\alpha_j)(1-\beta\rho)\tilde{\alpha}_j + \beta(1-\alpha)\alpha_j\tilde{\rho}_j}{(1-\beta\rho_j)(1-\beta\rho)\alpha\alpha_j}$$

As before, the residuals embed the dependent variables, since

$$\begin{aligned}\bar{\eta}_t &= \sum_{j=1}^J w_j \eta_{j,t} + \sum_{j=1}^J w_j \tilde{\xi}_j \pi_{j,t-1} + \sum_{j=1}^J w_j \tilde{\psi}_j s_{j,t} \\ \bar{u}_t &= \sum_{j=1}^J w_j u_{j,t} + \sum_{j=1}^J w_j \tilde{\rho}_j s_{j,t-1}\end{aligned}$$

Orthogonality conditions impose that the residuals should verify

$$\begin{aligned}E[(\pi_t - \xi_{ML} \pi_{t-1} - \psi_{ML} s_t) \pi_{t-1}] &= 0 \\ E[(\pi_t - \xi_{ML} \pi_{t-1} - \psi_{ML} s_t) s_t] &= 0\end{aligned}$$

where  $\xi^M$  and  $\psi^M$  denote ML estimates. The nature of  $\bar{\eta}_t$  under heterogeneity will induce biases in potentially all the coefficients in the Phillips curve. In probability limits, these biases write

$$\begin{aligned}\tilde{\Lambda} \text{plim}(\xi_{ML} - \xi) &= E(s_t^2) E(\pi_{t-1} \bar{\eta}_t) - E(s_t \pi_{t-1}) E(s_t \bar{\eta}_t) \\ \tilde{\Lambda} \text{plim}(\psi_{ML} - \psi) &= E(\pi_{t-1}^2) E(s_t \bar{\eta}_t) - E(s_t \pi_{t-1}) E(\pi_{t-1} \bar{\eta}_t) \\ \text{plim}(\rho_{ML} - \rho) &= E(s_{t-1} \bar{u}_t) / E(s_{t-1}^2)\end{aligned}$$

where

$$\tilde{\Lambda} = E(s_t^2) E(\pi_{t-1}^2) - (E(s_t \pi_{t-1}))^2$$

Since  $\tilde{\Lambda} > 0$ , the signs of the asymptotic biases are given by the right-hand side expression. Under assumptions H1-H4, the sign of the bias affecting each parameter is given in Proposition 2.

**Proposition 2** *Under H1-H4,*

- *The asymptotic bias of  $(\xi_{ML} - \xi)$  can be decomposed into the sum of two terms:*

$$\begin{aligned}C_1 &= C_{11} + C_{12} = \frac{E(s_t^2)}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\xi}_j E(\pi_{t-1} \pi_{j,t-1}) - \frac{E(s_t \pi_{t-1})}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\xi}_j E(s_t \pi_{j,t-1}) \\ C_2 &= C_{21} + C_{22} = \frac{E(s_t^2)}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\psi}_j E(\pi_{t-1} s_{j,t}) - \frac{E(s_t \pi_{t-1})}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\psi}_j E(s_t s_{j,t})\end{aligned}$$

where

$$\begin{aligned} C_{11} &> 0 & C_{12} < 0, \\ C_{21} &> 0 & C_{22} < 0. \end{aligned}$$

- The asymptotic bias of  $(\psi_{ML} - \psi)$  can be decomposed into the sum of two terms:

$$\begin{aligned} D_1 &= D_{11} + D_{12} = \frac{E(\pi_{t-1}^2)}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\xi}_j E(s_t \pi_{j,t-1}) - \frac{E(s_t \pi_{t-1})}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\xi}_j E(\pi_{t-1} \pi_{j,t-1}) \\ D_2 &= D_{21} + D_{22} = \frac{E(\pi_{t-1}^2)}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\psi}_j E(s_t s_{j,t}) - \frac{E(s_t \pi_{t-1})}{\tilde{\Lambda}} \sum_{j=1}^J w_j \tilde{\psi}_j E(\pi_{t-1} s_{j,t}) \end{aligned}$$

where

$$\begin{aligned} D_{11} &> 0 & D_{12} < 0, \\ D_{21} &> 0 & D_{22} < 0. \end{aligned}$$

- The asymptotic bias of  $(\rho_{ML} - \rho)$ , given by

$$\rho_{ML} - \rho = \sum_{j=1}^J w_j \tilde{\rho}_j E(s_t s_{j,t}) / E(s_{t-1}^2),$$

is positive.  $\square$

- *Proof: See Appendix 2.*

The biases on both reduced form coefficients have ambiguous signs. However, it is possible to conjecture the importance of their respective components. Once again, our simulation results suggest  $\xi_{ML} - \xi$  should be largest, and in addition should arise mostly from heterogeneity in  $\alpha_j$ . In terms of the analytical biases, this means relatively large values for  $C_2$ , since the heterogeneity in nominal rigidities only affects  $\tilde{\psi}_j$ . Why is  $C_1$  relatively small compared with  $C_2$ ?

Using again the terminology introduced by Angeloni et al (2005), industries with high extrinsic inflation (high realizations of  $\tilde{\psi}_j$ ) will naturally tend to display high correlations between marginal costs and inflation. This acts to increase the value of  $C_{21}$ . In contrast, under the Maximum Likelihood assumptions on marginal costs, there is not reason to expect extrinsic inflation to correlate in any systematic manner with the variance in marginal costs, which only depends on its autoregressive properties.  $C_{22}$  should ceteris paribus take low values. This accounts for the fact that  $C_2$  will tend to take positive, and large, values.

Similarly, industries with high intrinsic inflation (high realizations of  $\tilde{\xi}_j$ ) will ceteris paribus also display high inflation volatility, by virtue of the reduced form Phillips curve given in equation (10). This results in high values for  $C_{11}$ . Unless the cross correlation between inflation and marginal costs is low. In that case, it is possible to have simultaneously high  $\tilde{\xi}_j$ , low inflation volatility, and therefore low values for  $C_{11}$ . But then, it is also true that  $C_{12}$  takes low values, and so, possibly,  $C_1$  continues to be positive.



By virtue of exactly identical reasoning, it is likely that  $D_{22}$  will take large and negative values, and since, for similar reasons,  $D_{21}$  is unlikely to be highly positive,  $D_2$  may well take negative values. Similarly,  $D_1$  is affected by the extent of intrinsic inflation, which is only indirectly related with  $E(s_{j,t}\pi_{j,t-1})$ ; it is on the other hand directly affecting  $E(\pi_{j,t-1}\pi_{j,t-1})$ , and thus, ceteris paribus,  $D_{12}$ .

Put simply, the sub-components of the Maximum Likelihood bias on  $\psi$  can be expected to offset each other, with small end effects. On the other hand, the components of the bias on  $\xi$  act to reinforce each other's influence. This confirms analytically our simulation results that the ML bias on  $\xi$  is largest. The fact that a large part of this bias seems to originate from the dispersion in  $\alpha_j$  finds confirmation in the likely large (positive) magnitude taken by  $C_2$ .

The bias on  $\rho_{ML}$  is unambiguously positive, and unsurprisingly increases with dispersion in the sectoral values of  $\rho_j$ . This dispersion can have dramatic effects in aggregate Maximum Likelihood estimates. Consider for instance the simple case where  $\xi$  and  $\alpha$  are homogeneous across industries, but  $\rho_j$  is industry specific. A heterogeneity bias continues to potentially plague *all* estimates of the aggregate Phillips curve. The expressions for the asymptotic biases in this section do not involve the realizations of  $\tilde{\rho}_j$ , because they are dedicated to signing the biases. But Appendix 2 shows that the magnitude of both  $(\xi_{ML} - \xi)$  and  $(\psi_{ML} - \psi)$  is potentially affected by heterogeneity in the persistence of marginal costs. GMM does not suffer from such mis-specification of an auxiliary model.

### 3.3 Simulations

We now perform some simulations whose purpose is to confirm the nature of the various relevant heterogeneity biases, under alternative assumptions on the nature of heterogeneity. To facilitate comparison, we express all estimates in terms of the structural parameters ( $\xi$  and  $\alpha$ ), and infer the corresponding values for the reduced-form parameters. We use our analytical results to decompose all biases into their various components, and examine their relative signs and magnitudes.

We reproduce the experiment described in section 2, with the additional refinement that we now decompose all biases into their theoretical components. As before, the structural parameters ( $\xi$ ,  $\alpha$ , and  $\rho$ ) are initially set at the values implied by heterogeneous estimations performed on our French data. Cross-industry heterogeneity in the structural parameters is drawn from a normal distribution with variances  $\sigma_\alpha^2 = \sigma_\xi^2 = 0.1$ . We deduce sector-specific reduced-form estimates and simulate samples of sectoral inflation and marginal cost. We use these artificial data to compute the corresponding aggregate inflation and marginal cost, as well as the unbiased, sector-specific estimates of  $\xi_j$  and  $\alpha_j$ . We then estimate the aggregate Phillips Curve, using the GMM and ML estimators on simulated aggregate series. GMM implies estimates of  $\lambda^b$  and  $\theta$ , and ML implies estimates of  $\xi$ ,  $\psi$ , and  $\rho$ . These are then used to back out the corresponding structural estimates  $\xi$  and  $\alpha$ . We iterate the procedure, saving for each sample both the theoretical values of the structural parameters, and their empirical counterparts. We report the median value of the obtained estimates. We now have sixteen sectors, as in the French data, and 111 observations per sector.

Table 1a presents our baseline results. As expected the GMM estimator generates positive heterogeneity biases in both  $\xi$  and  $\theta$ , and the former is larger in absolute magnitude. What is more, both biases originate almost fully from large and positive values of  $A_3$  and  $B_3$ , which we argue stem from dispersion

in  $\tilde{\theta}_j$ , and ultimately in  $\alpha_j$ .  $(\theta_{GMM} - \theta)$  takes lower values than  $(\xi_{GMM} - \xi)$  because  $B_1$  is negative, as predicted by the previous section. It is also true that the bias on  $\alpha$  is negative, as it should given that both  $(\theta_{GMM} - \theta)$  and  $(\xi_{GMM} - \xi)$  are positive; it is however small in our simulations. The right panel of the table presents the biases implied by the ML estimator. The results are once again consistent with our analytical expressions. The ML heterogeneity bias on  $\xi$  is largest, and this happens mostly because  $C_2$  is large and positive. The ML bias on  $\psi$  is negligible, as  $D_1$  and  $D_2$  have opposite signs. As a result, ML barely biases aggregate estimates of  $\alpha$ .

These results underline the importance of heterogeneity in  $\alpha_j$ , as the largest component of all biases invariably involves either  $\tilde{\theta}_j$  or  $\tilde{\psi}_j$ , rather than  $\tilde{\xi}_j$ . We further illustrate this asymmetry in Table 1b, where we alternatively shut down heterogeneity in  $\alpha_j$  and in  $\xi_j$ , and calibrate larger values of the dispersion that subsists in the simulations. Table 1b confirms all heterogeneity biases are larger when heterogeneity arises from  $\sigma_\alpha^2$ . This is particularly true of GMM, for which the difference between the two calibrations is sizable, and originates almost exclusively from different values for  $A_3$ . By the same token, both  $C_2$  and  $D_2$  are larger when heterogeneity is modeled to arise from  $\alpha_j$  rather than  $\xi_j$ .

In summary, our simulations suggest that heterogeneity results in a large positive bias on the degree of indexation, especially under the ML approach. It is the cross-sectional dispersion in the Calvo parameter  $\alpha$  that has largest effects on the magnitude of heterogeneity biases, rather than heterogeneity in the extent of indexation.

## 4 Econometric Methods

We now introduce the estimators we use to account for sectoral heterogeneity. We discuss two estimators: the Mean Group (MG) and Random Coefficient (RC) models, introduced by Pesaran and Smith (1995). The main difference between the two estimators comes from their assumptions on the nature of heterogeneity. MG assumes sector-specific deviations from mean parameters are deterministic, whereas RC assumes they are random. As a result, MG implements a simple averaging of sector specific estimates, whereas RC requires a generalized least squares procedure that optimally accounts for the stochastic nature of heterogeneity.<sup>26</sup>

In other words, the RC estimator relaxes assumption H1. In this section, we also relax the other assumptions that afforded the analytical expressions of the biases. In particular, some of our estimates use the industry weights directly implied by French data (H2), and we allow for cross-industry interdependences (H3). We do this in two ways. First, we implement the Seemingly Unrelated Regression Estimation (SURE) correction to both heterogeneous estimators.<sup>27</sup> Second, we introduce the common correlated effects Mean Group estimator (CCE-MG) and its random coefficient version (CCE-RC) introduced by Pesaran (2006). Assumption H4 is eschewed naturally when we implement the GMM estimator, which exonerates from any assumption on the process driving marginal costs.

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<sup>26</sup>The difference is akin to that between fixed effect and random effect estimators, and can be tested accordingly. Hsiao (1986) shows that MG and RC estimators are equivalent in the limit. In other words, our analytical results, which were calculated on the basis of deterministic heterogeneity, become valid in the limit, even if heterogeneity is actually stochastic.

<sup>27</sup>This is directly applicable since we have sixteen sectors but 111 observations.

## 4.1 Mean Group

The MG estimator introduced in Pesaran and Smith (1995) simply consists in an arithmetic average of sector-specific parameter estimates. In particular, let  $\Upsilon_j$  denote the vector of sector-specific parameters. The MG estimator  $\hat{\Upsilon}_{MG}$  is given by

$$\hat{\Upsilon}_{MG} = \frac{1}{J} \sum_{j=1}^J \hat{\Upsilon}_j.$$

A consistent estimator of the covariance matrix of  $\hat{\Upsilon}_{MG}$  can be computed as

$$V(\hat{\Upsilon}_{MG}) = \frac{1}{T(T-1)} \sum_{j=1}^J \left( \hat{\Upsilon}_j - \hat{\Upsilon}_{MG} \right) \left( \hat{\Upsilon}_j - \hat{\Upsilon}_{MG} \right)'$$

Two complications arise in our case. First, the theory on MG was developed and applied to estimators of the ML type. Little is known about the applicability or the properties of a MG estimator combined with sectoral parameters estimated with GMM. In what follows, we therefore implement the MG estimator using sector-specific ML estimates (and report the latter). Second, the arithmetic average of sector-specific inflation is not the object whose dynamics we are interested in. Aggregate inflation is given by a weighted average of sector-specific price changes, with weights  $w_j$  corresponding to industry shares in the GDP deflator. We therefore amend the MG estimator and introduce weights  $w_j$  in computing aggregate estimates, which then indeed evaluate the dynamics of aggregate inflation as befits. Introducing these empirical weights does not affect the consistency of the MG estimator.

Third, the parameters estimated by ML are not all structural. To preserve coherence, we obtain industry-level estimates of the reduced form coefficients  $\xi_j$  and  $\psi_j$ , and perform the aggregation on the basis of these estimates. Then MG (or indeed RC) yields consistent estimates for the aggregate coefficients  $\xi$  and  $\psi$ , which we use to back out the aggregate structural parameters. Thus we avoid aggregating non-linear estimates of  $\alpha_j$ . When evaluating the existence of a heterogeneity bias, we therefore compare homogeneous estimates of  $\xi$  and  $\psi$  to the corresponding weighted average of their industry level values. The alternative would be to estimate structural parameters  $\xi$  and  $\alpha$  at both levels of aggregation, infer the true aggregate estimates from a weighted average of disaggregated results, and compare homogeneous and heterogeneous estimates. But this might conflate the heterogeneity bias with one induced by aggregating non-linearities. We only present results that pertain to the former approach, which in addition is more in line with the initial insight in Pesaran and Smith (1995). This is true of all our heterogeneous estimations.

## 4.2 Random Coefficients

Following Pesaran and Smith (1995), and Hsiao and Pesaran (2004), we define the RC estimator as a weighted average of least squares estimates, with weights inversely proportional to their covariance matrices. In particular, the best linear unbiased estimator of the mean coefficient vector is given by

$$\hat{\Upsilon}_{RC} = \sum_{j=1}^J W_j \hat{\Upsilon}_j$$

The weighting scheme is given by

$$W_j = \left[ \sum_{j=1}^J (\Delta + \Sigma_{\hat{Y}_j})^{-1} \right]^{-1} (\Delta + \Sigma_{\hat{Y}_j})^{-1}$$

where

$$\hat{\Delta} = \frac{1}{T-1} \sum_{j=1}^J \left( \hat{Y}_j - \hat{Y}_{MG} \right) \left( \hat{Y}_j - \hat{Y}_{MG} \right)' - \frac{1}{J} \sum_{j=1}^J \hat{\sigma}_j^2 (X_j' X_j)^{-1}$$

and  $\Sigma_{\hat{Y}_j} = \sigma_j^2 (X_j' X_j)^{-1}$ .  $X_{j,t} = (\pi_{j,t-1}, s_{j,t})'$  is the vector of regressors, and  $\hat{\sigma}_j^2 = \frac{1}{T-K} \pi_j' (I_T - X_j (X_j' X_j)^{-1} X_j') \pi_j$ . In words,  $\Delta + \Sigma_{\hat{Y}_j}$  captures the dispersion of the industry-specific estimates, so that  $W_j$  will optimally act to associate a large weight to sectors where the estimates are precise. The MG estimator is efficient when the optimal weights are not different from the arithmetic ones.

A weighting issue arises when implementing RC for our purposes. The optimal weights  $W_j$  are not necessarily aligned with the empirical sector shares,  $w_j$ . Although the exact pattern of weights does not matter in the limit, as exemplified by the asymptotic equivalence between RC and MG, we ascertain how much our results are affected by a particular patterns of industry weights. In what follows, we report RC estimates as implied by both optimal and observed weights. In particular, we constrain the weighing scheme used to compute  $\hat{Y}_{MG}$  to be either uniform (which boils down to standard RC) or to entail observed weights (which boils down to our augmented version of MG, as discussed in the previous section).

### 4.3 Cross-Industry Linkages

It is eminently likely that shocks to sectoral inflation or marginal costs be correlated across sectors. This would happen for instance in the presence of macroeconomic aggregate shocks, or input-output linkages between industries. We now discuss how to deal econometrically with this possibility, while preserving some level of generality in the nature of cross-industry linkages.

We first apply a SURE correction to both our heterogeneous estimators. In particular, assume now  $E[\varepsilon_{i,t} \varepsilon_{j,t}] = \sigma_{\varepsilon_i \varepsilon_j}$ ,  $E[u_{i,t} u_{j,t}] = \sigma_{u_i u_j}$ , and  $E[\varepsilon_{i,t} u_{j,t}] = \sigma_{\varepsilon_i u_j}$ , for  $i \neq j$ . Stacking all sectors,

$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_J \end{pmatrix} = \begin{pmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & X_J \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_J \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_J \end{pmatrix}$$

where  $Y_j = (\pi_j \quad s_j)'$ ,  $X_j = \begin{pmatrix} \pi_{j,-1} & s_j & 0 \\ 0 & 0 & s_{j,-1} \end{pmatrix}$ ,  $\pi_j = (\pi_{j,t})$ ,  $\pi_{j,-1} = (\pi_{j,t-1})$ ,  $s_j = (s_{j,t})$ , and  $s_{j,-1} = (s_{j,t-1})$ . The stacked disturbances have a covariance matrix,  $\Omega$ , which can be accounted for using standard Maximum Likelihood techniques. We correct both RC and MG accordingly.

The SURE correction requires the estimation of a large-dimensional covariance matrix, which may affect the finite-sample properties of the estimators. An alternative is proposed in Pesaran (2006), who

introduced a correction technique to account for unobserved common factors potentially correlated with sectoral-specific regressors. The sector-specific estimations are filtered by means of cross-section aggregate regressors, which purges the differential effects of unobserved common factors. The approach is particularly appealing because of its simplicity. It merely requires the addition of an auxiliary regressor, given by the cross-sectional average of the regressors, which suffices to filter the common correlated effect (CCE) out. In particular, the model rewrites

$$\begin{aligned}\pi_{j,t} &= \xi \pi_{j,t-1} + \psi s_{j,t} + f'_{j,t} \gamma_{\pi,j} + \bar{\eta}_{j,t} \\ s_{j,t} &= \rho s_{j,t-1} + f'_{j,t} \gamma_{s,j} + \bar{u}_{j,t}\end{aligned}$$

where  $f_t = (\bar{\pi}_t, \bar{\pi}_{t-1}, \bar{s}_t, \bar{s}_{t-1})'$  and  $\bar{x}_t$  is the cross-sectional average of  $x_{j,t}$ . We implement the CCE correction onto both our heterogeneous estimators.<sup>28</sup>

## 5 Results

We first introduce our sectoral data, which include production, prices, wages and employment in sixteen French sectors. We describe some summary statistics. Next we present the industry specific estimates of the Phillips curve, and identify the main sources of heterogeneity in our data. Finally, we discuss the heterogeneity bias.

### 5.1 Data

Our data is constructed by INSEE, the French statistical institute. We have observations on output, prices, wages and employment for sixteen sectors of the French economy, comprising all activities listed in the Appendix. Coverage includes agriculture, manufacturing (six sectors) and services (nine sectors). For each industry, the inflation rate is computed as the quarter-on-quarter growth rate of the value-added deflator. We follow Galí and Gertler (1999) and compute the marginal costs  $s_{j,t}$  as the (logarithm) deviation of the share of labor income in value added from its sample mean. Our data are quarterly from 1978:1 to 2005:2, for a total of 111 observations.

Table 2 presents some summary statistics. We report average inflation and average growth in marginal costs, their serial correlations, and the contemporaneous cross-correlation, at both industry and aggregate levels. There is extensive heterogeneity across sectors in both average measures. Annual inflation ranges between 0.2% and 5.5%, and the average annual growth in real marginal costs ranges between  $-3.6\%$  and  $0.1\%$ . The same is true of serial correlation in inflation, and the cross-correlation between  $s_{j,t}$  and  $\pi_{j,t}$ . In contrast marginal costs are consistently highly serially correlated. Figures 3 and 4 plot sectoral inflation rates and real marginal costs, using unfiltered data.<sup>29</sup> There is again considerable heterogeneity across sectors in the patterns of both variables, although they tend to track each other within each industry as testified in the industry-level Phillips curves we later estimate.

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<sup>28</sup>A consistent estimator of  $\Sigma_{\hat{\gamma}}$  is obtained using the Newey and West (1987) type procedure. The CCE estimator of the sectoral-specific parameters are consistent as  $J, T \rightarrow \infty$ . As a result, the CCE correction of the MG (or RC) estimator is asymptotically unbiased as  $J \rightarrow \infty$ , for  $T$  either fixed or  $T \rightarrow \infty$ . A rank condition is necessary regarding the factor loadings  $f_t$ . The asymptotic distribution can only be derived if  $\sqrt{T}/J \rightarrow 0$  as  $J, T \rightarrow \infty$ .

<sup>29</sup>Detrending has minimal effect.

Table 2 also reveals that aggregate inflation and real marginal costs are highly serially correlated, and covary to an extent that is much larger than most industry-level series. Figure 5 plots both aggregate series, and illustrates that both series experienced a similar downward trend over the sample. Aggregate inflation and marginal costs track each other quite closely, quite reassuringly given the existing empirical support for aggregate Phillips curves.

## 5.2 Individual Estimates

Table 3 presents the industry-level estimates of the New Keynesian Phillips curve, on the basis of the Maximum Likelihood approach. In other words, we maintain the assumption that industry-level marginal costs are well characterized by an AR(1), and estimate the resulting reduced form equation. There are several reasons why we focus on ML estimates of the Phillips curve at the industry level. First, how to aggregate heterogeneous coefficients obtained from GMM estimations is uncharted territory. Our MG and RC heterogeneous estimators (and their corrections) are only equipped to aggregate industry-level estimates that arise from a ML estimator. Second, GMM is well known to suffer from small sample biases, which may be particularly relevant to our present purposes when estimating an industry-level Phillips curve.<sup>30</sup>

Several results stand out from Table 3. First, our data are supportive of inflation dynamics at the industry level that are consistent with the New Keynesian framework. Both  $\alpha$  and  $\xi$  are adequately bounded, and marginal costs are unanimously persistent. There is however considerable heterogeneity, especially as pertains to the extent of backward indexation  $\xi_j$ . For instance, Energy or Business services seem to display purely forward-looking price setting behavior, with values of  $\xi_j$  not different from zero. Real estate and Government are at the other end of the spectrum, with  $\xi_j$  virtually equal to one. The dispersion in the estimated values for  $\alpha_j$  is substantially less marked, if still sizable. We find values for  $\alpha$  close to one for most of the service sectors, but smaller for food manufacturing, energy or transports - though still significantly different from zero. The fact that the heterogeneity in  $\alpha_j$  be small in our data does not preclude large heterogeneity biases. Our simulations and analytics have shown that large inconsistencies are still possible even when  $\xi_j$  are virtually homogeneous, provided some differences in nominal rigidities still prevail.<sup>31</sup>

Table 3 is broadly consistent with the estimates based on French data reported in Fougère et al (2005) or Beaudry et al. (2005). Neither paper is directly comparable because of different data sources, aggregation levels and modeling strategies, but both of them find substantial heterogeneity, in particular between services and manufacturing industries.<sup>32</sup> Table 3 is consistent with relatively homogeneous estimates

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<sup>30</sup>In fact, Fuhrer, Moore, and Schuh (1995) have forcefully developed this argument in the context of the New Keynesian Phillips curve. Admittedly, ML does not go without problems either, since it requires the imposition of an auxiliary regression on marginal costs. In practice, we have little leeway given the heterogeneous estimators we propose to implement. We note that the two estimators are asymptotically equivalent.

<sup>31</sup>We have implemented Fisher tests with null hypothesis coefficient equality across sectors. We used the industry-level estimates in Table 3, and found overwhelming rejection of the homogeneity assumption across all three parameters  $\alpha$ ,  $\xi$  and  $\rho$ . When using instead (unreported) industry-level estimates that implement SURE to correct for cross-industry correlations, rejection becomes even stronger.

<sup>32</sup>For instance, they use CPI data, and focus on consumption goods only, whereas we have information on producers goods as well.

within services, for instance.<sup>33</sup> Interestingly, Fougère et al (2005) also estimate long-lived nominal rigidities in services on average, but especially short ones in Food Manufacturing or Energy.<sup>34</sup>

### 5.3 Heterogeneity Bias

We now quantify the heterogeneity bias in our data, and compare estimates of the structural parameters implied by standard GMM or ML estimators performed on aggregated data, with the corresponding estimates implied by heterogeneous estimators that make use of sectoral information. To facilitate comparison and interpretation, in what follows, we discuss implied estimates for  $\lambda^b$ ,  $\theta$  and  $\psi$ .<sup>35</sup> For completeness, we also report estimates of  $\rho$ .

Table 4 reports our results. The first two rows report estimates for the aggregate New Keynesian Phillips curve, as implied by both GMM and ML. The lower four rows present the results that pertain to variants of our heterogeneous estimators, depending on what weighting pattern is used in aggregating. In particular,  $MG^*$  and  $RC^*$  correspond to effectively observed GDP weights, whereas  $MG^{**}$  and  $RC^{**}$  assume uniform weights.<sup>36</sup>

We find a large and positive heterogeneity bias on  $\xi$ . The estimated coefficient is up to twice larger when based on homogeneous estimators implemented on aggregate data. ML implies  $\xi$  close to 0.89, GMM estimates it to be around 0.74, while heterogeneous estimators imply values between 0.4 and 0.47. This is far from negligible, for it implies a large positive bias on the extent of backward looking behavior in the data.  $\lambda^b$  is effectively substantially lower than what aggregate estimators suggest, from 0.43 down to 0.29 in the  $MG^{**}$  case.

We also uncover some evidence that the reduced form coefficient on marginal costs suffer from a negative heterogeneity bias under both estimators.  $\theta_{GMM}$  is not different from zero, and  $MG^{**}$ ,  $RC^*$  and  $RC^{**}$  all imply larger estimates than  $\psi_{ML}$ . These discrepancies are not always significant, but they correspond to upward biases on  $\alpha$ , and indeed on the duration of price rigidities. For instance,  $ML$  yields estimates of  $\alpha$  slightly above what is implied by  $RC$  or  $MG$ . Aggregate duration as implied by ML is around eight quarters, but closer to seven according to the RC estimator, and six with  $MG^{**}$ . We note the relative magnitudes of GMM and ML biases on the structural parameters are perfectly in line with our simulations, even though these results were obtained under more restrictive assumptions. In particular, the bias on  $\xi$  is the largest under both estimators, while the bias on  $\alpha$  is smaller and barely significant. This arises in a dataset where there is less dispersion in  $\alpha_j$  than in  $\xi_j$ .<sup>37</sup>

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<sup>33</sup>In fact, we ran our heterogeneous estimators within both manufacturing industries and services, and in each case found barely any evidence of a heterogeneity bias. In contrast as will become clearer, the bias is large when considering the complete sample. In other words, the extent of heterogeneity in our data is largest between manufacturing industries and services.

<sup>34</sup>Fougère et al (2005) report an average duration of price stickiness about twice larger in services than in manufactures. Our heterogeneous estimators imply a duration around four quarters in manufactures, and up to nine in services.

<sup>35</sup>The standard errors are obtained using a simple delta method.

<sup>36</sup>To be precise, the use of different weights only matters for computing  $\hat{Y}_{MG}$ . The Random Coefficient model continues to use the optimal weights  $W_j$  when aggregating.

<sup>37</sup>We implemented simple Hausman tests to investigate the significance of the heterogeneity bias, comparing results implied by aggregate ML with the RC model. The bias on  $\xi$  was significant at any level of confidence, with a P value equal to zero. But the differences in  $\alpha$  and  $\rho$  were insignificant when comparing these two estimators.

## 5.4 Correcting for Cross-Industry Linkages

We now implement two corrections to our heterogeneous estimators, that allow for the possibility that shocks be correlated across industries. Our corrections are general enough to account for common macroeconomic shocks, input-output production linkages or indeed anything that would engender influences on sectoral prices or marginal costs that are contemporaneously correlated across industries.

In Appendix 3, we report various estimates that help quantifying cross-industry linkages in our data. Table A1 reports the cross-correlations between the residuals  $\eta_{j,t}$  of each industry-specific Phillips curve estimate, as implied by equation (10). Table A2 focuses instead on the cross-industry correlations in  $u_{j,t}$ , the cost-push shock in equation (9). On average, the values in Table A1 are positive, though relatively few are significant. This is consistent with price pressures affecting simultaneously several industries. We note significantly positive correlations tend to occur within manufactures, and within services. The same can be said of shocks to marginal costs in Table A2, providing support that cost-push shocks tend to affect more than one sector. This would characterize aggregate macroeconomic shocks, or perhaps production linkages across industries.

In short, shocks are correlated across sectors. Table 5 presents the two corrections we implement, SURE and CCE in the two lower panels; the top two panels reproduce the aggregate estimates from Table 4 for comparison purposes. The corrections only act to reinforce our conclusions. The corrected biases on  $\xi$  become substantially larger for both corrections and both estimators, and heterogeneous estimates are as low as 0.27 under SURE-RC, as compared with 0.89 under homogeneous ML. This is a large bias, with considerable impact on the estimated role for backward looking pricing. Unbiased estimates of  $\lambda^b$  are as low as 0.21, as compared with 0.47 when imposing homogeneity.

The bias on  $\alpha$  also increases slightly in magnitude, especially as implied by the CCE correction. The implied duration of rigidities falls to between six and seven quarters, down from eight under the homogeneous ML estimator. To this correspond significant estimates for  $\psi$ , that is for the role of marginal cost in driving inflation. These differences continue to be small in magnitude, and indeed still barely significant. Even though our simulations (and analytical expressions for heterogeneity biases) suggested the biases on  $\xi$  ought to be larger than on  $\alpha$ , we stress that this may well be accentuated in the specific dataset we are using here, where, in particular, the dispersion in  $\alpha_j$  is relatively low.

## 6 Conclusion

We show estimates of the aggregate Phillips Curve are biased in the presence of heterogeneity in firms pricing behavior. We let the extent of nominal rigidities and indexation vary from sector to sector, and derive analytical expressions for the biases induced by standard GMM and ML estimators. We show that the degree of price indexation is systematically biased upward, which translates into higher estimates of backward-looking behavior than is effectively present in the data. Under the same assumptions, the duration of nominal rigidities is weakly biased upwards in standard estimations. Our analytics further suggest that it is dispersion in the extent of nominal rigidities at the industry level that is most likely to engender a heterogeneity bias.



We verify these analytics in French sectoral data. We document a large positive bias in the backward looking component of the Phillips curve that measures intrinsic persistence, which more than halves when heterogeneity is accounted for. Our corrected estimates point to more than eighty percent forward looking behavior, as compared with less than half when homogeneity is imposed. The result stems from a large positive bias in the extent of backward looking indexation. We do also find a (slight) negative bias on the coefficient measuring the importance of marginal costs in driving inflation, that measures extrinsic persistence. This corresponds to a (slight) positive bias on the extent of nominal rigidities, and their duration. In our data, correcting for the heterogeneity bias reduces the estimated duration of rigidities from about eight quarters down to about six. It decreases to four quarters when the focus is on manufacturing industries only. This arises in a dataset where price stickiness varies less across industries than the estimated extent of backward indexation, and where sixteen sectors are observed separately.

Our results withstand the possibility that inflation and marginal costs be driven by common causes across industries, because of common shocks or inter-industry production linkages. In fact, they become even stronger once the adequate corrections for correlated residuals are implemented. But we can not predict what would happen in a finer dataset. Our analytics suggest however the heterogeneity will always be largest on the measure of intrinsic inflation, and would arise mostly because of industry-specific price stickiness.

## Appendix 1: Derivation of the reduced form (10)

The full model of (sectoral) inflation rests on the following system

$$\begin{aligned}\pi_{j,t} &= \lambda_j^b \pi_{j,t-1} + \lambda_j^f E_t \pi_{j,t+1} + \theta_j s_{j,t} + \varepsilon_{j,t} \\ s_{j,t} &= \rho_j s_{j,t-1} + u_{j,t}.\end{aligned}$$

The characteristic equation of the Phillips Curve writes

$$\left(1 - \lambda_j^b L - \lambda_j^f L^{-1}\right) \pi_{j,t} = \theta_j s_{j,t} + \varepsilon_{j,t}$$

where  $L$  denotes the lag operator. The two roots are  $\xi_j < 1$  and  $\frac{1}{\beta} > 1$ . The dynamics of inflation therefore rewrites

$$\frac{1}{\beta} (1 - \xi_j L) (1 - \beta L^{-1}) \pi_{j,t} = \frac{\theta_j}{\lambda_{j,f}} s_{j,t} + \frac{\varepsilon_{j,t}}{\lambda_{j,f}}.$$

After some manipulations, this implies

$$\pi_{j,t} = \xi_j \pi_{j,t-1} + \psi_j s_{j,t} + \eta_{j,t}$$

with

$$\psi_j = \frac{1 - \beta \alpha_j}{1 - \beta \rho_j} \frac{1 - \alpha_j}{\alpha_j} \quad \text{and} \quad \eta_{j,t} = (1 + \beta \xi_j) \varepsilon_{j,t}.$$

The aggregate dynamics is then obtained by imposing that the aggregate Phillips Curve is true on average whenever  $\xi_j = \xi + \tilde{\xi}_j$ ,  $\alpha_j = \alpha + \tilde{\alpha}_j$  and  $\rho_j = \rho + \tilde{\rho}_j$ . Then we have

$$\begin{aligned}\pi_t &= \xi \pi_{t-1} + \psi s_t + \bar{\eta}_t \\ s_t &= \rho s_{t-1} + \bar{u}_t\end{aligned}$$

where  $\psi = (1 - \beta \alpha) (1 - \alpha) / [(1 - \beta \rho) \alpha]$ .

## Appendix 2: Proofs of Propositions 1 and 2

Before showing propositions 1 and 2, we need the two following lemmas

**Lemma 1** *Let  $X$  be a random variable and,  $f$  and  $g$  be two increasing functions, then*

$$\text{cov}(f(X), g(X)) \geq 0.$$

□

Proof: Assume two independent draws,  $X_i$  and  $X_j$  ( $i \neq j$ ), from the distribution of  $X$ . Since  $f$  and  $g$  are increasing, we have

$$[f(X_i) - f(X_j)] [g(X_i) - g(X_j)] \geq 0.$$

Using the expectation operator, we obtain

$$E([f(X_i) - f(X_j)] [g(X_i) - g(X_j)]) \geq 0$$

which, since  $X_i$  and  $X_j$  are independent, is equivalent to

$$E[f(X_i)h(X_i)] - Ef(X_j)Eh(X_i) - Ef(X_i)Eg(X_j) + E[f(X_j)h(X_j)] \geq 0$$

or equivalently, since  $X_i$  and  $X_j$  have the same distribution,

$$2(E[f(X)g(X)] - Ef(X)Eg(X)) \geq 0$$

Thus

$$\text{cov}(f(X), g(X)) \geq 0.$$

□

**Lemma 2** *Assume a reduced form given by*

$$\begin{aligned} \pi_t &= \xi \pi_{t-1} + \psi s_t + \bar{\eta}_t \\ s_t &= \rho s_{t-1} + \bar{u}_t \end{aligned}$$

where  $\psi = \frac{1-\beta\alpha}{1-\beta\rho} \frac{1-\alpha}{\alpha}$ . Then,

$$\begin{aligned} E(s_{j,t}^2) &= \frac{\sigma_{u_j}^2}{1-\rho_j^2} = \sigma_{S_j}^2 \\ E(\pi_{j,t} s_{j,t}) &= \frac{\psi_j}{1-\xi_j \rho_j} \sigma_{S_j}^2 \\ E(\pi_{j,t-1} s_{j,t}) &= \frac{\psi_j \rho_j}{1-\xi_j \rho_j} \sigma_{S_j}^2 \\ E(\pi_{j,t+1} s_{j,t}) &= \left( \frac{\xi_j \psi_j}{1-\xi_j \rho_j} + \psi_j \rho_j \right) \sigma_{S_j}^2 \\ E(\pi_{j,t}^2) &= \frac{\sigma_{\eta_j}^2}{1-\xi_j^2} + \frac{1+\xi_j \rho_j}{(1-\xi_j^2)(1-\xi_j \rho_j)} \psi_j^2 \sigma_{S_j}^2 \\ E(\pi_{j,t+1} \pi_{j,t-1}) &= \xi_j^2 \frac{\sigma_{\eta_j}^2}{1-\xi_j^2} + \left[ \frac{\xi_j^2 (1+\xi_j \rho_j)}{(1-\xi_j^2)(1-\xi_j \rho_j)} + \frac{\rho_j (\xi_j + \rho_j)}{(1-\xi_j \rho_j)} \right] \psi_j^2 \sigma_{S_j}^2 \end{aligned}$$

and

$$\begin{aligned}
E(\pi_{j,t-1}(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) &= \frac{\sigma_{\eta_j}^2}{1 - \xi_j^2} (1 - \beta^2 \xi_j^2) \\
&\quad + \left( \frac{(1 + \xi_j \rho_j)(1 - \beta^2 \xi_j^2)}{(1 - \xi_j^2)(1 - \xi_j \rho_j)} - \beta^2 \frac{\rho_j(\xi_j + \rho_j)}{(1 - \xi_j \rho_j)} \right) \psi_j^2 \sigma_{S_j}^2 \\
E(s_{j,t}(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) &= (\psi_j \rho_j (1 - \beta^2) - \beta^2 \xi_j \psi_j (1 - \rho_j^2)) \frac{\sigma_{S_j}^2}{1 - \xi_j \rho_j}.
\end{aligned}$$

□

Proof: Assuming that all stochastic processes are weakly stationary, we have

$$\begin{aligned}
E(\pi_{j,t} s_{j,t}) &= E(\xi_j \pi_{j,t-1} + \psi_j s_{j,t} + \eta_{j,t}, s_{j,t}) = \frac{\psi_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \\
E(\pi_{j,t-1} s_{j,t}) &= E(\pi_{j,t-1}(\rho_j s_{j,t-1} + u_{j,t})) = \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \\
E(\pi_{j,t+1} s_{j,t}) &= E((\xi_j \pi_{j,t} + \psi_j s_{j,t+1} + \eta_{j,t+1}) s_{j,t}) = \left( \frac{\xi_j \psi_j}{1 - \xi_j \rho_j} + \psi_j \rho_j \right) \sigma_{S_j}^2 \\
E(\pi_{j,t}^2) &= \xi_j^2 E(\pi_{j,t-1}^2) + \psi_j^2 E(s_{j,t}^2) + 2\xi_j \psi_j E(\pi_{j,t-1} s_{j,t}) + \sigma_{\eta_j}^2 \\
&= \frac{1}{1 - \xi_j^2} \left[ \psi_j^2 E(s_{j,t}^2) + 2\xi_j \psi_j E(\pi_{j,t-1} s_{j,t}) + \sigma_{\eta_j}^2 \right] \\
&= \frac{\sigma_{\eta_j}^2}{1 - \xi_j^2} + \frac{1 + \xi_j \rho_j}{(1 - \xi_j^2)(1 - \xi_j \rho_j)} \psi_j^2 \sigma_{S_j}^2 \\
E(\pi_{j,t+1} \pi_{j,t-1}) &= E((\xi_j \pi_{j,t} + \psi_j s_{j,t+1} + \eta_{j,t+1}) \pi_{j,t-1}) \\
&= \xi_j E((\xi_j \pi_{j,t-1} + \psi_j s_{j,t} + \eta_{j,t}) \pi_{j,t-1}) + \psi_j E((\rho_j s_{j,t} + u_{j,t+1}) \pi_{j,t-1}) \\
&= \xi_j^2 E(\pi_{j,t}^2) + \xi_j \psi_j E(s_{j,t} \pi_{j,t-1}) + \psi_j \rho_j E(s_{j,t} \pi_{j,t-1}) \\
&= \xi_j^2 \left[ \frac{\sigma_{\eta_j}^2}{1 - \xi_j^2} + \frac{1 + \xi_j \rho_j}{(1 - \xi_j^2)(1 - \xi_j \rho_j)} \psi_j^2 \sigma_{S_j}^2 \right] + \frac{\rho_j \psi_j^2 (\xi_j + \rho_j)}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \\
&= \xi_j^2 \frac{\sigma_{\eta_j}^2}{1 - \xi_j^2} + \left[ \frac{\xi_j^2 (1 + \xi_j \rho_j)}{(1 - \xi_j^2)(1 - \xi_j \rho_j)} + \frac{\rho_j (\xi_j + \rho_j)}{(1 - \xi_j \rho_j)} \right] \psi_j^2 \sigma_{S_j}^2.
\end{aligned}$$

□

### Proof of Proposition 1

The GMM estimator of  $(\lambda^b, \theta)$  is given by the two moment conditions

$$\begin{aligned}
E \left[ \pi_{t-1} \left( \pi_t - \beta \pi_{t+1} - \lambda^b (\pi_{t-1} - \beta^2 \pi_{t+1}) - \theta s_t \right) \right] &= 0 \\
E \left[ s_t \left( \pi_t - \beta \pi_{t+1} - \lambda^b (\pi_{t-1} - \beta^2 \pi_{t+1}) - \theta s_t \right) \right] &= 0.
\end{aligned}$$

The probability limits of the estimators given by these moments conditions are

$$\begin{aligned}\text{plim}(\lambda_{GMM}^b) &= \frac{1}{\Lambda} [E(s_t^2) E(\pi_{t-1}(\pi_t - \beta\pi_{t+1})) - E(s_t\pi_{t-1}) E(s_t(\pi_t - \beta\pi_{t+1}))] \\ \text{plim}(\theta_{GMM}) &= \frac{1}{\Lambda} [E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) E(s_t(\pi_t - \beta\pi_{t+1})) \\ &\quad - E(s_t(\pi_{t-1} - \beta^2\pi_{t+1})) E(\pi_{t-1}(\pi_t - \beta\pi_{t+1}))] \\ \text{plim}(\xi_{GMM}) &= \text{plim}(\lambda_{GMM}^b)/(1 - \beta \text{plim}(\lambda_{GMM}^b))\end{aligned}$$

where

$$\Lambda = E(s_t^2) E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) - E(s_t\pi_{t-1}) E(s_t(\pi_{t-1} - \beta^2\pi_{t+1}))$$

As a consequence, the asymptotic bias on the three estimators is given by

$$\begin{aligned}\Lambda \text{plim}(\lambda_{GMM}^b - \lambda^b) &= E(s_t^2) E(\pi_{t-1}\bar{\varepsilon}_t) - E(s_t\pi_{t-1}) E(s_t\bar{\varepsilon}_t) \\ \Lambda \text{plim}(\xi_{GMM} - \xi) &= (1 + \beta\hat{\xi})(1 + \beta\xi) [E s_t^2 E \pi_{t-1} \bar{\varepsilon}_t - E \pi_{t-1} s_t E s_t \bar{\varepsilon}_t] \\ \Lambda \text{plim}(\theta_{GMM} - \theta) &= E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) E(s_t\bar{\varepsilon}_t) \\ &\quad - E(s_t(\pi_{t-1} - \beta^2\pi_{t+1})) E(\pi_{t-1}\bar{\varepsilon}_t)\end{aligned}$$

where

$$\bar{\varepsilon}_t = \sum_{j=1}^J w_j \varepsilon_{j,t} + \sum_{j=1}^J w_j \tilde{\lambda}_j^b (\pi_{j,t-1} - \beta^2\pi_{j,t+1}) + \sum_{j=1}^J w_j \tilde{\theta}_j s_{j,t}$$

and

$$\Lambda = E(s_t^2) E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) - E(s_t\pi_{t-1}) E(s_t(\pi_{t-1} - \beta^2\pi_{t+1})).$$

Note that

$$\lambda_{GMM}^b - \lambda^b = \frac{\xi_{GMM}}{1 + \beta\xi_{GMM}} - \frac{\xi}{1 + \beta\xi} = \frac{\xi_{GMM} - \xi}{(1 + \beta\xi_{GMM})(1 + \beta\xi)}$$

Under H1, we have

$$\begin{aligned}\Lambda \text{plim}(\lambda_{GMM}^b - \lambda^b) &= E(s_t^2) \sum_{j=1}^J w_j [\tilde{\lambda}_j^b E(\pi_{t-1}(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) + \tilde{\theta}_j E(\pi_{t-1}s_{j,t})] \\ &\quad - E(s_t\pi_{t-1}) \sum_{j=1}^J w_j [\tilde{\lambda}_j^b E(s_t(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) + \tilde{\theta}_j E(s_t s_{j,t})] \\ \Lambda \text{plim}(\theta_{GMM} - \theta) &= E(\pi_{t-1}(\pi_{t-1} - \beta^2\pi_{t+1})) \sum_{j=1}^J w_j [\tilde{\lambda}_j^b E(s_t(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) + \tilde{\theta}_j E(s_t s_{j,t})] \\ &\quad - E(s_t(\pi_{t-1} - \beta^2\pi_{t+1})) \sum_{j=1}^J w_j [\tilde{\lambda}_j^b E(\pi_{t-1}(\pi_{j,t-1} - \beta^2\pi_{j,t+1})) + \tilde{\theta}_j E(\pi_{t-1}s_{j,t})].\end{aligned}$$

The expression for  $\text{plim}(\lambda_{GMM}^b - \lambda^b)$  can be rewritten as the sum of three terms

$$\begin{aligned} A_1 &= \frac{1}{\Lambda} E(s_t^2) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(\pi_{t-1}(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\ A_2 &= -\frac{1}{\Lambda} E(s_t \pi_{t-1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(s_t(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\ A_3 &= \frac{1}{\Lambda} E(s_t^2) \sum_{j=1}^J w_j \tilde{\theta}_j E(\pi_{t-1} s_{j,t}) - \frac{1}{\Lambda} E(s_t \pi_{t-1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(s_t s_{j,t}). \end{aligned}$$

And, similarly,  $\text{plim}(\theta_{GMM} - \theta)$  decomposes into

$$\begin{aligned} B_1 &= \frac{1}{\Lambda} E(\pi_{t-1}^2 - \beta^2 \pi_{t-1} \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(s_t(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\ B_2 &= -\frac{1}{\Lambda} E(s_t \pi_{t-1} - \beta^2 s_t \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\lambda}_j^b E(\pi_{t-1}(\pi_{j,t-1} - \beta^2 \pi_{j,t+1})) \\ B_3 &= \frac{1}{\Lambda} E(\pi_{t-1}^2 - \beta^2 \pi_{t-1} \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(s_t s_{j,t}) \\ &\quad - \frac{1}{\Lambda} E(s_t \pi_{t-1} - \beta^2 s_t \pi_{t+1}) \sum_{j=1}^J w_j \tilde{\theta}_j E(\pi_{t-1} s_{j,t}). \end{aligned}$$

We can characterize the sign of  $A_j$  under H1, H2 and H3 and when  $\beta$  is close to one. In particular, we use H2 to simplify away the production weights  $w_j$  by assuming they are equal across all sectors. Using Lemma 2 and H4,  $A_1$  becomes

$$A_1 = \frac{E(s_t^2)}{\Lambda J^2} \sum_{j=1}^J \tilde{\lambda}_j^b \left( \sigma_{\eta_j}^2 + \frac{1 - \rho_j^2}{1 - \xi_j \rho_j} \psi_j^2 \sigma_{S_j}^2 \right).$$

Under the weak stationary assumption,

$$\lim_{J \rightarrow \infty} \frac{1}{J^2} \sum_{j=1}^J \tilde{\lambda}_j^b \left( \sigma_{\eta_j}^2 + \frac{1 - \rho_j^2}{1 - \xi_j \rho_j} \psi_j^2 \sigma_{S_j}^2 \right) = \text{cov}(\tilde{\lambda}_j^b, \tilde{A}_{1j})$$

where  $\tilde{A}_{1j} = \sigma_{\eta_j}^2 + \frac{1 - \rho_j^2}{1 - \xi_j \rho_j} \psi_j^2 \sigma_{S_j}^2$ , and

$$\lim_{J \rightarrow \infty} A_1 = \frac{1}{\Lambda} V(s_t) \text{cov}(\tilde{\lambda}_j^b, \tilde{A}_{1j}).$$

Since  $\tilde{\lambda}_j^b$  only depends on  $\xi_j$  and the variance of the aggregate marginal cost is positive, the previous expression simplifies to  $\text{cov}(f(\xi_j), h(\xi_j))$ , where  $f(\xi_j) = \xi_j / (1 + \beta \xi_j)$  and  $h(\xi_j) = \tilde{A}_{1j}$ . It is straightforward to show that  $f$  is increasing in  $\xi_j$ , the first term of  $h$  is independent from  $\xi_j$  while the second term  $(1 - \rho_j^2) \psi_j^2 \sigma_{S_j}^2 / (1 - \xi_j \rho_j)$  is increasing in  $\xi_j$ . Using Lemma 1, it follows that  $A_1$  is positive, as  $J \rightarrow \infty$ .

The same reasoning can show that

$$\begin{aligned}\lim_{J \rightarrow \infty} A_2 &= \frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov} \left( \tilde{\lambda}_j^b, \frac{\xi_j \psi_j (1 - \rho_j^2)}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) > 0 \\ \lim_{J \rightarrow \infty} A_3 &= \frac{1}{\Lambda} V(s_t) \operatorname{cov} \left( \tilde{\theta}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) - \frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov}(\tilde{\theta}_j, \sigma_{S_j}^2) \\ &= \frac{1}{\Lambda} V(s_t) \operatorname{cov} \left( \tilde{\theta}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) > 0.\end{aligned}$$

Invoking (H2) to assume equal sectoral weights and imposing (H1) we have

$$B_1 = \frac{E(\pi_{t-1}^2 - \pi_{t-1} \pi_{t+1})}{\Lambda J^2} \sum_{j=1}^J \tilde{\lambda}_j^b E(s_t (\pi_{j,t-1} - \pi_{j,t+1})).$$

Using (H4) and Lemma 2, it follows that

$$\lim_{J \rightarrow \infty} B_1 = -\frac{1}{\Lambda} E(\pi_{t-1} (\pi_{t-1} - \pi_{t+1})) \operatorname{cov}(\tilde{\lambda}_j^b, \tilde{B}_{1j})$$

where  $\tilde{B}_{1j} = \xi_j \psi_j (1 - \rho_j^2) \sigma_{S_j}^2 / (1 - \xi_j \rho_j)$ . Given that  $E(\pi_{t-1} (\pi_{t-1} - \pi_{t+1}))$  is positive and the covariance term is positive from Lemma 1, it follows that the asymptotic bias of  $B_1$  is negative.

A similar argument shows that

$$\lim_{J \rightarrow \infty} B_2 = -\frac{1}{\Lambda} E(s_t (\pi_{t-1} - \pi_{t+1})) \operatorname{cov}(\tilde{\lambda}_j^b, \tilde{B}_{2j})$$

where  $\tilde{B}_{2j} = \sigma_{\eta_j}^2 + \psi_j^2 (1 - \rho_j^2) \sigma_{S_j}^2 / (1 - \xi_j \rho_j)$ . Since the first covariance term  $E(s_t (\pi_{t-1} - \pi_{t+1}))$  is negative and the second term,  $\operatorname{cov}(\tilde{\lambda}_j^b, \tilde{B}_{2j})$ , is positive, it follows that the asymptotic bias of  $B_2$  is positive

Finally, we have

$$\begin{aligned}\lim_{J \rightarrow \infty} B_3 &= \frac{1}{\Lambda} E(\pi_{t-1} (\pi_{t-1} - \pi_{t+1})) \operatorname{cov}(\tilde{\theta}_j, \tilde{B}_{31j}) - \frac{1}{\Lambda} E(s_t (\pi_{t-1} - \pi_{t+1})) \operatorname{cov}(\tilde{\theta}_j, \tilde{B}_{32j}) \\ &= -\frac{1}{\Lambda} E(s_t (\pi_{t-1} - \pi_{t+1})) \operatorname{cov}(\tilde{\theta}_j, \tilde{B}_{32j})\end{aligned}$$

where  $\tilde{B}_{31j} = \sigma_{S_j}^2$  and  $\tilde{B}_{32j} = \psi_j \rho_j \sigma_{S_j}^2 / (1 - \xi_j \rho_j)$ . Since the first term is asymptotically equal to 0 and  $\operatorname{cov}(\tilde{\theta}_j, \tilde{B}_{32j})$  is positive, the asymptotic bias of  $B_3$  is positive.

□

## Proof of Proposition 2

The proof is the same. The overall sign of  $C_1$ ,  $C_2$ ,  $D_1$ , and  $D_2$  cannot be determined without further assumptions, but their respective sub-components can be signed. We first have

$$\lim_{J \rightarrow \infty} C_{11} = \frac{1}{\Lambda} E(s_t^2) \operatorname{cov}(\tilde{\xi}_j, \tilde{C}_{11})$$

where  $\tilde{C}_{11} = \sigma_{\eta_j}^2 / (1 - \xi_j^2)$ . Using Lemma 1, one obtains that the covariance is positive and thus the asymptotic bias of  $C_{11}$  is positive. The same reasoning shows that

$$\begin{aligned}\lim_{J \rightarrow \infty} C_{12} &= \frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov} \left( \tilde{\xi}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) < 0 \\ \lim_{J \rightarrow \infty} C_{21} &= \frac{1}{\Lambda} E(s_t^2) \operatorname{cov} \left( \tilde{\psi}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) > 0 \\ \lim_{J \rightarrow \infty} C_{22} &= \frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov} \left( \tilde{\psi}_j, \frac{\sigma_{u_j}^2}{1 - \rho_j^2} \right) < 0.\end{aligned}$$

The last two equations require that each term of the covariance be increasing in  $\rho_j$ , which is trivially true. Similarly, the asymptotic biases on  $D_i$ s are given by

$$\begin{aligned}\lim_{J \rightarrow \infty} D_{11} &= \frac{1}{\Lambda} E(\pi_{t-1}^2) \operatorname{cov} \left( \tilde{\xi}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) > 0 \\ \lim_{J \rightarrow \infty} D_{12} &= -\frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov} \left( \tilde{\xi}_j, \frac{\sigma_{\eta_j}^2}{1 - \xi_j^2} + \frac{1 + \xi_j \rho_j}{(1 - \xi_j^2)(1 - \xi_j \rho_j)} \psi_j^2 \sigma_{S_j}^2 \right) < 0 \\ \lim_{J \rightarrow \infty} D_{21} &= \frac{1}{\Lambda} E(\pi_{t-1}^2) \operatorname{cov} \left( \tilde{\psi}_j, \frac{\sigma_{u_j}^2}{1 - \rho_j^2} \right) > 0 \\ \lim_{J \rightarrow \infty} D_{22} &= -\frac{1}{\Lambda} E(s_t \pi_{t-1}) \operatorname{cov} \left( \tilde{\psi}_j, \frac{\psi_j \rho_j}{1 - \xi_j \rho_j} \sigma_{S_j}^2 \right) < 0.\end{aligned}$$

Finally, the bias on  $\rho_{ML}$  depends on the covariance between the persistence of the marginal cost  $\tilde{\rho}_j = (\rho_j - \rho)$  and the parameters  $\delta_j = w_j E(s_t s_{j,t})$ . Under (H2) and (H3), we have  $\delta_j = w_j \sigma_{u_j}^2 / (1 - \rho_j^2)$ , which is clearly an increasing function of  $\rho_j$ . As a consequence, the asymptotic bias on  $\rho_{ML}$  is positive.

□



### Appendix 3: Cross-Industry Linkages

Table A1: Correlations of sectoral Phillips Curve residuals

Industry	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[1]	1.000							
[2]	0.098	1.000						
[3]	0.222*	-0.019	1.000					
[4]	-0.081	0.079	0.293*	1.000				
[5]	0.094	0.113	0.333*	0.422*	1.000			
[6]	0.035	0.075	-0.127	0.025	0.089	1.000		
[7]	0.044	0.256*	-0.042	-0.084	0.198*	-0.059	1.000	
[8]	-0.017	0.202*	0.078	0.151	0.120	0.171	0.038	1.000
[9]	-0.064	0.186	-0.103	-0.229*	-0.206*	0.070	-0.007	-0.145
[10]	0.191	-0.021	0.193	0.091	0.004	0.173	-0.016	0.121
[11]	0.030	-0.068	0.122	0.114	0.120	-0.217*	0.031	-0.129
[12]	-0.119	0.094	0.086	-0.047	0.033	-0.116	0.313*	0.213*
[13]	0.112	-0.062	0.048	-0.068	0.033	-0.009	-0.673*	0.034
[14]	0.140	0.076	0.305*	0.152	0.260*	0.071	0.178	0.262*
[15]	-0.010	0.017	0.283*	0.216*	0.281*	-0.031	-0.023	0.142
[16]	-0.035	0.173	0.005	0.065	0.034	0.024	0.061	0.069

Industry	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
[9]	1.000							
[10]	-0.034	1.000						
[11]	-0.281*	-0.034	1.000					
[12]	-0.104	0.040	-0.085	1.000				
[13]	0.067	0.022	-0.025	-0.261*	1.000			
[14]	-0.288*	0.111	0.027	0.439*	-0.080	1.000		
[15]	-0.116	-0.013	0.018	0.167	-0.052	0.235*	1.000	
[16]	0.045	-0.011	-0.037	0.088	0.028	0.072	0.259*	1.000

Note: \* means that the correlation is statistically significant at 5% level.

Table A2: Correlations of sectoral real marginal cost residuals

Industry	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
[1]	1.000							
[2]	-0.008	1.000						
[3]	0.209*	-0.003	1.000					
[4]	0.233*	0.092	0.177	1.000				
[5]	0.247*	-0.014	0.098	0.398*	1.000			
[6]	0.244*	0.109	0.289*	0.259*	0.309*	1.000		
[7]	-0.038	0.193	-0.013	-0.064	0.074	0.015	1.000	
[8]	0.099	0.003	0.252*	0.188	0.088	0.349*	-0.019	1.000
[9]	0.120	0.210*	0.311*	0.206*	0.076	0.346*	-0.022	0.140
[10]	0.137	0.100	0.177	0.169	0.025	0.331*	0.070	0.095
[11]	0.006	-0.054	0.091	0.180	0.041	0.073	-0.018	0.055
[12]	-0.060	0.082	0.188	-0.179	0.019	0.246*	0.152	0.106
[13]	0.068	-0.042	0.208*	0.078	0.070	0.056	-0.672*	0.066
[14]	0.013	0.100	0.135	0.115	-0.050	-0.005	0.056	0.275*
[15]	-0.090	-0.079	0.066	0.140	0.116	0.047	-0.062	0.084
[16]	0.085	-0.113	-0.097	0.000	0.111	-0.102	-0.046	-0.113
Industry	[9]	[10]	[11]	[12]	[13]	[14]	[15]	[16]
[9]	1.000							
[10]	0.191	1.000						
[11]	-0.080	0.133	1.000					
[12]	0.145	-0.056	-0.005	1.000				
[13]	0.089	-0.070	0.085	0.055	1.000			
[14]	0.031	0.115	0.169	-0.087	0.005	1.000		
[15]	0.054	-0.011	-0.026	-0.047	-0.032	0.198	1.000	
[16]	-0.101	-0.086	0.162	-0.267*	-0.023	0.159	0.493*	1.000

Note: The notation \* means that the correlation is statistically significant at 5% level.

## Appendix 4: List of Industries

- Agriculture [1]
  
- Manufacturing
  - Food manufacturing [2]
  - Consumption goods [3]
  - Car industry [4]
  - Equipment goods [5]
  - Intermediary goods [6]
  - Energy [7]
  
- Service
  - Construction [8]
  - Trade [9]
  - Transportation [10]
  - Financial activities [11]
  - Real estate [12]
  - Business services [13]
  - Personal services [14]
  - Education and health services [15]
  - Government [16]

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Table 1a: GMM and ML estimates (baseline case)

	True Value	GMM	Decomposition			ML	Decomposition	
			<i>A1</i>	<i>A2</i>	<i>A3</i>		<i>C1</i>	<i>C2</i>
$\xi$	0.412	0.660	0.011	0.004	0.171	0.867	0.006	0.424
$\alpha$	0.858	0.738				0.846		
$\rho$	0.920	–				0.904		
$\lambda^b$	0.291	0.399				0.467		
$\lambda^f$	0.705	0.599				0.533		
			<i>B1</i>	<i>B2</i>	<i>B3</i>			
$\theta$	0.023	0.058	–0.002	0.000	0.032	0.016		
							<i>D1</i>	<i>D2</i>
$\psi$	0.349	0.923				0.292	–0.011	0.021

Note: The “true values” used for the simulations correspond to the random coefficient estimation of the aggregate Phillips Curve reported in Table 4 (with uniform weights). The variances of the random parameters are 0.05 for  $\rho$  (ruling out explosive paths), and 0.1 for  $\xi$  and  $\alpha$ . Uniform weights are used to compute estimates.



Table 1b: GMM and ML estimates - Alternative Sources of Heterogeneity

	True value	GMM	Decomposition			ML	Decomposition	
$\sigma_\xi^2 = 0, \sigma_\alpha^2 = 0.2$								
			<i>A1</i>	<i>A2</i>	<i>A3</i>		<i>C1</i>	<i>C2</i>
$\xi$	0.412	0.877	0.023	0.001	0.343	0.909	-0.002	0.496
$\alpha$	0.839	0.608				0.800		
$\rho$	0.921	–				0.901		
$\lambda^b$	0.293	0.470				0.478		
$\lambda^f$	0.703	0.530				0.521		
			<i>B1</i>	<i>B2</i>	<i>B3</i>			
$\theta$	0.024	0.144	-0.001	0.001	0.090	0.028		
							<i>D1</i>	<i>D2</i>
$\psi$	0.437	2.422				0.482	-0.007	0.209
$\sigma_\xi^2 = 0.2, \sigma_\alpha^2 = 0$								
			<i>A1</i>	<i>A2</i>	<i>A3</i>		<i>C1</i>	<i>C2</i>
$\xi$	0.412	0.532	0.007	0.014	0.053	0.826	0.073	0.300
$\alpha$	0.858	0.803				0.853		
$\rho$	0.921	–				0.901		
$\lambda^b$	0.293	0.348				0.454		
$\lambda^f$	0.702	0.649				0.545		
			<i>B1</i>	<i>B2</i>	<i>B3</i>			
$\theta$	0.018	0.033	-0.003	0.000	0.016	0.016		
							<i>D1</i>	<i>D2</i>
$\psi$	0.285	0.545				0.273	-0.036	0.052

Note: The “true values” used for the simulations correspond to the random coefficient estimation of the aggregate Phillips Curve reported in Table 4 (with uniform weights). Uniform weights are used to compute estimates.

Table 2: Summary statistics on French data

Industry	Weights	$\bar{\pi}$	$\bar{s}$	$corr(\pi_{t-1}, \pi_t)$	$corr(s_{t-1}, s_t)$	$corr(\pi_t, s_t)$
Aggregate	100.00	3.996	-0.095	0.921	0.984	0.887
Agriculture	2.92	1.255	-0.276	0.782	0.977	-0.247
Food Mfg	2.33	3.477	-0.102	-0.075	0.778	-0.320
Cons. Goods	3.02	2.639	-0.087	0.620	0.939	0.367
Car	0.96	3.293	-3.616	0.291	0.981	0.198
Equip. Goods	2.96	0.237	-0.128	0.041	0.915	-0.412
Inter. Goods	5.72	2.788	-1.007	0.725	0.988	0.600
Energy	2.18	5.393	-0.934	-0.281	0.683	-0.449
Constr.	6.67	4.889	-0.327	0.511	0.977	0.389
Trade	10.57	4.241	-0.253	0.760	0.974	0.662
Transport	3.76	2.935	-0.112	0.027	0.777	0.034
Finance	5.01	3.366	-0.410	0.600	0.971	0.143
Real Estate	11.82	5.023	-0.272	0.864	0.983	-0.683
Business Serv.	14.19	3.635	-0.021	-0.290	0.946	-0.362
Personal Serv.	5.75	5.486	0.062	0.758	0.961	-0.707
Educ. & Health	13.94	5.542	-0.261	0.933	0.986	0.848
Government.	8.21	4.419	-0.050	0.954	0.917	0.484

Table 3: Sectoral ML estimates

Industry	$\xi$	$\alpha$	$\rho$	$\psi$
Agriculture	0.718 (0.069)	0.982 (0.044)	0.981 (0.021)	0.017 (0.066)
Food Mfg.	0.157 (0.056)	0.695 (0.087)	0.795 (0.062)	0.642 (0.266)
Cons. Goods	0.613 (0.073)	0.845 (0.050)	0.938 (0.033)	0.419 (0.140)
Car	0.166 (0.071)	0.938 (0.027)	0.979 (0.019)	0.150 (0.048)
Equip. Goods	0.157 (0.067)	0.992 (0.708)	0.868 (0.040)	0.001 (0.130)
Inter. Goods	0.375 (0.056)	0.908 (0.023)	0.971 (0.015)	0.265 (0.043)
Energy	0.001 (0.034)	0.502 (0.103)	0.673 (0.070)	1.501 (0.619)
Constr.	0.353 (0.069)	0.919 (0.036)	0.982 (0.021)	0.286 (0.066)
Trade	0.434 (0.058)	0.891 (0.036)	0.972 (0.022)	0.378 (0.060)
Transport	0.126 (0.076)	0.595 (0.075)	0.777 (0.061)	1.210 (0.324)
Finance	0.184 (0.064)	0.906 (0.038)	0.969 (0.023)	0.263 (0.075)
Real Estate	0.862 (0.070)	0.997 (0.074)	0.971 (0.018)	0.001 (0.031)
Business Serv.	0.016 (0.062)	0.970 (0.338)	0.886 (0.039)	0.010 (0.197)
Personal Serv.	0.718 (0.082)	0.994 (0.627)	0.917 (0.029)	0.001 (0.147)
Educ. & Health	0.722 (0.061)	0.925 (0.020)	0.970 (0.016)	0.174 (0.044)
Govrnmnt.	0.925 (0.033)	0.935 (0.044)	0.921 (0.039)	0.058 (0.074)

Note: Standard deviation in parentheses.

Table 4: Aggregate estimates

Method	$\xi$	$\alpha$	$\lambda^b$	$\theta$	Duration	
GMM	0.738 (0.036) [0.000]	0.784 (0.112) [0.000]	0.426 (0.012) [0.000]	0.036 (0.025) [0.160]	4.636 (2.410) [0.057]	
Methods	$\xi$	$\alpha$	$\lambda^b$	$\psi$	Duration	$\rho$
ML	0.889 (0.038) [0.000]	0.880 (0.041) [0.000]	0.473 (0.011) [0.000]	0.235 (0.092) [0.012]	8.339 (2.831) [0.004]	0.935 (0.034) [0.000]
MG*	0.475 (0.038) [0.000]	0.880 (0.006) [0.000]	0.323 (0.012) [0.000]	0.224 (0.017) [0.000]	8.336 (0.401) [0.000]	0.931 (0.034) [0.000]
MG**	0.408 (0.038) [0.000]	0.838 (0.010) [0.000]	0.290 (0.010) [0.000]	0.336 (0.027) [0.000]	6.166 (0.371) [0.000]	0.911 (0.034) [0.000]
RC*	0.412 (0.081) [0.000]	0.861 (0.045) [0.000]	0.292 (0.044) [0.000]	0.273 (0.119) [0.024]	7.180 (2.324) [0.003]	0.922 (0.025) [0.000]
RC**	0.412 (0.079) [0.000]	0.862 (0.044) [0.000]	0.292 (0.043) [0.000]	0.271 (0.116) [0.021]	7.220 (2.282) [0.002]	0.922 (0.024) [0.000]

Note: Standard deviation in parentheses, p-values in brackets.

\*: Estimation from the reduced form using sector weights.

\*\* : Estimation from the reduced form using uniform weights.

All industries are used in the estimation.

Table 5: Aggregate estimates with contemporaneous correlations across sectors

	Method	$\xi$	$\alpha$	$\lambda^b$	$\theta$	Duration	
Aggregate	GMM	0.738 (0.036) [0.000]	0.784 (0.112) [0.000]	0.426 (0.012) [0.000]	0.036 (0.025) [0.160]	4.636 (2.410) [0.057]	
		$\xi$	$\alpha$	$\lambda^b$	$\psi$	Duration	$\rho$
Aggregate	ML	0.889 (0.038) [0.000]	0.880 (0.041) [0.000]	0.473 (0.011) [0.000]	0.235 (0.092) [0.012]	8.339 (2.831) [0.004]	0.935 (0.034) [0.000]
		$\xi$	$\alpha$	$\lambda^b$	$\psi$	Duration	$\rho$
SURE	MG*	0.357 (0.021) [0.000]	0.878 (0.007) [0.000]	0.264 (0.011) [0.000]	0.190 (0.016) [0.000]	8.166 (0.458) [0.000]	0.913 (0.004) [0.000]
	RC*	0.268 (0.065) [0.000]	0.870 (0.036) [0.000]	0.215 (0.044) [0.000]	0.197 (0.075) [0.010]	7.701 (2.127) [0.001]	0.904 (0.023) [0.000]
CCE	MG	0.326 (0.030) [0.000]	0.837 (0.016) [0.000]	0.260 (0.010) [0.000]	0.254 (0.041) [0.000]	6.127 (0.591) [0.000]	0.877 (0.008) [0.000]
	RC	0.293 (0.085) [0.001]	0.851 (0.046) [0.000]	0.222 (0.052) [0.000]	0.202 (0.097) [0.040]	6.692 (2.039) [0.001]	0.871 (0.037) [0.000]

Note: Standard deviation in parentheses, p-value in brackets.

\*: Estimation from the reduced form using sector weights.

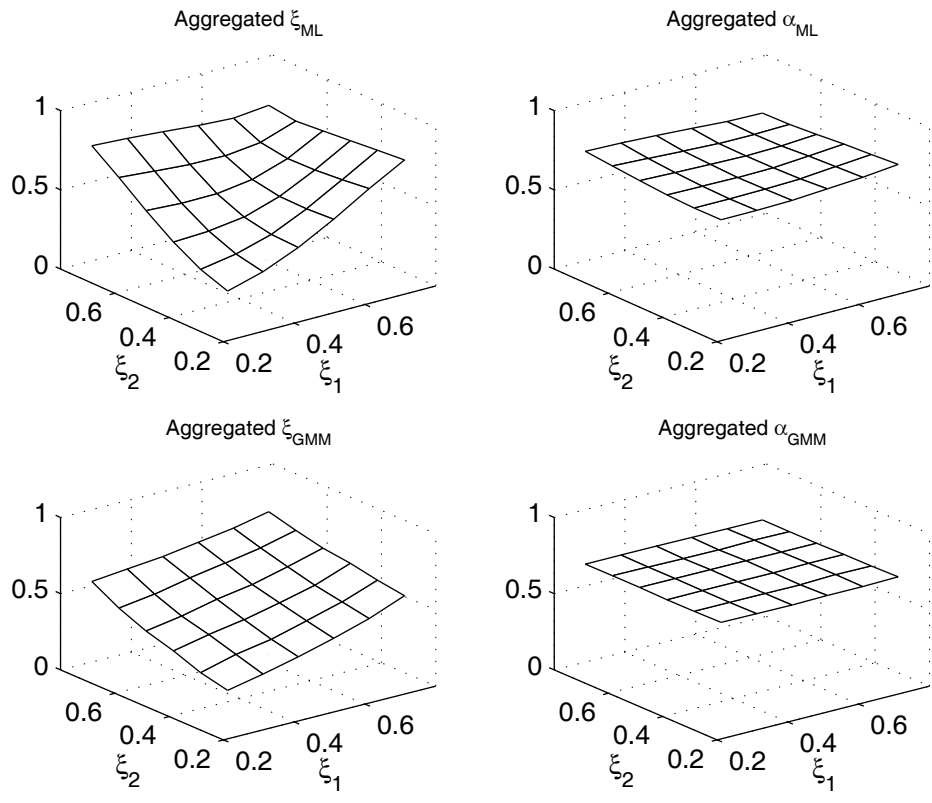


Figure 1: Two-Sector Model: Heterogeneity on  $\xi_j$

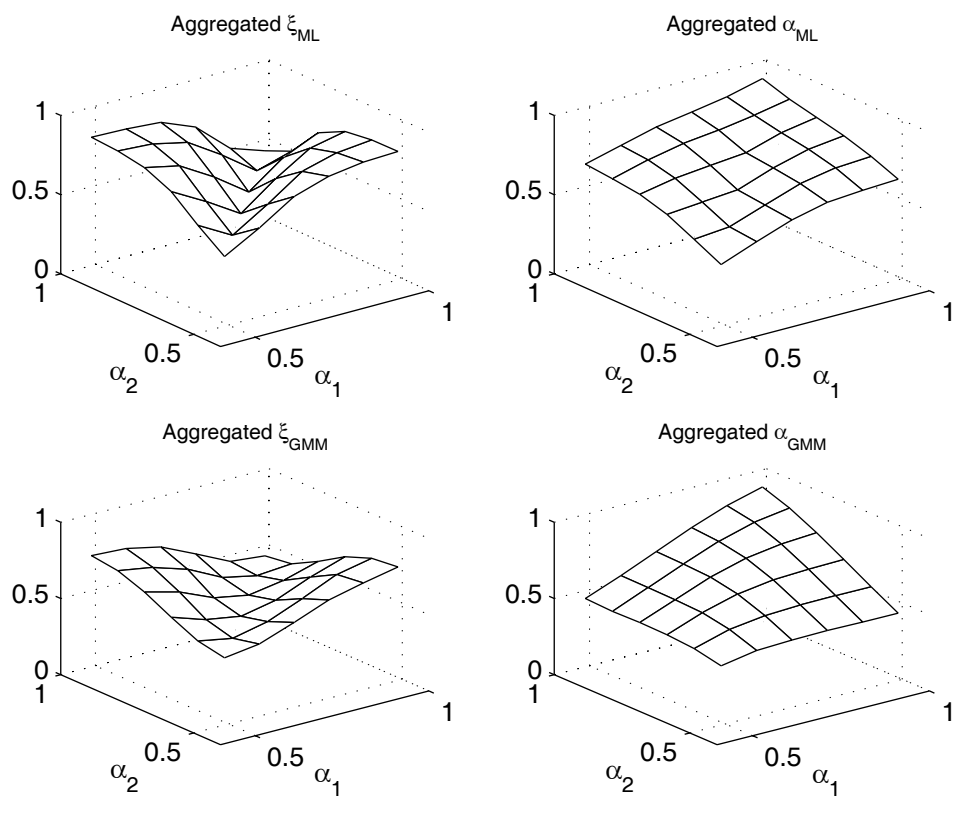


Figure 2: Two-Sector Model: Heterogeneity on  $\alpha_j$

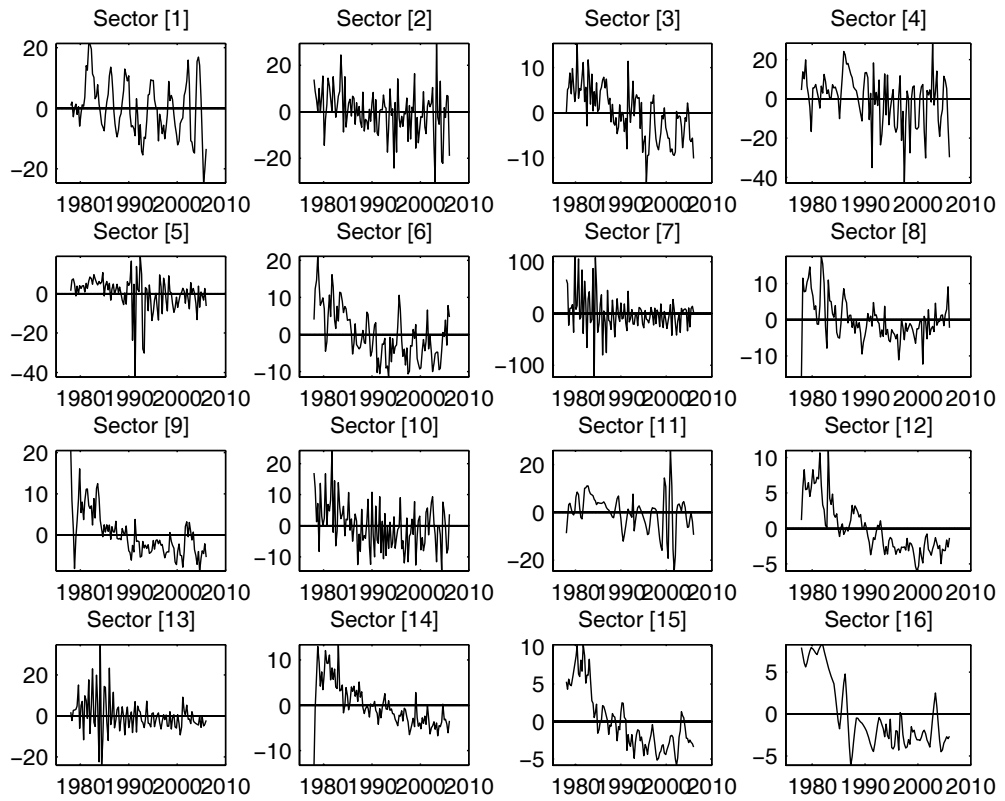


Figure 3: Sectoral Inflation



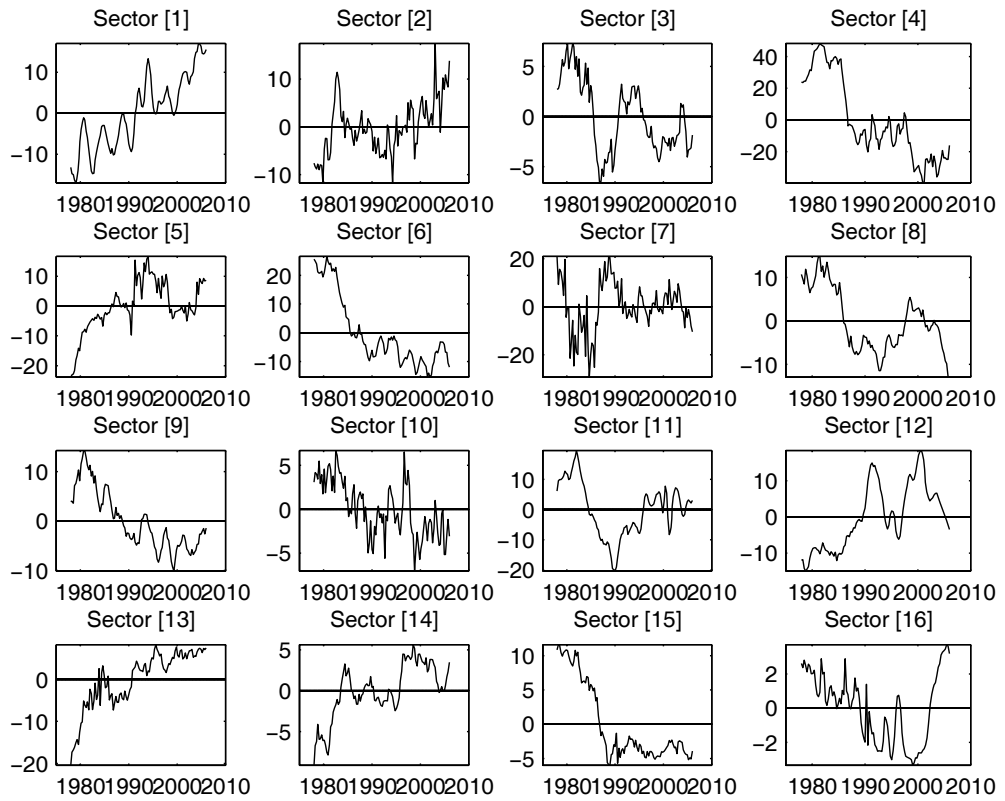


Figure 4: Sectoral Real Marginal Cost

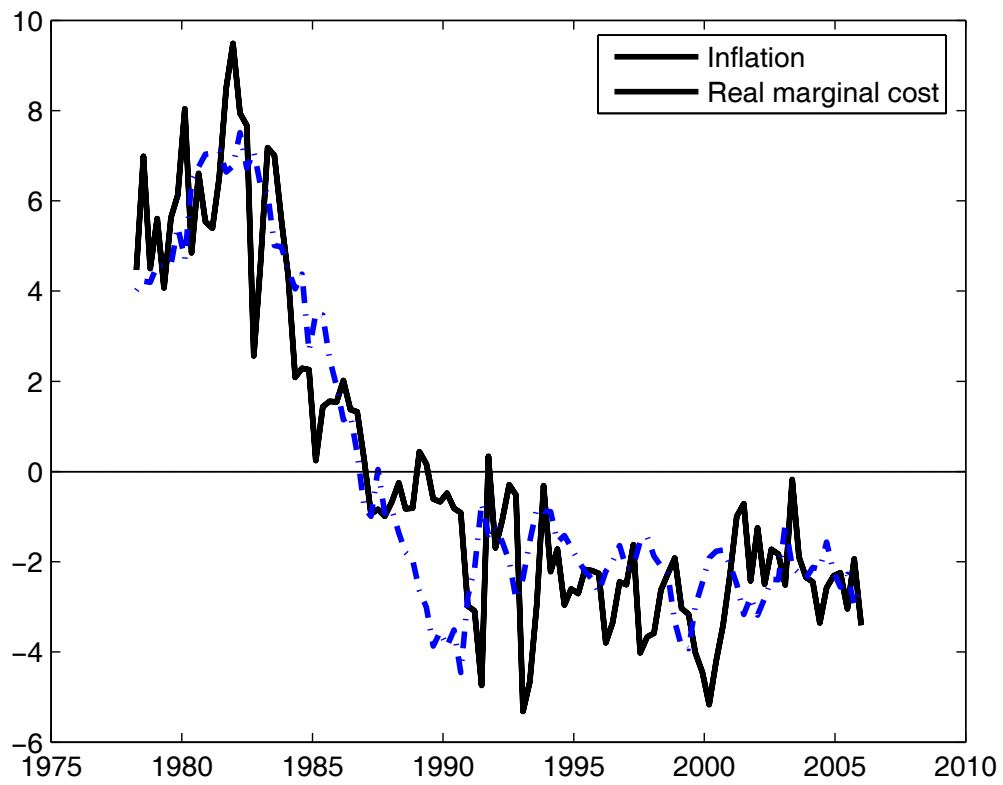


Figure 5: Aggregate Inflation and Real Marginal Cost