Business Cycle Dynamics under Rational Inattention*

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Abstract

This paper studies a dynamic stochastic general equilibrium model with rational inattention. Decisionmakers have limited attention. Decisionmakers choose the optimal allocation of their attention. We study the implications of rational inattention for business cycle dynamics. For example, we study how rational inattention affects impulse responses of prices and quantities to monetary policy shocks, aggregate technology shocks and micro-level shocks.

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1 Introduction

This paper studies a dynamic stochastic general equilibrium model with rational inattention. We model the idea that agents cannot attend perfectly to all available information. Following Sims (2003), we model attention as a flow of information, and we model agents' limited attention as a bound on information flow. We let decisionmakers choose the optimal allocation of attention. For example, agents decide how to allocate their attention across their different decision problems. Furthermore, agents decide how to attend to the different factors determining an optimal decision.

The economy consists of households, firms and a government. Households consume a variety of goods, can hold nominal government bonds and supply differentiated types of labor. Firms hire labor and produce differentiated goods. The central bank sets the nominal interest rate according to a Taylor rule. There are no adjustment costs. Every period, households take consumption and wage setting decisions. Each period, firms take input and price setting decisions. We compute the impulse responses of prices and quantities to monetary policy shocks, aggregate technology shocks and micro-level shocks under both perfect information and rational inattention.

The impulse responses under rational inattention have several properties of empirical impulse response functions.

2 Model

2.1 Households

There are J households. Households supply differentiated types of labor, consume a variety of goods and can hold nominal government bonds.

Each household seeks to maximize the expected discounted sum of period utility. The discount factor is $\beta \in (0,1)$. The period utility function is

$$U\left(C_{jt}, L_{j1t}, \ldots, L_{jNt}\right) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi \sum_{n=1}^{N} e^{-\chi_{jnt}} \frac{L_{jnt}^{1+\psi}}{1+\psi}, \tag{1}$$

where C_{jt} is composite consumption by household j in period t, L_{jnt} is supply of household j's nth type of labor in period t, and χ_{jnt} is a preference shock affecting the disutility of supplying the nth type of labor. We introduce preference shocks in order to generate variation in relative wage rates. We assume that each household supplies N types of labor in order to allow for a certain degree of risk sharing within the household. The parameter $\gamma > 0$ is the coefficient of relative risk aversion, the parameter $\varphi > 0$ affects the disutility of supplying labor and the parameter $\psi > 0$ is the inverse of the Frisch elasticity of labor supply. Composite consumption is given by the usual Dixit-Stiglitz aggregator

$$C_{jt} = \left(\sum_{i=1}^{I} C_{ijt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where C_{ijt} is consumption of good i by household j in period t. We assume that the elasticity of substitution between different goods exceeds one, $\theta > 1$.

Households can save by holding nominal government bonds. The flow budget constraint of household j in period t is

$$\sum_{i=1}^{I} P_{it} C_{ijt} + B_{jt} = R_{t-1} B_{jt-1} + (1 + \tau_w) \sum_{n=1}^{N} W_{jnt} L_{jnt} + \frac{D_t}{J} - \frac{T_t}{J}, \tag{3}$$

where P_{it} is the price of good i in period t, B_{jt} are bond holdings by household j between period t and period $t+1$, R_{t-1} is the nominal interest rate on bond holdings between period $t-1$ and period t, τ_w is a wage subsidy, W_{jnt} is the nominal wage rate for household j's nth type of labor, (D_t/J) is a pro-rata share of nominal aggregate profits and (T_t/J) is a pro-rata share of nominal lump-sum taxes. We assume that all J households have the same initial bond holdings. We assume a natural debt limit.

Every period each household chooses a consumption vector, $(C_{1jt}, \ldots, C_{Ijt})$, and a vector of nominal wage rates, $(W_{j1t}, \ldots, W_{jNt})$.[1] Each household commits to supply any quantity of labor at the chosen nominal wage rates.

We will solve the household problem under two alternative assumptions. First, we will assume that households have perfect information. Afterwards, we will assume that households have limited attention, that is, households take their decisions subject to a constraint on information flow.

[1] Bond holdings then follow from the labor demand function derived below and the flow budget constraint (3).

2.2 Firms

There are I firms in the economy. Firms hire labor in order to produce differentiated goods.

The technology of firm i is given by

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^{\alpha}, \tag{4}$$

where Y_{it} is output, $(e^{a_t} e^{a_{it}})$ is total factor productivity and L_{it} is composite labor input of firm i in period t. Total factor productivity has an aggregate component, e^{a_t}, and a firm-specific component, $e^{a_{it}}$. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to composite labor. Composite labor is given by the following constant elasticity aggregator

$$L_{it} = \left(\sum_{j=1}^{J} \sum_{n=1}^{N} L_{ijnt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{5}$$

where L_{ijnt} is firm i's input of type jn labor in period t. Recall that type jn labor is household j's nth type of labor. We assume that the elasticity of substitution between different types of labor exceeds one, $\eta > 1$.

The nominal profits of firm i in period t equal

$$(1 + \tau_p) P_{it} Y_{it} - \sum_{j=1}^{J} \sum_{n=1}^{N} W_{jnt} L_{ijnt}, \tag{6}$$

where τ_p is a production subsidy.

Every period each firm chooses a labor mix and a price, P_{it}. Each firm commits to supply any quantity of the good at the chosen price.

We will solve the firm problem under two alternative assumptions. First, we will assume that decisionmakers in firms have perfect information. Afterwards, we will assume that decisionmakers in firms have limited attention, that is, they take their decisions subject to a constraint on information flow.

2.3 Government

There is a monetary authority and a fiscal authority. Let $\Pi_t = (P_t / P_{t-1})$ denote inflation where P_t is a price index that will be defined later. Let $Y_t = \sum_{i=1}^{I} Y_{it}$ denote aggregate output. The central bank sets the nominal interest rate according to the rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_R} e^{\varepsilon_t^R}, \tag{7}$$

where R, Π and Y are the values of the nominal interest rate, inflation and output in the non-stochastic steady state, and ε_t^R is a monetary policy shock. The policy parameters satisfy $\rho_R \in [0,1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

The government budget constraint in period t reads

$$T_t + (B_t - B_{t-1}) = (R_{t-1} - 1) B_{t-1} + \tau_w \left(\sum_{j=1}^{J} \sum_{n=1}^{N} W_{jnt} L_{jnt}\right) + \tau_p \left(\sum_{i=1}^{I} P_{it} Y_{it}\right). \tag{8}$$

The government has to finance interest on nominal government bonds, the wage subsidy and the production subsidy. The government can collect taxes or issue new government bonds.

2.4 Shocks

There are four types of shocks in the economy: monetary policy shocks, aggregate productivity shocks, firm-specific productivity shocks and labor-specific preference shocks. We assume that, for all i and jn, the processes $\{\varepsilon_t^R\}$, $\{a_t\}$, $\{a_{it}\}$ and $\{\chi_{jnt}\}$ are independent. Furthermore, we assume that all the firm-specific productivity processes, $\{a_{it}\}$, are independent across firms and all the labor-specific preference shocks, $\{\chi_{jnt}\}$, are independent across types of labor. Finally, we assume that all these processes are stationary Gaussian processes with mean zero. In the following, we denote the period t innovation to a_t, a_{it} and χ_{jnt} by ε_t^A, ε_{it}^I and ε_{jnt}^χ, respectively.

3 Solution under perfect information

In this section we derive the equilibrium under perfect information, that is, we assume that in period t all households and all firms know all variables up to and including period t. We will show that under perfect information the classical dichotomy holds. Monetary policy has no real effects. Quantities and relative prices depend only on aggregate productivity (a_t), firm-specific productivity (a_{it}) and labor-specific disutility of work (χ_{jnt}).

3.1 Equations characterizing equilibrium

Cost minimization implies that the demand for type jn labor in period t is given by

$$L_{jnt} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta} L_t, \tag{9}$$

where W_t is the following wage index

$$W_t = \left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{10}$$

and L_t is the aggregate composite labor input

$$L_t = \sum_{i=1}^{I} L_{it}. \tag{11}$$

The problem of household j is to choose a contingent plan for the consumption vector and for the vector of nominal wage rates so as to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}^{1-\gamma}-1}{1-\gamma} - \varphi\sum_{n=1}^{N} e^{-\chi_{jnt}}\frac{L_{jnt}^{1+\psi}}{1+\psi}\right)\right], \tag{12}$$

subject to the consumption aggregator (2), the flow budget constraint (3), the natural debt limit and the labor demand function (9). The first-order conditions for the household problem are:

$$C_{jt}^{-\gamma} = E_t\left[\beta\frac{R_t}{\Pi_{t+1}}C_{jt+1}^{-\gamma}\right], \tag{13}$$

for all i

$$\frac{C_{ijt}}{C_{jt}} = \left(\frac{P_{it}}{P_t}\right)^{-\theta}, \tag{14}$$

and for all n

$$\frac{W_{jnt}}{P_t} = \frac{1}{1+\tau_w}\frac{\eta}{\eta-1}\varphi e^{-\chi_{jnt}}\left[\left(\frac{W_{jnt}}{W_t}\right)^{-\eta}L_t\right]^{\psi}C_{jt}^{\gamma}, \tag{15}$$

where E_t is the expectation operator conditioned on information in period t and P_t is the following price index

$$P_t = \left(\sum_{i=1}^{I} P_{it}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{16}$$

Equation (13) is the consumption Euler equation. Equation (14) characterizes the optimal consumption basket. Equation (15) characterizes the optimal wage setting behavior.

Throughout the paper we will assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. Here this implies that

$$1 + \tau_w = \frac{\eta}{\eta - 1}. \tag{17}$$

Multiplying equation (14) by C_{jt} and summing over all households yields the demand for good i in period t

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t, \tag{18}$$

where C_t is aggregate composite consumption

$$C_t = \sum_{j=1}^{J} C_{jt}. \tag{19}$$

The problem of firm i under perfect information is to choose a price and a labor mix so as to maximize profits (6) subject to the technology (4)-(5), the requirement that output has to equal demand and the demand function (18). The firm problem is a static decision problem, because there are no adjustment costs and the demand function (18) is static. The first-order conditions for the firm problem are:

$$P_{it} = \frac{1}{1+\tau_p}\frac{\theta}{\theta-1}W_t\frac{1}{\alpha}\frac{\left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t\right]^{\frac{1}{\alpha}-1}}{\left(e^{a_t}e^{a_{it}}\right)^{\frac{1}{\alpha}}}, \tag{20}$$

and for all jn

$$\frac{L_{ijnt}}{L_{it}} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta}, \tag{21}$$

where W_t is the wage index (10). Equation (20) characterizes the profit-maximizing price. Throughout the paper we will assume that the government sets the production subsidy τ_p so as to correct the distortion arising from firms' market power on the goods market. This now implies that

$$1 + \tau_p = \frac{\theta}{\theta - 1}. \tag{22}$$

Equation (21) characterizes the profit-maximizing labor mix. Multiplying equation (21) by L_{it} and summing over all firms yields the labor demand function (9).

3.2 Non-stochastic steady state

We call the following situation a non-stochastic steady state: there are no shocks; all equations characterizing equilibrium are satisfied; and quantities, relative prices, the nominal interest rate and inflation are constant over time. Here we report some relationships in the non-stochastic steady state that we will use below.

Equation (20) implies that in the non-stochastic steady state all firms set the same price. Thus households choose a consumption basket with equal weights, implying that all firms produce the same amount and have the same composite labor input. It follows from the price index (16), the consumption aggregator (2), the definition of aggregate output and the definition of aggregate composite labor input (11) that

$$\left(\frac{P_i}{P}\right)^{1-\theta} = \left(\frac{C_{ij}}{C_j}\right)^{\frac{\theta-1}{\theta}} = \frac{Y_i}{Y} = \frac{L_i}{L} = \frac{1}{I}, \tag{23}$$

where (P_i/P) denotes the value of (P_{it}/P_t) in the non-stochastic steady state etc.

Since all households face the same decision problem, all households choose the same composite consumption. It follows from the definition of aggregate composite consumption (19) that

$$\frac{C_j}{C} = \frac{1}{J}. \tag{24}$$

Furthermore, equation (15) implies that in the non-stochastic steady state all households set the same wage rate for all different types of labor. Thus firms choose a labor mix with equal weights. It follows from the wage index (10) and the labor aggregator (5) that

$$\left(\frac{W_{jn}}{W}\right)^{1-\eta} = \left(\frac{L_{ijn}}{L_i}\right)^{\frac{\eta-1}{\eta}} = \frac{1}{JN}, \tag{25}$$

where (W_{jn}/W) denotes the value of (W_{jnt}/W_t) in the non-stochastic steady state etc.

3.3 Log-linearization

In this subsection, we log-linearize the equations characterizing equilibrium. Afterwards we report the log-linear equilibrium dynamics under perfect information. In the following, $\tilde{P}_{it} = (P_{it}/P_t)$ denotes the relative price of good i, $\tilde{W}_{jnt} = (W_{jnt}/P_t)$ denotes the real wage rate for type jn labor and $\tilde{W}_t = (W_t/P_t)$ denotes the real wage index. Furthermore, small letters denote log-deviations from the non-stochastic steady state.

Log-linearizing the households' first-order conditions yields

$$c_{jt} = E_t \left[-\frac{1}{\gamma} \left(r_t - \pi_{t+1} \right) + c_{jt+1} \right], \tag{26}$$

$$c_{ijt} - c_{jt} = -\theta \tilde{p}_{it}, \tag{27}$$

and

$$\tilde{w}_{jnt} = -\frac{1}{1+\eta\psi}\chi_{jnt} + \frac{\eta\psi}{1+\eta\psi}\tilde{w}_t + \frac{\psi}{1+\eta\psi}l_t + \frac{\gamma}{1+\eta\psi}c_{jt}. \tag{28}$$

Furthermore, dividing the definition of the price index (16) by P_t, log-linearizing and using (23) yields

$$\sum_{i=1}^{I} \tilde{p}_{it} = 0. \tag{29}$$

Log-linearizing both the demand function (18) and the definition of aggregate composite consumption (19) and using (24) yields

$$c_{it} = -\theta \tilde{p}_{it} + c_t, \tag{30}$$

and

$$c_t = \frac{1}{J} \sum_{j=1}^{J} c_{jt}. \tag{31}$$

Log-linearizing the firms' first-order conditions yields

$$\tilde{p}_{it} = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right), \tag{32}$$

and

$$l_{ijnt} - l_{it} = -\eta \left(\tilde{w}_{jnt} - \tilde{w}_t \right). \tag{33}$$

Dividing the definition of the wage index (10) by W_t, log-linearizing and using (25) yields

$$\tilde{w}_t = \frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} \tilde{w}_{jnt}. \tag{34}$$

Log-linearizing the production function (4) as well as the labor aggregator (5) and using (25) yields

$$y_{it} = a_t + a_{it} + \alpha l_{it}, \tag{35}$$

and

$$l_{it} = \frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} l_{ijnt}. \tag{36}$$

Log-linearizing the labor demand function (9) as well as the definition of aggregate composite labor input (11) and using (23) yields

$$l_{jnt} = -\eta \left(\tilde{w}_{jnt} - \tilde{w}_t \right) + l_t, \tag{37}$$

and

$$l_t = \frac{1}{I} \sum_{i=1}^{I} l_{it}. \tag{38}$$

Log-linearizing the monetary policy rule (7) yields

$$r_t = \rho_R r_{t-1} + (1 - \rho_R) \left(\phi_\pi \pi_t + \phi_y y_t \right) + \varepsilon_t^R. \tag{39}$$

Finally, log-linearizing the definition of aggregate output and using (23) yields

$$y_t = \frac{1}{I} \sum_{i=1}^{I} y_{it}. \tag{40}$$

3.4 Log-linearized solution

Assume that I and N are sufficiently large so that[2]

$$\frac{1}{I} \sum_{i=1}^{I} a_{it} = 0, \tag{41}$$

and

$$\frac{1}{N} \sum_{n=1}^{N} \chi_{jnt} = 0. \tag{42}$$

Then the log-linearized aggregate dynamics under perfect information are given by

$$y_t = c_t = \frac{1+\psi}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{43}$$

$$l_t = \frac{1-\gamma}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{44}$$

$$\tilde{w}_t = \frac{\gamma+\psi}{1-\alpha+\alpha\gamma+\psi} a_t, \tag{45}$$

$$r_t - E_t \left[\pi_{t+1} \right] = \gamma \frac{1+\psi}{1-\alpha+\alpha\gamma+\psi} E_t \left[a_{t+1} - a_t \right]. \tag{46}$$

[2] Up to this point $N = 1$ is a special case of the model. Now we are making the assumption that N is sufficiently large so that households can insure against labor-specific preference shocks within the household. This assumption implies that all households have the same consumption level.

The proof is in Appendix A. Under perfect information aggregate output, aggregate employment, the real wage index and the real interest rate depend only on aggregate productivity. Monetary policy has no real effects. The nominal interest rate and inflation follow from the monetary policy rule (39) and the real interest rate (46). Since $(1-\rho_R)\,\phi_\pi > 0$ and $(1-\rho_R)\,\phi_\pi + \rho_R > 1$, the equilibrium paths of the nominal interest rate and inflation are locally determinate.[3]

Substituting the solution (43) and (45) into the price setting equation (32) yields

$$\tilde{p}_{it} = -\frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} a_{it}, \tag{47}$$

which from (27) implies that

$$c_{ijt} - c_{jt} = \frac{\theta\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}} a_{it}. \tag{48}$$

The relative price of good i and relative consumption of good i depend only on the firm-specific component of the productivity of firm i.

Since all households face the same decision problem and labor-specific preference shocks average out within the household, all households choose the same composite consumption. Thus $c_{jt} = c_t$. Substituting $c_{jt} = c_t$ and the solution (43)-(45) into the wage setting equation (28) yields

$$\tilde{w}_{jnt} - \tilde{w}_t = -\frac{1}{1+\eta\psi} \chi_{jnt}, \tag{49}$$

which from (33) implies that

$$l_{ijnt} - l_{it} = \frac{\eta}{1+\eta\psi} \chi_{jnt}. \tag{50}$$

The relative wage rate for type jn labor and the relative input of type jn labor depend only on the labor-specific disutility of work.

In summary, in this model monetary policy has no real effects under perfect information. Under perfect information fluctuations in quantities and in relative prices are driven by aggregate productivity shocks, firm-specific productivity shocks and labor-specific preference shocks. Next we will solve the model assuming that decisionmakers have limited attention.

[3] See Woodford (2003), chapter 2, Proposition 2.8.

4 Case 1: Firms rational inattention, households perfect information

In this section, we assume that decisionmakers in firms have limited attention. For the moment, we continue to assume that households have perfect information in order to isolate the role of limited attention on the side of firms.

4.1 Firms' objective

We assume that firm i chooses the allocation of attention so as to maximize the expected discounted sum of profits. Nominal profits are given by (6). Technology is given by (4)-(5). The demand function is given by (18), because households have perfect information. Substituting the technology (4)-(5) and the demand function (18) into the expression for nominal profits (6) and dividing by P_t yields the real profit function. Computing a log-quadratic approximation of the real profit function around the non-stochastic steady state yields the following expression for (minus) the expected discounted sum of losses in profits due to suboptimal behavior:

$$E\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left(x_t-x_t^*\right)'H\left(x_t-x_t^*\right)\right],$$

where

$$\underset{(JN\times 1)}{x_t}=\begin{pmatrix}\tilde{p}_{it}\\ \hat{l}_{i11t}\\ \vdots\\ \hat{l}_{iJ(N-1)t}\end{pmatrix},$$

$$\underset{(JN\times JN)}{H}=\tilde{W}L_i\begin{bmatrix}\frac{\theta(\theta-1)}{\alpha}-\frac{\theta^2}{\alpha^2} & 0 & \cdots & \cdots & 0\\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN}\\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots\\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN}\\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN}\end{bmatrix},$$

and the optimal behavior x_t^* is given by equations (32), (33) and (34). Here $\hat{l}_{ijnt}\equiv l_{ijnt}-l_{it}$. The derivation is in Appendix B. Note that, after the log-quadratic approximation of the profit function, losses in profits due to suboptimal behavior depend only on the deviation from the optimal behavior. Furthermore, the optimal behavior is given by the usual log-linearized first-order conditions. The H matrix contains all the information (up to second order) about how costly different types of mistakes are. The H matrix is the matrix of second derivatives of the real profit function with respect to x_t and x_t' evaluated at the non-stochastic steady state. The diagonal elements of the H matrix contain information about the cost of a mistake in a single variable. The off-diagonal elements of the H matrix contain information about how a mistake in one variable affects the cost of a mistake in another variable.

4.2 Firms' attention problem

Next we formalize the idea that decisionmakers in firms have limited attention. Following Sims (2003), we model decisionmakers' limited attention as a bound on information flow. In particular, we place a bound on the information flow between the factors driving the optimal behavior and the actual behavior. In other words, the factors driving the optimal behavior cannot contain too much information about the actual behavior, and vice versa. This implies that the actual behavior will differ from the optimal behavior. Agents will make mistakes. We assume that decisionmakers choose the allocation of attention so as to maximize the expected discounted sum of profits, or equivalently, so as to minimize the expected discounted sum of losses in profits due to suboptimal behavior. Formally, the attention problem of firm i reads

$$\min_{B(L),C(L)} -E\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right], \tag{51}$$

subject to

$$p_{it}^* = A_{p1}(L)\,\varepsilon_t^A + A_{p2}(L)\,\varepsilon_t^R + A_{p3}(L)\,\varepsilon_{it}^I \tag{52}$$

$$\hat{l}_{ijnt}^* = A_l(L)\,\varepsilon_{jnt}^{\chi}, \tag{53}$$

$$p_{it} = \underbrace{B_{p1}(L)\,\varepsilon_t^A + C_{p1}(L)\,\nu_{it}^A}_{p_{it}^A} + \underbrace{B_{p2}(L)\,\varepsilon_t^R + C_{p2}(L)\,\nu_{it}^R}_{p_{it}^R} + \underbrace{B_{p3}(L)\,\varepsilon_{it}^I + C_{p3}(L)\,\nu_{it}^I}_{p_{it}^I} \tag{54}$$

$$\hat{l}_{ijnt} = B_l(L)\,\varepsilon_{jnt}^{\chi} + C_l(L)\,\nu_{ijnt}^{\chi}, \tag{55}$$

$$\mathcal{I}\left(\left\{\varepsilon_t^A, \varepsilon_t^R, \varepsilon_{it}^I, \varepsilon_{11t}^\chi, \ldots, \varepsilon_{J(N-1)t}^\chi\right\}; \left\{p_{it}^A, p_{it}^R, p_{it}^I, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right\}\right) \leq \kappa. \tag{56}$$

Furthermore,

$$\tilde{p}_{it} - \tilde{p}_{it}^* = p_{it} - p_{it}^*. \tag{57}$$

Here ν_{it}^A, ν_{it}^R, ν_{it}^I and ν_{ijnt}^χ follow Gaussian white noise processes that are mutually independent and independent of all other shocks in the economy. Equations (52) and (53) characterize the optimal behavior. $A_{p1}(L)$, $A_{p2}(L)$, $A_{p3}(L)$ and $A_l(L)$ are infinite-order lag polynomials. Equations (54) and (55) specify the actual behavior. Choosing the process for the actual behavior is formalized as choosing the lag polynomials $B_{p1}(L)$, $C_{p1}(L)$, etc. If the decisionmaker had unlimited attention, the actual behavior would equal the optimal behavior. Formally, $B_{p1}(L) = A_{p1}(L)$ and $C_{p1}(L) = 0$. The constraint on information flow (56) implies that this is not possible. The operator $\mathcal{I}$ measures the information flow between stochastic processes.[4] The information flow constraint states that the information flow between the shocks driving the optimal behavior and the actual behavior cannot exceed the parameter κ. Finally, equation (57) gives the relationship between the mistake in the dollar price of good i, $p_{it} - p_{it}^*$, and the mistake in the relative price of good i, $\tilde{p}_{it} - \tilde{p}_{it}^*$. Real profits depend on the relative price of good i while the firm chooses the dollar price of good i.

4.3 Computing the equilibrium

We use an iterative procedure to solve for the equilibrium of the model. First, we make a guess concerning the process for the profit-maximizing price (52) and the process for the profit-maximizing labor mix (53). Second, we solve the firms' attention problem (51)-(57). Third, we aggregate the individual prices to obtain the aggregate price level

$$p_t = \frac{1}{I}\sum_{i=1}^{I} p_{it}. \tag{58}$$

Fourth, we compute the aggregate dynamics implied by the price level dynamics. The following equations have to be satisfied in equilibrium:

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left[\phi_\pi (p_t - p_{t-1}) + \phi_y y_t\right] + \varepsilon_t^R, \tag{59}$$

[4]For a definition of the operator $\mathcal{I}$, see equations (1)-(4) in Section 2 of Maćkowiak and Wiederholt (2007).

$$c_t = E_t\left[-\frac{1}{\gamma}\left(r_t - p_{t+1} + p_t\right) + c_{t+1}\right], \tag{60}$$

$$\tilde{w}_t = \psi l_t + \gamma c_t, \tag{61}$$

$$y_t = c_t, \tag{62}$$

$$y_t = a_t + \alpha l_t, \tag{63}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_t^A. \tag{64}$$

The first equation is the Taylor rule. The second equation is the consumption Euler equation. The third equation follows from optimal wage setting by households. See Appendix C. The fourth equation follows from the requirement that output equals demand. The fifth equation follows from the production function and the sixth equation is the process for aggregate productivity. We employ a standard solution method for linear rational expectations models to solve the system of equations containing the price level dynamics and these six equations. We obtain the law of motion for $(r_t, c_t, y_t, l_t, \tilde{w}_t)$ implied by the price level dynamics. Fifth, we compute the law of motion for the profit-maximizing price from

$$p_{it}^* = p_t + \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right). \tag{65}$$

If the process for the profit-maximizing price differs from our guess, we update our guess. We iterate until we reach a fixed point. Finally, we compute the fixed point for the profit-maximizing labor mix. This is explained in Appendix C.

4.4 Benchmark parameter values and solution

In this section we report the numerical solution of the model for the following parameter values. We set $\alpha = 2/3$, $\beta = 0.99$, $\gamma = 1$, $\psi = 1$, $\theta = 3$, $\eta = 3$, $\phi_\pi = 1.5$, $\phi_y = 0.5$, and $\rho_R = 0.95$. To calibrate the exogenous process for aggregate productivity we make use of Fernald's (2007) quarterly data on total factor productivity growth rate. We construct from Fernald's data a quarterly time series for the level of total factor productivity. We detrend this time series with a linear trend, and fit a first-order autoregression to the detrended total factor productivity data. This yields $\rho_A = 0.95$ and the standard deviation of the innovation in aggregate productivity equal to 0.0085. To calibrate the standard deviation of the innovation in the Taylor rule, we make use of the estimates reported in Justiniano and Primiceri (2006). Justiniano and Primiceri allow for time variation in the size of monetary policy shocks. Based on the average estimate of Justiniano and Primiceri we set the standard deviation of the innovation in the Taylor rule equal to 0.002.

We assume that labor-specific preference shocks follow a white noise process. This implies that solving for relative wages and labor inputs is straightforward. See Appendix C. In order to calibrate the standard deviation of labor-specific preference shocks we proceed as follows. Autor, Katz and Kearney (2005) report the variance of log hourly wages of men in the U.S. between 1975 and 2003. The average variance of log hourly wages of men in this period was 0.32. We choose the variance of χ_{jnt} such that the variance of $\tilde{w}_{jnt}$ in our model equals 0.32 under perfect information. This yields a standard deviation of labor-specific preference shocks equal to 2.26. See equation (49) and recall that $\psi = 1$ and $\eta = 3$. Furthermore, we set $J = 50$ and $N = 50$.

We assume that firm-specific productivity shocks follow a first-order autoregressive process. Recent papers calibrate the autocorrelation of firm-specific productivity to be about one-half: Burstein and Hellwig (2007) use 0.5, Golosov and Lucas (2007) use 0.55, Klenow and Willis (2007) use 0.46, Midrigan (2006) uses 0.5, and Nakamura and Steinsson (2007) use 0.66. We set the autocorrelation of firm-specific productivity equal to 0.5. Furthermore, we choose the standard deviation of the innovation to firm-specific productivity such that the average absolute size of price changes in our model equals 13.3% under perfect information. 13.3% is the average absolute size of price changes including sales reported in Klenow and Kryvtsov (2005). This yields a standard deviation of the innovation to firm-specific productivity equal to 0.22.

We compute the solution of the model by fixing the marginal value of information flow. The total information flow is then determined within the model. It turns out that taking the marginal value of information flow exogenously simplifies the solution. We set the marginal value of information flow equal to 2 percent of the steady state wage bill. We think that this is a reasonable number.

At the fixed point firms allocate 0.5 bits to tracking aggregate technology, 0.25 bits to tracking monetary policy, 1.43 bits to tracking firm-specific technology, and 0.09 bits to tracking each labor-specific preference shock. The total information flow at the solution equals 230 bits. Expected per period loss from imperfect tracking of aggregate conditions equals about 1 percent of the steady state wage bill.[5] We think that this is a reasonable number. Expected per period loss from imperfect tracking of firm-specific technology equals about 1.4 percent of the steady state wage bill. Expected per period loss from imperfect tracking of each labor-specific preference shock equals about 0.05 percent of the steady state wage bill.

Figures 1 and 2 show impulse responses of the price level, inflation, consumption (output), and the nominal interest rate at the fixed point (green lines with circles). For comparison, the figures also show impulse responses of the same variables in equilibrium under perfect information (blue lines with points). Figures 3 and 4 reproduce the impulse responses at the fixed point (green lines with circles) and also show impulse responses of the same variables in the Calvo model (red lines with points). We solved the Calvo model for the same parameter values and assuming that prices change after three quarters on average. All impulse responses are drawn such that the impulse response equal to one means "a one percentage point deviation from the non-stochastic steady state". Time is measured in quarters along horizontal axes.

Consider Figure 1. The price level shows a dampened and delayed response to a monetary policy shock. The impulse response of inflation to a monetary policy shock is persistent. Output falls after a positive innovation in the Taylor rule, and the decline in output is persistent. The nominal interest rate increases on impact and converges slowly to zero. The impulse responses to a monetary policy shock when firms face an information flow constraint differ a great deal from the impulse responses to a monetary policy shock under perfect information. Under perfect information the price level follows a random walk after a monetary policy shock, there are no real effects, and the nominal interest rate fails to change.

Consider Figure 2. The price level and inflation show a dampened and delayed response

[5] Expected per period loss from imperfect tracking of aggregate technology equals about 0.8 percent of the steady state wage bill. Expected per period loss from imperfect tracking of monetary policy equals about 0.3 percent of the steady state wage bill.

to an aggregate productivity shock. There is less delay in the impulse response of the price level and there is less persistence in the impulse response of inflation compared with the case of a monetary policy shock. The reason is that firms allocate more attention to tracking aggregate productivity than to tracking monetary policy. Output and the nominal interest rate show hump-shaped impulse responses to an aggregate productivity shock.

Figure 3 shows that the impulse responses to a monetary policy shock in the benchmark economy are similar to the impulse responses in the Calvo model. The long-run impulse response of the price level in the Calvo model is smaller, and the impulse response of output in the Calvo model is larger in the first few quarters. Figure 4 shows that the impulse responses to an aggregate productivity shock in the benchmark economy are similar to the impulse responses in the Calvo model. The deviations of the price level and output from the frictionless case ("perfect information" and "all prices change each quarter") are somewhat larger in the Calvo model than in the benchmark economy for our parameterization. Recall that we assume that prices in the Calvo model change after three quarters on average. Consider the impulse responses of inflation to a monetary policy shock and an aggregate productivity shock in Figures 3 and 4. The absolute response of inflation to a monetary policy shock is smaller in the benchmark economy compared with the Calvo model, and the absolute response of inflation to an aggregate productivity shock is larger in the benchmark economy compared with the Calvo model. The reason is that firms in the benchmark economy allocate more attention to tracking aggregate productivity than to tracking monetary policy.

Figure 5 shows the impulse response of an individual price to a firm-specific productivity shock. Firms track the profit-maximizing impulse response very well.

5 Conclusion

We have introduced rational inattention on the side of firms into a dynamic stochastic general equilibrium model. The impulse responses under rational inattention have several features of empirical impulse response functions. The next step is to introduce rational inattention on the side of households into this model.

A Solution under perfect information

First, $y_{it} = c_{it}$ and equations (29), (30) and (40) imply that

$$y_t = c_t.$$

Second, computing the average of the production function (35) over all i and using (38), (40) and (41) yields

$$y_t = a_t + \alpha l_t.$$

Third, computing the average of the price setting equation (32) over all i and using (29), (41) and $l_t = \frac{1}{\alpha}(c_t - a_t)$ yields

$$\tilde{w}_t = c_t - l_t.$$

The real wage index equals output per labor input. Fourth, computing the average of the wage setting equation (28) over all jn and using (31), (34) and (42) yields

$$\tilde{w}_t = \psi l_t + \gamma c_t.$$

The real wage index equals the marginal rate of substitution of consumption for leisure. When we solve the last four equations for y_t, c_t, l_t and $\tilde{w}_t$, we arrive at equations (43)-(45). Finally, computing the average of the Euler equation (26) over all j and using (31) yields

$$c_t = E_t\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{t+1}\right].$$

Substituting the solution for c_t into the last equation yields equation (46).

B The firms' objective

The nominal profits of firm i in period t equal

$$(1+\tau_p) P_{it} Y_{it} - L_{it}\left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}\hat{L}_{ijnt}\right),$$

where $\hat{L}_{ijnt} = (L_{ijnt}/L_{it})$. The term in brackets is the wage bill per unit of composite labor. The production function (4) implies that

$$L_{it} = \left(\frac{Y_{it}}{e^{a_t + a_{it}}}\right)^{\frac{1}{\alpha}}.$$

The labor aggregator (5) implies that

$$1=\sum_{j=1}^{J}\sum_{n=1}^{N}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}},$$

or equivalently

$$\hat{L}_{iJNt}=\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

Furthermore, since households have perfect information, the demand function equals

$$C_{it}=\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t.$$

Substituting the production function, the labor aggregator and the demand function into the expression for nominal profits yields the profit function

$$(1+\tau_p)\,P_{it}\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t-\left(\frac{\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}W_{jnt}\hat{L}_{ijnt}+W_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right].$$

Dividing by P_t yields the real profit function

$$(1+\tau_p)\,\tilde{P}_{it}^{1-\theta}C_t-\left(\frac{\tilde{P}_{it}^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}\tilde{W}_{jnt}\hat{L}_{ijnt}+\tilde{W}_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right],\quad (66)$$

where $\tilde{P}_{it}=(P_{it}/P_t)$ and $\tilde{W}_{jnt}=(W_{jnt}/P_t)$. One can express the real profit function in terms of log-deviations from the non-stochastic steady state

$$\begin{aligned}&(1+\tau_p)\,\tilde{P}_iC_ie^{(1-\theta)\tilde{p}_{it}+c_t}\\&-L_ie^{\frac{1}{\alpha}(-\theta\tilde{p}_{it}+c_t-a_t-a_{it})}\tilde{W}\frac{1}{JN}\left[\sum_{jn\neq JN}e^{\tilde{w}_{jnt}+\hat{l}_{ijnt}}+e^{\tilde{w}_{JNt}}\left(JN-\sum_{jn\neq JN}e^{\frac{\eta-1}{\eta}\hat{l}_{ijnt}}\right)^{\frac{\eta}{\eta-1}}\right]\end{aligned}\quad (67)$$

Here we have used equation (25).

We assume that firm i chooses the allocation of attention so as to maximize the expected discounted sum of profits

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}F\left(\tilde{P}_{it},C_t,e^{a_t},e^{a_{it}},\tilde{W}_{11t},\ldots,\tilde{W}_{JNt},\hat{L}_{i11t},\ldots,\hat{L}_{iJ(N-1)t}\right)\right],\quad (68)$$

where $\beta^t \left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}$ is the stochastic discount factor and F is the real profit function (66). Two remarks concerning firm i's objective may be helpful. First, C_{jt} is composite consumption by household j in period t. Since in equilibrium all households have the same composite consumption, the stochastic discount factor does not depend on j. Second, E is the unconditional expectation operator. Here we are using the assumption that firms choose the allocation of attention before receiving any information. This assumption slightly simplifies the computation of the equilibrium. The assumption can be relaxed.

One can express the objective (68) in terms of log-deviations from the non-stochastic steady state

$$E\left[\sum_{t=0}^{\infty}\beta^t e^{-\gamma(c_{jt}-c_{j0})} f\left(\tilde{p}_{it}, c_t, a_t, a_{it}, \tilde{w}_{11t}, \ldots, \tilde{w}_{JNt}, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right)\right], \tag{69}$$

where f is the real profit function (67).

Next we compute a second-order Taylor approximation around the non-stochastic steady state of the term inside the expectation operator of (69). Afterwards, we deduct from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior $\left\{\tilde{p}^*_{it}, \hat{l}^*_{i11t}, \ldots, \hat{l}^*_{iJ(N-1)t}\right\}_{t=0}^{\infty}$. This yields the following expression for (minus) the expected discounted sum of losses in profits due to suboptimal behavior:

$$E\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{2}\left(x_t - x^*_t\right)' H \left(x_t - x^*_t\right)\right],$$

where

$$\underset{(JN\times 1)}{x_t} = \begin{pmatrix} \tilde{p}_{it} \\ \hat{l}_{i11t} \\ \vdots \\ \hat{l}_{iJ(N-1)t} \end{pmatrix},$$

$$\underset{(JN\times JN)}{H} = \tilde{W}L_i \begin{bmatrix} \frac{\theta(\theta-1)}{\alpha} - \frac{\theta^2}{\alpha^2} & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} \\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN} \\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN} \end{bmatrix},$$

and x_t^* is given by the following two equations:

$$\tilde{p}_{it}^* = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right),$$

and

$$\hat{l}_{ijnt}^* = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right).$$

Here $\tilde{w}_t = \frac{1}{JN}\sum_{j=1}^{J}\sum_{n=1}^{N}\tilde{w}_{jnt}$. Note that deducting from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior is simply a monotone transformation of the quadratic objective. This transformation simplifies the quadratic objective without affecting the solution to the optimization problem.

C Firms rational inattention, households perfect information: solving for relative wages

In this subsection, we solve for the relative wage rate for type jn labor and the relative input of type jn labor at firm i. Solving for relative wage rates is more complicated than solving for relative prices, because firms' inattention lowers the wage elasticity of labor demand, which affects households' wage setting behavior. In order to make the derivation as clear as possible, we assume that all the χ_{jnt} follow a common white noise process.

Let $\hat{w}_{jnt} = \tilde{w}_{jnt} - \tilde{w}_t$ denote the relative wage rate for type jn labor. We guess that in equilibrium

$$\hat{w}_{jnt} = A\chi_{jnt}, \tag{70}$$

where A is an unknown coefficient. Let $\hat{l}_{ijnt} = l_{ijnt} - l_{it}$ denote the relative input of type jn labor at firm i. The profit-maximizing relative input of type jn labor at firm i is given by equation (33):

$$\hat{l}_{ijnt}^* = -\eta\hat{w}_{jnt}. \tag{71}$$

Since χ_{jnt} follows a Gaussian white noise process, both $\hat{w}_{jnt}$ and $\hat{l}_{ijnt}^*$ follow Gaussian white noise processes. Tracking an optimal decision that follows a Gaussian white noise process with an information flow equal to κ_χ yields the following decision under rational inattention when the aim is to minimize the mean squared error

$$\hat{l}_{ijnt}^{RI} = \left(1 - \frac{1}{2^{2\kappa_\chi}}\right)\hat{l}_{ijnt}^* + \sqrt{\frac{1}{2^{2\kappa_\chi}} - \frac{1}{2^{4\kappa_\chi}}}\sqrt{Var\left(\hat{l}_{ijnt}^*\right)}\nu_{ijnt}^\chi, \tag{72}$$

where ν_{ijnt}^{χ} follows an independent Gaussian white noise process with unit variance. See, for example, Proposition 3 in Maćkowiak and Wiederholt (2007). Using equation (71) to substitute for $\hat{l}_{ijnt}^{*}$ in equation (72), we arrive at

$$\hat{l}_{ijnt}^{RI} = -\eta \left(1 - \frac{1}{2^{2\kappa_\chi}}\right) \left(\hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_\chi} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu_{ijnt}^{\chi}\right). \tag{73}$$

Now it is easy to verify that the signal

$$s_{ijnt} = \hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_\chi} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu_{ijnt}^{\chi} \tag{74}$$

has the property

$$\hat{l}_{ijnt}^{RI} = E\left[\hat{l}_{ijnt}^{*} | s_{ijnt}, s_{ijnt-1}, \ldots\right].$$

Hence, one can interpret the decision under rational inattention as being due to the fact that firms pay limited attention to the relative wage rate for type jn labor. Furthermore, comparing (71) to (73) one can see that firms' limited attention lowers the wage elasticity of labor demand from η to $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)$.

Computing the average of (73) over all firms and using the fact that noise is idiosyncratic yields

$$l_{jnt} - l_t = -\eta \left(1 - \frac{1}{2^{2\kappa_\chi}}\right) \hat{w}_{jnt}, \tag{75}$$

where $l_{jnt} = \frac{1}{I}\sum_{i=1}^{I} l_{ijnt}$ and $l_t = \frac{1}{I}\sum_{i=1}^{I} l_{it}$. Exponentiating both sides of equation (75), multiplying by L_{jn} and using the fact that $L_{jn} = \left(\frac{W_{jn}}{W}\right)^{-\eta} L$ yields

$$L_{jnt} = \left(\frac{W_{jn}}{W}\right)^{-\frac{\eta}{2^{2\kappa_\chi}}} \left(\frac{W_{jnt}}{W_t}\right)^{-\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)} L_t. \tag{76}$$

The first term on the RHS is due to the fact that in the non-stochastic steady state the wage elasticity of labor demand equals η rather than $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)$.

When firms have limited attention, the household problem is to maximize (12) subject to (2), (3) and the new labor demand function (76). The optimality conditions for consumption are still equations (13) and (14). So long as $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right) > 1$, the optimal wage rate for type jn labor is given by

$$\frac{W_{jnt}}{P_t} = \frac{1}{1 + \tau_w} \frac{\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)}{\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right) - 1} \varphi e^{-\chi_{jnt}} L_{jnt}^{\psi} C_{jt}^{\gamma}.$$

We continue to assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. This now implies that

$$1+\tau_w = \frac{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)}{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)-1}.$$

Therefore the wage setting equation reduces to

$$\frac{W_{jnt}}{P_t} = \varphi e^{-\chi_{jnt}} L_{jnt}^{\psi} C_{jt}^{\gamma}. \tag{77}$$

Taking logs on both sides of (77) and using the fact that (77) with $\chi_{jnt}=0$ also holds in the non-stochastic steady state yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi l_{jnt} + \gamma c_{jt}.$$

Using the labor demand function (75) and $\hat{w}_{jnt} = \tilde{w}_{jnt} - \tilde{w}_t$ to substitute for l_{jnt} in the last equation yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi\left[-\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)(\tilde{w}_{jnt}-\tilde{w}_t) + l_t\right] + \gamma c_{jt}. \tag{78}$$

Computing the average of (78) over all types of labor and using (34), (42) and $c_{jt} = c_t$ yields the following expression for the real wage index

$$\tilde{w}_t = \psi l_t + \gamma c_t. \tag{79}$$

Computing the difference between (78) and (79) and using again $c_{jt} = c_t$ yields the following expression for the relative wage rate for type jn labor

$$\tilde{w}_{jnt} - \tilde{w}_t = -\frac{1}{1+\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\psi}\chi_{jnt}. \tag{80}$$

Comparing (70) and (80) shows that the guess (70) was correct. Firm i's profit-maximizing labor mix then follows from equation (71) and firm i's actual labor mix under rational inattention follows from equation (73).

We still need to solve for the equilibrium attention allocated to the profit-maximizing relative input of type jn labor. Equations (71), (73) and (80) imply that the mean squared error in the relative input of type jn labor equals

$$E\left[\left(\hat{l}_{ijnt}^{*} - \hat{l}_{ijnt}^{RI}\right)^2\right] = \frac{1}{2^{2\kappa_\chi}} \frac{\eta^2}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^2}\sigma_\chi^2.$$

The derivative of the mean squared error with respect to κ_χ equals

$$\frac{\partial E\left[\left(\hat{l}^*_{ijnt} - \hat{l}^{RI}_{ijnt}\right)^2\right]}{\partial \kappa_\chi} = -2\ln(2)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^3}\eta^2\sigma_\chi^2.$$

It follows from the objective (51) that the marginal value of paying attention to the profit-maximizing relative input of type jn labor equals

$$\lambda_\chi = \frac{1}{1-\beta}\frac{1}{\eta JN}2\ln(2)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^3}\eta^2\sigma_\chi^2. \tag{81}$$

By equating the marginal value of attention across different activities we obtain the equilibrium κ_χ.

D Solving the Calvo model

If we assume that firms and households have perfect information but firms face a Calvo friction, we obtain the following version of the New Keynesian Phillips curve

$$\pi_t = \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}\frac{\frac{\psi}{\alpha}+\gamma+\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\theta}\left(c_t - c_t^f\right) + \beta E_t\left[\pi_{t+1}\right], \tag{82}$$

where $(1-\lambda)$ is the fraction of goods prices that change every period and c_t^f is the flexible price solution given by equation (43). The aggregate dynamics are obtained by solving the system containing equations (59)-(64) and equation (82). The solution of the Calvo model reported in Figures 3-4 assumes that $\lambda = 2/3$.

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Figure 1: Impulse responses, benchmark economy

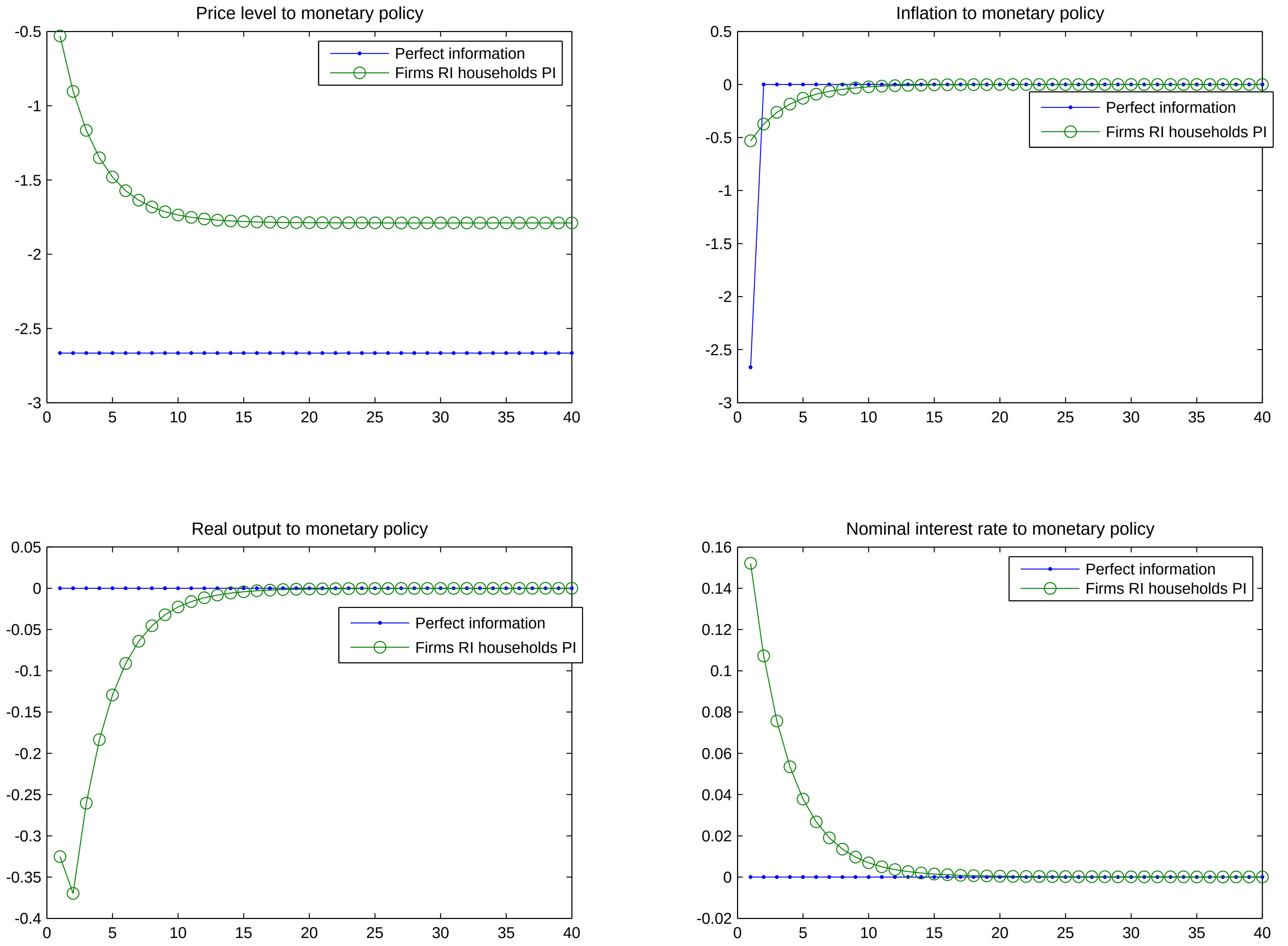

Figure 2: Impulse responses, benchmark economy (continued)

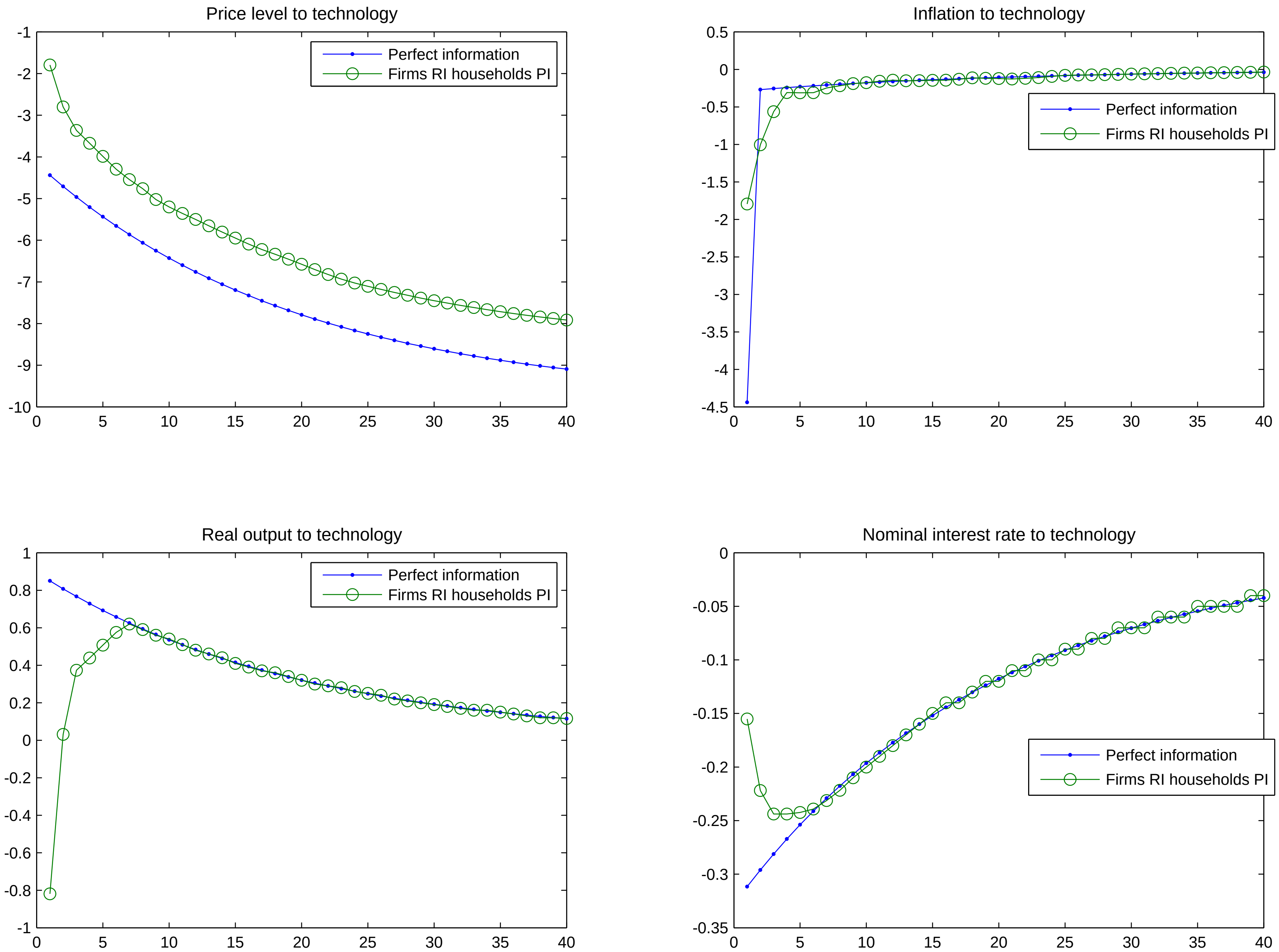

Figure 3: Benchmark economy vs. the Calvo model

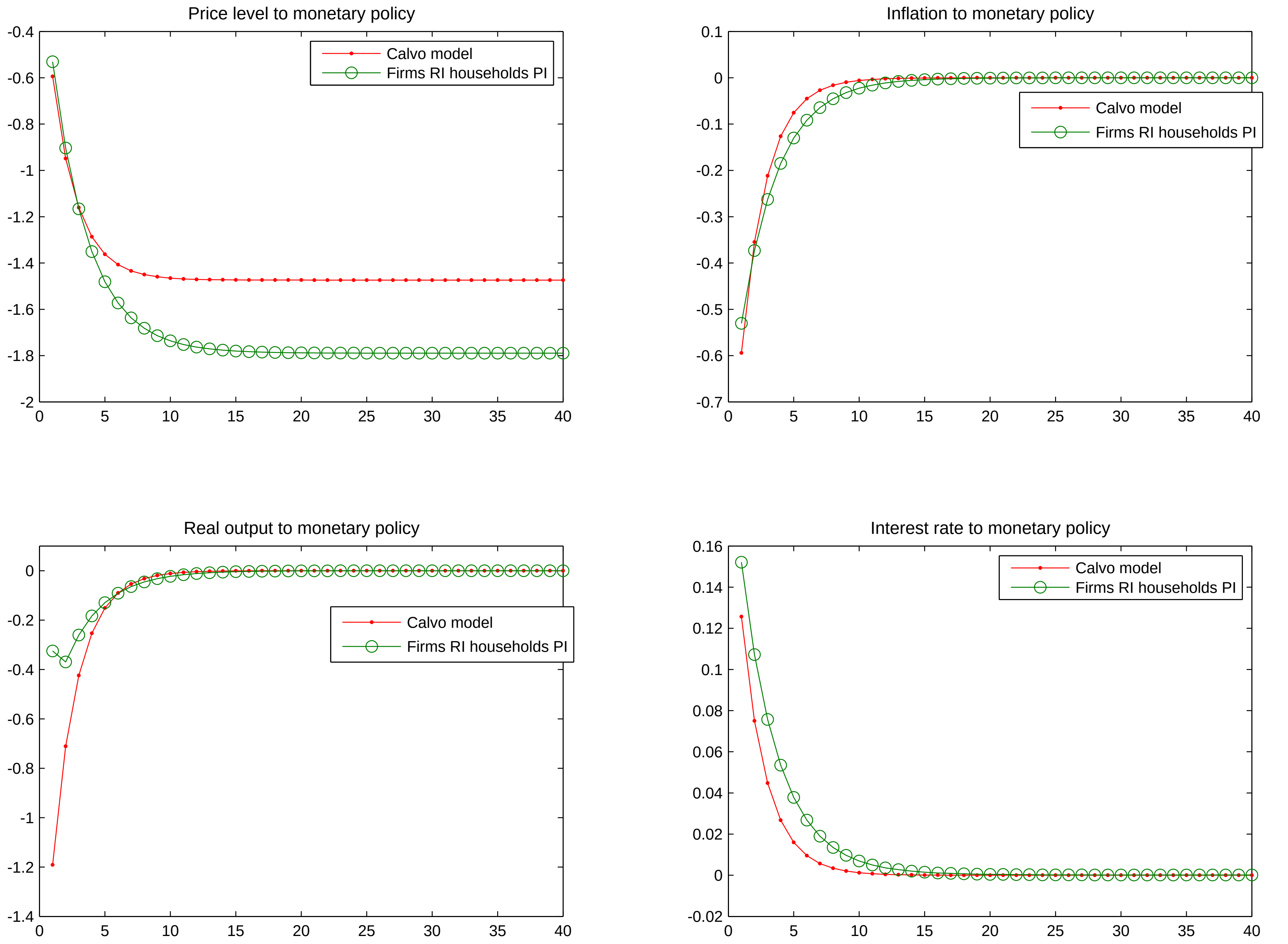

Figure 4: Benchmark economy vs. the Calvo model (continued)

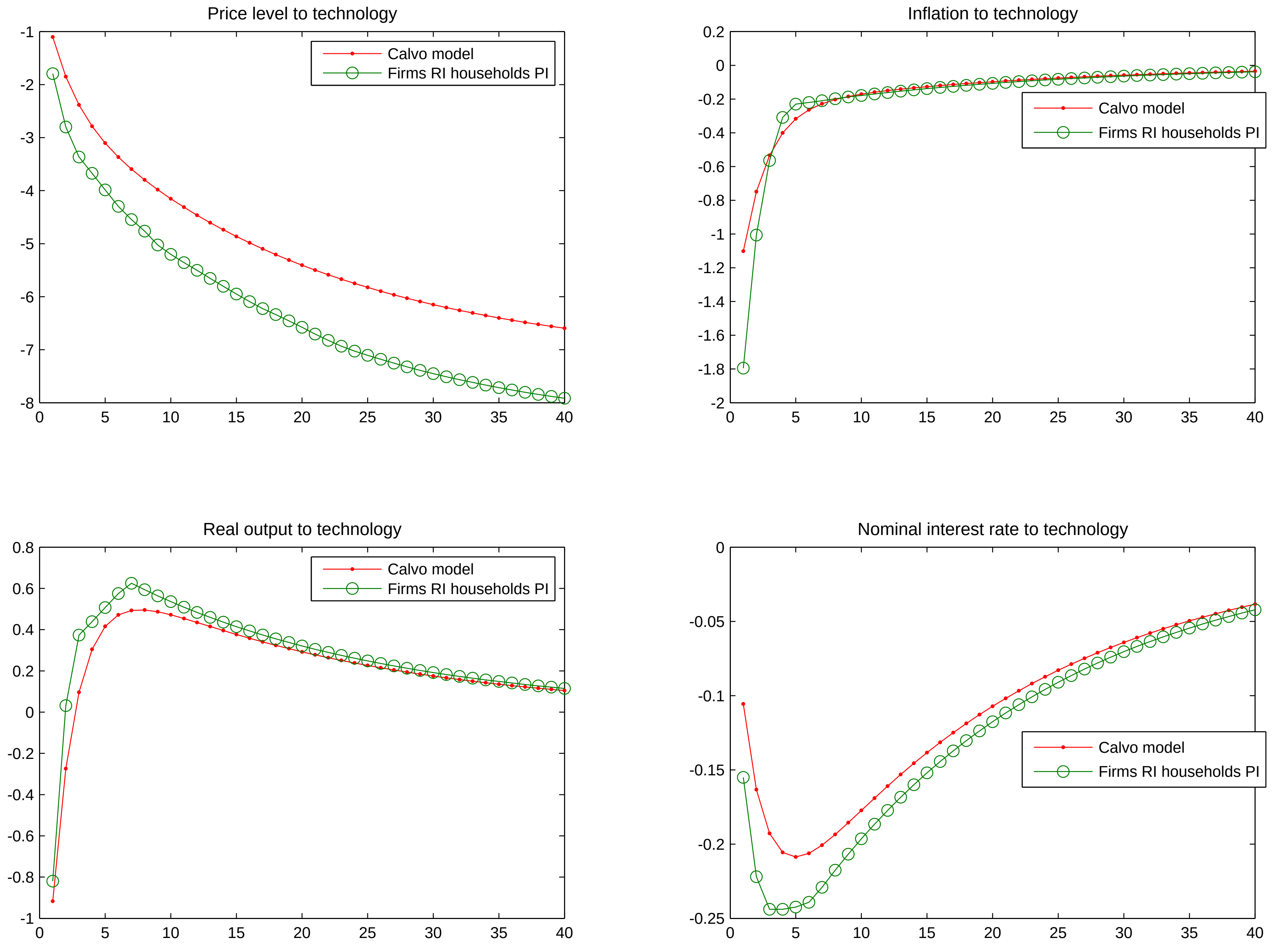

Figure 5: Impulse response of an individual price to a firm-specific productivity shock, benchmark economy