The Great Depression and the Friedman-Schwartz Hypothesis (Preliminary and Incomplete. Please do not quote without the permission of the authors.)

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1. Introduction

Was the US Great Depression of the 1930s due to bungling at the Fed? In their classic analysis of US monetary history, Friedman and Schwartz (1963) conclude that the answer is 'yes'. To be sure, they do admit that if the Fed had not been part of the problem we would have seen recessions. But, they would have been the usual garden-variety slowdowns, not the spectacular collapse that actually occurred. The Friedman and Schwarz answer is a comforting one. Under the assumption that the Fed is smarter now than it was then, we don't have to worry about the possibility of a repeat.

Or do we? Is there anything the Fed can do that has consequences on the order of magnitude of the Great Depression? A recent analysis by Sims (1999) concludes 'no'. He argues that if a modern central banker had somehow been transported back into the 1930s and made chairman of the Fed, the Great Depression would have unfolded pretty much the way it did. For example, using a similar style of reasoning as Sims, Christiano (1999) argued that it would have made little difference if the Fed had acted to prevent the fall in $M1$. This seems inconsistent with a centerpiece of Friedman and Schwartz's argument: that the Great Depression was so severe, in part because the Fed allowed $M1$ to collapse. Although this argument creates a doubt, it is at best only suggestive because it is made by manipulating a subset of equations in a vector autoregression, without worrying about the possible consequences for other equations.

Our purpose is to do the relevant experiment 'right'. For this, we require a structural model of the economy that captures the essential features emphasized by Friedman and Schwartz. There is a variety of elements that this model must incorporate, to be interesting. First, there must be some model of credit market frictions that allow us to capture the effects of the enormous fall in stock market value that occurred. For this, we incorporate the credit market frictions described in Bernanke, Gertler and Gilchrist (1999) (BGG).[1] Second, an important component of the Friedman and Schwartz argument is that the Fed did not act to prevent the decline in M1 that occurred as people converted demand deposits into currency. Also, Friedman and Schwartz argue that later in the depression, the Fed failed to appreciate the fact that banks wanted to hold excess reserves in conducting monetary policy. Thinking that the high levels of reserves the banks held were potentially inflationary, they increased reserve requirements. This was highly contractionary, when it turned out that the excess reserves banks were holding were desired. To model these features of the time, we need to incorporate a banking sector with demand deposits, currency, bank reserves and bank excess reserves. For this, we use the banking model of Chari, Christiano and Eichenbaum (1995)

[1]This work builds on Townsend (1979), Gale and Hellwig (1985), Williamson (1987). Other recent contributions to this literature include Fisher (1996) and Carlstrom and Fuerst (1997, 2000).

(CCE). Finally, we incorporate these banking and net worth considerations into the model environment described in Altig, Christiano, Eichenbaum and Linde (2002) (ACEL). This model seems appropriate for the task, since it captures key features of aggregate data, as well as of the monetary transmission mechanism.

This draft provides a description of the model and the solution method. In addition, a set of preliminary parameter values are reported, together with the associated steady state properties as well as some impulse responses. The full analysis will appear in the next draft.

2. The Model Economy

In this section we describe our model economy and display the problems solved by intermediate and final good firms, entrepreneurs, producers of physical capital, banks and households. Final output is produced using the usual Dixit-Stiglitz aggregator of intermediate inputs. Intermediate inputs are produced by monopolists who set prices using a variant of the approach described in Calvo (1983). These firms use the services of capital and labor. We assume that a fraction of these variable costs ('working capital') must be financed in advance through banks.

Labor services are an aggregate of specialized services, each of which is supplied by a monopolist household. Households set wages, subject to the type of frictions modeled in Calvo (1983).[2] Capital services are supplied by entrepreneurs who own the physical capital and determine its rate of utilization. Our model of the entrepreneurs follows BGG. In particular, the entrepreneurs only have enough net worth to finance a part of their holdings of physical capital. The rest must be financed by loans from a financial intermediary. Entrepreneurs are risky because they are subject to idiosyncratic productivity shocks. Moreover, while the realization of an individual entrepreneur's productivity shock is observed freely by the entrepreneur, the intermediary must pay a cost to observe it. The contract extended by the intermediary to the entrepreneur is a standard debt contract. As is standard in the costly state verification (CSV) framework with net worth, we need to make assumptions to guarantee that entrepreneurs do not accumulate enough net worth to make the CSV technology irrelevant. We accomplish this by assuming that a part of net worth is exogenously destroyed in each period.

The actual production of physical capital is carried out by capital producing firms, who combine old capital and investment goods to produce new, installed, capital. The capital owned by entrepreneurs is purchased from these firms.

All financial intermediation activities occur in a 'bank'. They receive two types of deposits from households. Demand deposits are used to finance the working capital loans.

[2]This aspect of the model follows CCE, who in turn build on Erceg, Henderson and Levin (2000).

To maintain deposits requires the use of capital and labor resources. This aspect of the model follows CCE. The bank also handles the intermediation activities associated with the financing of entrepreneurs. To finance this, the bank issues 'time deposits' to households. The maturity structure of bank liabilities match those of bank assets exactly. There is no risk in banking.

The timing of decisions during a period is important in the model. At the beginning of the period, shocks to the various technologies are realized. Then, wage, price, consumption, investment and capital utilization decisions are made. In addition, households decide how to split their financial assets between currency and deposits at this time.[3] After this, various financial market shocks are realized and the monetary action occurs. Finally, goods and asset markets meet and clear. See Figure 1 for reference.

2.1. Information

We divide up the shocks in the model into financial market shocks - money demand (by banks, households and firms) and monetary policy shocks - and non-financial market shocks (technology, government spending, preference for leisure, elasticities of demand for differentiated products and labor, etc.). The time t information set which includes period $t-s$, $s>0$, and period t observations on the non-financial shocks is denoted Ω_t. The information set which includes Ω_t plus the current period financial market shocks is denoted Ω_t^μ. Also,

$$\begin{aligned} E\left[X_t|\Omega_t\right] &= E_t X_t \\ E\left[X_t|\Omega_t^\mu\right] &= E_t^\mu X_t. \end{aligned}$$

2.2. Firm Sector

We adopt the variant on the standard Dixit-Stiglitz setup for our firm sector that was used in CEE. At time t, a final consumption good, Y_t, is produced by a perfectly competitive firm. The firm does so by combining a continuum of intermediate goods, indexed by $j\in[0,1]$, using the technology

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj\right]^{\lambda_f}$$

where $1 \leq \lambda_f < \infty$, and Y_{jt} denotes the time t input of intermediate good j. Let P_t and P_{jt} denote the time t price of the consumption good and intermediate good j, respectively.

[3] By adopting this timing convention for household portfolio allocation, we follow the literature on limited participation models, as discussed in CCE.

Profit maximization implies the Euler equation

$$\left(\frac{P_t}{P_{jt}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{jt}}{Y_t}, \tag{2.1}$$

which leads to the following relationship between the aggregate price level and individual prices:

$$P_t = \left[\int_0^1 P_{jt}^{\frac{1}{1-\lambda_f}} dj\right]^{(1-\lambda_f)}. \tag{2.2}$$

The j^{th} intermediate good is produced by a monopolist who sets its price, P_{jt}, after the realization of non-financial market shocks, but before the realization of financial market shocks. In addition to this information constraint, there are also Calvo-style frictions in setting prices that we will describe shortly. The intermediate good producer is assumed to satisfy whatever demand materializes at its posted price. Once prices have been set, and after the realization of current period uncertainty, the intermediate good producer selects inputs to minimize costs. The production function of the j^{th} intermediate good firm is:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} - \Phi z_t & \text{if } \epsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} > \Phi z_t \\ 0, & \text{otherwise} \end{cases}, \ 0 < \alpha < 1,$$

where Φ is a fixed cost and K_{jt} and l_{jt} denote the services of capital and labor. The variable, z_t, is a shock to technology, which has a covariance stationary growth rate, μ_{zt}, where

$$\mu_{zt} = \frac{z_t}{z_{t-1}}.$$

The variable, ϵ_t, is a stationary shock to technology. The time series representations for z_t and ϵ_t are discussed below. Firms are competitive in factor markets, where they confront a rental rate, Pr_t^k, on capital services and a wage rate, W_t, on labor services. Each of these is expressed in units of money. Also, each firm must finance a fraction, $\psi_{k,t}$, of its capital services expenses in advance. Similarly, it must finance a fraction, $\psi_{l,t}$, of its labor services in advance. The interest rate it faces is R_t. Working capital includes the wage bill, $W_t l_{jt}$, and the rent on capital services, $P_t r_t^k K_t$. As a result, the marginal cost - after dividing by P_t - of producing one unit of Y_{jt} is:

$$s_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(r_t^k \left[1 + \psi_{k,t} R_t\right]\right)^{\alpha} \left(w_t \left[1 + \psi_{l,t} R_t\right]\right)^{1-\alpha}}{\epsilon_t}, \tag{2.3}$$

where

$$w_t = \frac{W_t}{z_t P_t}.$$

Efficient input choice by firms also leads to the following condition:

$$s_t = \frac{r_t^k \left[1 + \psi_{k,t} R_t\right]}{\alpha \epsilon_t \left(\frac{z_t l_t}{K_t}\right)^{1-\alpha}}, \tag{2.4}$$

where ν is the share of aggregate labor and capital services in the intermediate good sector. The complementary share, $1 - \nu$, is used in the banking sector. We impose equality of the share of capital and labor in their respective aggregates to save notation and because this is a property of equilibrium, given that we adopt the same production function for the intermediate good and banking sectors. Finally, l_t and K_t are the unweighted integrals of employment and capital services hired by individual intermediate good producers.

We adopt the variant of Calvo pricing proposed in CEE. In each period, t, a fraction of intermediate good firms, $1 - \xi_p$, can reoptimize its price. The complementary fraction must set its price equal to what it was in period $t - 1$, scaled up by the inflation rate from $t - 2$ to $t - 1$. After linearizing (2.2) and the optimizing firms' first order condition about steady state, we obtain the following law of motion for aggregate inflation:

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}\left[E_t\left(\hat{s}_t\right) + \hat{\lambda}_{f,t}\right]. \tag{2.5}$$

In the usual way, $\hat{x}_t = (x_t - x)/x_t$, where x is the value of x_t in nonstochastic steady state, and x_t is a small deviation from that steady state. Also, π_t denotes the aggregate inflation rate, $\pi_t = P_t/P_{t-1}$. Finally, the stochastic process, $\hat{\lambda}_{f,t}$, is a shock to the parameter, λ_f, in the final good production function. In the linearization of our economy, the only place this shock shows up is (2.5).

2.3. Capital Producers

There is a large, fixed, number of identical capital producers, who take prices as given. They are owned by households and any profits or losses are transmitted in a lump-sum fashion to households. The capital producer must commit to a level of investment, I_t, before the period t realization of the monetary policy shock and after the period t realization of the other shocks. Investment goods are actually purchased in the goods market which meets after the monetary policy shock. The price of investment goods in that market is P_t, and this is a function of the realization of the monetary policy shock. The capital producer also purchases old capital in the amount, x, at the time the goods market meets. Old capital and investment goods are combined to produce new capital, x', using the following technology:

$$x' = x + F(I_t, I_{t-1}),$$

where the presence of lagged investment reflects that there are costs to changing the flow of investment. We denote the price of new capital by $Q_{\bar{K}',t}$, and this is a function of the realized value of the monetary policy shock. Since the marginal rate of transformation from old capital into new capital is unity, the price of old capital is also $Q_{\bar{K}',t}$. The firm's time t profits, after the realization of the monetary policy shock are:

$$\Pi_t^k = Q_{\bar{K}',t}\left[x + F(I_t, I_{t-1})\right] - Q_{\bar{K}',t}x - P_t I_t.$$

This expression for profits is a function of the realization of the period t monetary policy shock, because $Q_{\bar{K}',t}$, x, and P_t are. Since the choice of I_t influences profits in period $t+1$, the firm must incorporate that into the objective as well. But, that term involves I_{t+1} and x_{t+1}. So, state contingent choices for those variables must be made for the firm to be able to select I_t and x_t. Evidently, the problem choosing x_t and I_t expands into the problem of solving an infinite horizon optimization problem:

$$\max_{\{I_{t+j}, x_{t+j}\}} E\left\{\sum_{j=0}^{\infty} \beta^j \lambda_{t+j}\left(Q_{\bar{K}',t+j}\left[x_{t+j} + F(I_{t+j}, I_{t+j-1})\right] - Q_{\bar{K}',t+j}x_{t+j} - P_{t+j}I_{t+j}\right)|\Omega_t\right\},$$

where it is understood that I_{t+j} is a function of all shocks up to period $t+j$ except the $t+j$ financial market shocks and x_{t+j} is a function of all the shocks up to period $t+j$. Also, Ω_t includes all shocks up to period t, except the period t financial market shocks. These are composed of shocks to monetary policy and to money demand.

From this problem it is evident that any value of x_{t+j} whatsoever is profit maximizing. Thus, setting $x_{t+j} = (1-\delta)\bar{K}_{t+j}$ is consistent with both profit maximization by firms and with market clearing.

The first order necessary condition for maximization of I_t is:

$$E\left[\lambda_t P_t q_t F_{1,t} - \lambda_t P_t + \beta\lambda_{t+1}P_{t+1}q_{t+1}F_{2,t+1}|\Omega_t\right] = 0,$$

where q_t is Tobin's q :

$$q_t = \frac{Q_{\bar{K},t}}{P_t}.$$

The physical stock of capital evolves as follows

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t,$$

where S is a function that is concave in the neighborhood of steady state.

2.4. Entrepreneurs

There is a large population of entrepreneurs. Consider the j^{th} entrepreneur (see Figure 2). During the period t goods market, the j^{th} entrepreneur accumulates net worth, N_{t+1}^{j}. This abstract purchasing power, which is denominated in units of money, is determined as follows. The sources of funds are the rent earned as a consequence of supplying capital services to the period t capital rental market, the sales proceeds from selling the undepreciated component of the physical stock of capital to capital goods producers. The uses of funds include repayment on debt incurred on loans in period $t-1$ and expenses for capital utilization. Net worth is composed of these sources minus these uses of funds.

At this point, $1-\gamma$ entrepreneurs die and γ survive to live another day. The newly produced stock of physical capital is purchased by the γ entrepreneurs who survive and $1-\gamma$ newly-born entrepreneurs. The surviving entrepreneurs finance their purchases with their net worth and loans from the bank. The newly-born entrepreneurs finance their purchases with a transfer payment received from the government and a loan from the bank. We actually allow γ to be a random variable, but we delete the time subscript here to keep from cluttering the notation too much.

The j^{th} entrepreneur who purchases capital, $\bar{K}_{t+1}^{j}$, from the capital goods producers at the price, $Q_{\bar{K}',t}$ in period t experiences an idiosyncratic shock to the size of his purchase. Just after the purchase, the size of capital changes from $\bar{K}_{t+1}^{j}$ to $\omega\bar{K}_{t+1}^{j}$. Here, ω is a unit mean, non-negative random variable distributed independently across entrepreneurs. After observing the realization of the non-financial market shocks, but before observing the financial market shock, the j^{th} entrepreneur decides on the level of capital utilitzation in period $t+1$, and then rents capital services. At the end of the period $t+1$ goods market, the entrepreneur sells its undepreciated capital. At this point, the entrepreneur's net worth, N_{t+2}^{j}, is the rent earned in period $t+1$, minus the utilization costs on capital, minus debt repayment, plus the proceeds of the sale of the undepreciated capital, $(1-\delta)\omega\bar{K}_{t+1}^{j}$. As indicated above, the entrepreneur then proceeds to die with probability $1-\gamma$, and to survive to live another day with the complementary probability, γ.

The $1-\gamma$ entrepreneurs who are born and the γ who survive receive a subsidy, W_t^e. There is a technical reason for this. The standard debt contract in the entrepreneurial loan market has the property that entrepreneurs with no net worth receive no loans. If new-born entrepreneurs received no transfers, they would have no net worth and would therefore not be able to purchase any capital. In effect, without the transfer they could not enter the population of entrepreneurs. Regarding the surviving entrepreneurs, in each period a fraction loses everything, and they would have no net worth in the absence of a transfer. Absent a transfer, these entrepreneurs would in effect leave the population of entrepreneurs. Absent transfers, the population of entrepreneurs would be empty. The transfers are designed to avoid this. They are financed by a lump sum tax on households.

Entrepreneurial death in the model is a device to ensure that net worth does not grow to the point where the CSV setup becomes irrelevant. Presumably, this corresponds to the real-world observation that enormous concentrations of wealth, for various reasons, do not survive for long.

We need to allocate the net worth of the entrepreneurs who die. We assume that a fraction, Θ, of a dead entrepreneur's net worth is used to finance the purchase of C_t^e of final output. The complementary fraction is redistributed as a lump-sum transfer to the household. In practice, Θ will be small or zero.

2.4.1. The Production Technology of the Entrepreneur

We now go into the details of the entrepreneur's situation. The j^{th} entrepreneur produces capital services, K_{t+1}^j, from physical capital using using the following technology:

$$K_{t+1}^j = u_{t+1}^j \omega \bar{K}_{t+1}^j,$$

where u_{t+1}^j denotes the capital utilization rate chosen by the j^{th} entrepreneur. Here, ω is drawn from a distribution with mean unity and distribution function, F :

$$\Pr\left[\omega \leq x\right] = F(x).$$

Each entrepreneur draws independently from this distribution immediately after $\bar{K}_{t+1}^j$ has been purchased. Capital services are supplied to the capital services market in period $t+1$, where they earn the rental rate, r_{t+1}^k.

The capital utilization rate chosen by the j^{th} entrepreneur, u_{t+1}^j, must be chosen before period $t+1$ financial market shocks, and after the other period $t+1$ shocks. Higher rates of utilization are associated with higher costs as follows:

$$P_{t+1} a(u_{t+1}^j) \omega \bar{K}_{t+1}^j, \;\; a', a'' > 0.$$

As in BGG, we suppose that the entrepreneur is risk neutral. As a result, the j^{th} entrepreneur chooses u_{t+1}^j to solve:

$$\max_{u_{t+1}^j} E\left\{\left[u_{t+1}^j r_{t+1}^k - a(u_{t+1}^j)\right] \omega \bar{K}_{t+1}^j P_{t+1} | \Omega_{t+1}\right\}.$$

The first order necessary condition for optimization is:

$$E_t\left[r_t^k - a'(u_t)\right] = 0.$$

This reflects that $\bar{K}_{t+1}^{j}\ P_{t+1}$ are contained in Ω_{t+1}. After the capital has been rented in period $t+1$, the j^{th} entrepreneur sells the undepreciated part, $(1-\delta)\omega\bar{K}_{t+1}^{j}$, to the capital goods producer.

Below we introduce taxation on capital income. This does not enter into the above first order condition because capital income taxation affects rental income and the cost of utilization symmetrically. In addition, the capital income tax rate that applies to the utilization rate at time $t+1$ is contained in the information set, Ω_{t+1}.

2.4.2. Taxation of Capital Income

We adopt the following simple, tractable treatment of taxation on capital income. We suppose that the after-tax rate of return to capital, for an entrepreneur with productivity ω, is:

$$\begin{aligned} 1+R_{t+1}^{k,\omega} &= \left\{ \frac{(1-\tau_t^k)\left[u_{t+1}r_{t+1}^k - a(u_{t+1})\right] + (1-\delta)q_{t+1}}{q_t}\frac{P_{t+1}}{P_t} + \tau_t^k\delta \right\}\omega \\ &= (1+R_{t+1}^k)\omega. \end{aligned}$$

Note how after tax rate of return on capital for an individual entrepreneur is proportional to ω. A drawback of this specification is the implication that one cannot depreciate the full amount of the initial capital purchase, when ω is low. An interpretation is that depreciation allowances are lost when the level of income is too low to deduct the full amount.

2.4.3. The Financing Arrangement for the Entrepreneur

How is the j^{th} entrepreneur's level of capital, $\bar{K}_{t+1}^{j}$, determined? At the moment the entrepreneur enters the loan market, it's state variable is its net worth. It is has nothing else. It owns no capital, for example. Apart from net worth, no other aspect of the entrepreneur's history is relevant at this point.

There are many entrepreneurs, all with different amounts of net worth. We imagine that corresponding to each possible value of net worth, there are many entrepreneurs. They participate in a competitive loan market with banks. That is, there is a competitive loan market corresponding to each different level of net worth, N_{t+1}. In the usual CSV way, the contracts traded in the loan market specify an interest rate and a loan amount. The contracts are competitively determined. This means that they must satisfy a zero profit condition on banks and they must be utility maximizing for entrepreneurs. Equilibrium is incompatible with positive profits because of free entry and incompatible with negative profits because of free exit. In addition, contracts must be utility maximizing (subject to zero profits) for entrepreneurs because of competition. Equilibrium is incompatible with contracts that fail to do so, because in any candidate equilibrium like this, an individual bank could offer a better contract, one that makes positive profits, and take over the market.

The CSV contracts that we study are known to be optimal when there is no aggregate uncertainty. However, the way we have set up our environment, there is such uncertainty. We do this in part because we are interested exploring phenomena like the 'debt deflation hypothesis' discussed by Irving Fisher. We interpret this hypothesis as corresponding to a situation in which a shock (in this case, to the price level) occurs after entrepreneurs have borrowed from banks, but before they have paid back what they owe. A problem with what we do is that the contract we study is not known to be the optimal one. However, we suspect that in fact the contract is optimal, at least for sufficiently risk averse households. This is because the contract has the property that uncertainty associated with an aggregate shock is absorbed by entrepreneurs, while households receive a state-noncontingent rate of return on their loans to entrepreneurs (these loans actually are intermediated by banks). The reason this arrangement may not be optimal is as follows. We have not ruled out the possibility that there could be a return for households which is state contingent but compensates them for this, and which permits a CSV loan contract to entrepreneurs that increases their welfare. An alternative interpretation of our results is that there are other, nonmodeled reasons for assuming that the rate of return paid to households by banks are non-statecontingent. Subject to this restriction, the contracts we work with are optimal.

We now discuss the contracts offered in equilibrium to entrepreneurs with level of net worth, N_{t+1}. Denote the level of capital purchases by such an entrepreneur by $\bar{K}^N_{t+1}$. To finance such a purchase an $N_{t+1}-$type entrepreneur must borrow

$$B^N_{t+1} = Q_{\bar{K}',t}\bar{K}^N_{t+1} - N_{t+1}. \tag{2.6}$$

The standard debt contract specifies a loan amount, B^N_{t+1}, and a gross rate of interest, Z^N_{t+1}, to be paid if ω is high enough that the entrepreneur can do so. Entrepreneurs who cannot pay this interest rate, because they have a low value of ω must give everything they have to the bank. The parameters of the $N_{t+1}-$type standard debt contract, B^N_{t+1} Z^N_{t+1}, imply a cutoff value of ω, $\bar{\omega}^N_{t+1}$, as follows:[4]

$$\bar{\omega}^N_{t+1}\left(1 + R^k_{t+1}\right) Q_{\bar{K}',t}\bar{K}^N_{t+1} = Z^N_{t+1}B^N_{t+1}. \tag{2.7}$$

The amount of the loan, B^N_{t+1}, extended to an $N_{t+1}-$type entrepreneur is obviously not dependent on the realization of the period $t+1$ shocks. For reasons explained below, the

[4]With the alternative treatment of depreciation, this expression becomes:

$$\left(\left[1 + \tilde{R}^k_{t+1}\right]\bar{\omega}^N_{t+1} + \tau^k_t\delta\right) Q_{\bar{K}',t}\bar{K}^N_{t+1} = Z^N_{t+1}B^N_{t+1}.$$

interest rate on the loan, Z_{t+1}^N, is dependent on those shocks. Since R_{t+1}^k and Z_{t+1}^N are dependent on the period $t+1$ shocks, it follows from the previous expression that $\bar{\omega}_{t+1}^N$ is in principle also dependent upon those shocks.

For $\omega < \bar{\omega}_{t+1}^N$, the entrepreneur pays all its revenues to the bank:

$$\left(1 + R_{t+1}^k\right) \omega Q_{\bar{K}',t} \bar{K}_{t+1}^N,$$

which is less than $Z_{t+1}^N B_{t+1}^N$. In this case, the bank must monitor the entrepreneur, at cost

$$\mu \left(1 + R_{t+1}^k\right) \omega Q_{\bar{K}',t} \bar{K}_{t+1}^N.$$

We now describe how the parameters, B_{t+1}^N and Z_{t+1}^N, of the standard debt contract that is offered in equilibrium to entrepreneurs with net worth N_{t+1} are chosen.

We suppose that banks have access to funds at the end of the period t goods market at a nominal rate of interest, R_{t+1}^e. This interest rate is contingent on all shocks realized in period t, and is not contingent on the realization of the idiosyncratic shocks to individual $N_{t+1}-$type entrepreneurs, and is also not contingent on the $t+1$ aggregate shocks. Banks obtain these funds for lending to entrepreneurs by issuing time deposits at the end of the goods market in period t, which is when the entrepreneurs need funds for the purchase of $\bar{K}_{t+1}^N$. Zero profits for banks implies:

$$\left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] Z_{t+1}^N B_{t+1}^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) \left(1 + R_{t+1}^k\right) Q_{\bar{K}',t} \bar{K}_{t+1}^N = \left(1 + R_{t+1}^e\right) B_{t+1}^N, \tag{2.8}$$

or,

$$\left[1 - F\left(\bar{\omega}_{t+1}^N\right)\right] \bar{\omega}_{t+1}^N + (1-\mu) \int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^k} \frac{B_{t+1}^N}{Q_{\bar{K}',t} \bar{K}_{t+1}^N}. \tag{2.9}$$

BGG argue that, given a mild regularity condition on F, the expression on the left of the equality has an inverted U shape. There is some unique interior maximum, $\bar{\omega}^*$. It is increasing for $\bar{\omega}_{t+1}^N < \bar{\omega}^*$ and decreasing for $\bar{\omega}_{t+1}^N > \bar{\omega}^*$. Conditional on a given ratio, $B_{t+1}^N / \left(Q_{\bar{K}',t} \bar{K}_{t+1}^N\right)$, the right side fluctuates with R_{t+1}^k. The setup resembles the usual Laffer-curve setup, with the right side playing the role of the financing requirement and the left the role of tax revenues as a function of function of the 'tax rate', $\bar{\omega}_{t+1}^N$. So, we see that, generically, there are two $\bar{\omega}_{t+1}^N$'s that solve the above equation for given $B_{t+1}^N / \left(Q_{\bar{K}',t} \bar{K}_{t+1}^N\right)$. Between these two, the smaller one is preferred to entrepreneurs, so this is a candidate CSV. The implication is that in a CSV, $\bar{\omega}_{t+1}^N \leq \bar{\omega}^*$. Since, for $\bar{\omega}_{t+1}^N < \bar{\omega}^*$ the left side is increasing in a CSV, we conclude that any shock that drives up R_{t+1}^k will simultaneously drive down $\bar{\omega}_{t+1}^N$.

From (2.8), it is possible to see why Z_{t+1}^N must be dependent upon the realization of the period $t+1$ shocks. Substitute out for $\left(1+R_{t+1}^k\right)Q_{\bar{K}',t}\bar{K}_{t+1}^N$ using (2.7), to obtain:

$$\left[1-F(\bar{\omega}_{t+1}^N)+\frac{1-\mu}{\bar{\omega}_{t+1}^N}\int_0^{\bar{\omega}_{t+1}^N}\omega dF(\omega)\right]Z_{t+1}^N=\left(1+R_{t+1}^e\right),$$

after dividing both sides by B_{t+1}^N. Recall our specification that R_{t+1}^e is not dependent on the period $t+1$ realization of shocks. The last expression then implies that if Z_{t+1}^N is not dependent on the period $t+1$ shocks, then $\bar{\omega}_{t+1}^N$ must not be either. In this case, it is impossible for (2.7) to hold for all date $t+1$ states of nature. So, Z_{t+1}^N must be dependent on the period $t+1$ shocks.[5] Of course, if R_{t+1}^e were state dependent, then perhaps we could specify Z_{t+1}^N to be period $t+1$ state independent.

Substituting out for $Z_{t+1}^N B_{t+1}^N$ from (2.7) in the bank's zero profit condition, we obtain:[6]

$$\left(1+R_{t+1}^e\right)B_{t+1}^N = \left[1-F(\bar{\omega}_{t+1}^N)\right]\bar{\omega}_{t+1}^N\left(1+R_{t+1}^k\right)Q_{\bar{K}',t}\bar{K}_{t+1}^N \qquad (2.10)$$

[5] This may appear implausible, at first glance. In practice, when banks extend loans the rate of interest that is to be paid is specified in advance. One interpretation of the fact that Z_t^N is contingent on the realization of the aggregate shock is that banks are unwilling to extend loans whose duration spans the whole period of the entrepreneur's project. Instead, they extend the loan for a part of the period, and that allows them to back out before too many funds are commited, in case it looks like the project is going bad. This is closely related to the interpretation offered in Bernanke, Gertler and Gilchrist (1999, footnote 10).

[6] Under the alternative treatment of depreciation,

$$\begin{aligned}
\left(1+R_{t+1}^e\right)B_{t+1}^N &= \left[1-F(\bar{\omega}_{t+1}^N)\right]\left[(1+\tilde{R}_{t+1}^k)\bar{\omega}_{t+1}+\tau_t^k\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^N \\
&\quad +\int_0^{\bar{\omega}_{t+1}^N}(1-\mu)\left[(1+\tilde{R}_{t+1}^k)\omega+\tau_t^k\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^j dF(\omega) \\
&= \left[1-F(\bar{\omega}_{t+1}^N)\right]\left[(1+\tilde{R}_{t+1}^k)\bar{\omega}_{t+1}+\tau_t^k\delta\right]Q_{\bar{K}',t}\bar{K}_{t+1}^N \\
&\quad +G(\bar{\omega}_{t+1}^N)(1-\mu)(1+\tilde{R}_{t+1}^k)Q_{\bar{K}',t}\bar{K}_{t+1}^j+F(\bar{\omega}_{t+1}^N)(1-\mu)\tau_t^k\delta Q_{\bar{K}',t}\bar{K}_{t+1}^j \\
&= \left[\left(1-F(\bar{\omega}_{t+1}^N)\right)\bar{\omega}_{t+1}+G(\bar{\omega}_{t+1}^N)(1-\mu)\right](1+\tilde{R}_{t+1}^k)Q_{\bar{K}',t}\bar{K}_{t+1}^N+\tau_t^k\delta Q_{\bar{K}',t}\bar{K}_{t+1}^N\left[1-F(\bar{\omega}_{t+1}^N)\mu\right] \\
&= \left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right](1+\tilde{R}_{t+1}^k)Q_{\bar{K}',t}\bar{K}_{t+1}^N+\tau_t^k\delta Q_{\bar{K}',t}\bar{K}_{t+1}^N\left[1-F(\bar{\omega}_{t+1}^N)\mu\right]
\end{aligned}$$

or, after dividing:

$$\frac{\left(1+R_{t+1}^e\right)B_{t+1}^N}{(1+\tilde{R}_{t+1}^k)Q_{\bar{K}',t}\bar{K}_{t+1}^N}=\left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right]+\frac{\tau_t^k\delta\left[1-F(\bar{\omega}_{t+1}^N)\mu\right]}{(1+\tilde{R}_{t+1}^k)}$$

$$
\begin{aligned}
&+\int_0^{\bar{\omega}_{t+1}^N}(1-\mu)\left(1+R_{t+1}^k\right)\omega Q_{\bar{K}',t}\bar{K}_{t+1}^j dF(\omega)\\
=&\left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right]\left(1+R_{t+1}^k\right)Q_{\bar{K}',t}\bar{K}_{t+1}^N,
\end{aligned}
$$

where $\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)$ is the expected share of profits, net of monitoring costs, accruing to the bank and

$$
\begin{aligned}
G(\bar{\omega}_{t+1}^N) &= \int_0^{\bar{\omega}_{t+1}^N}\omega dF(\omega).\\
\Gamma(\bar{\omega}_{t+1}^N) &= \bar{\omega}_{t+1}^N\left[1-F(\bar{\omega}_{t+1}^N)\right]+G(\bar{\omega}_{t+1}^N)
\end{aligned}
$$

It is useful to work out the derivative of Γ :

$$
\begin{aligned}
\Gamma'(\bar{\omega}_{t+1}^N) &= 1-F(\bar{\omega}_{t+1}^N)-\bar{\omega}_{t+1}^N F'(\bar{\omega}_{t+1}^N)+G'(\bar{\omega}_{t+1}^N)\\
&= 1-F(\bar{\omega}_{t+1}^N)>0.
\end{aligned}
\tag{2.11}
$$

Dividing both sides of (2.10) by $Q_{\bar{K}',t}\bar{K}_{t+1}^N\left(1+R_{t+1}^k\right)$:

$$
\frac{1+R_{t+1}^e}{1+R_{t+1}^k}\left(1-\frac{N_{t+1}}{Q_{\bar{K}',t}\bar{K}_{t+1}^N}\right)=\left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right]
$$

Multiply this expression by $\left(Q_{\bar{K}',t}\bar{K}_{t+1}^N/N_{t+1}\right)(1+R_{t+1}^k)/(1+R_{t+1}^e)$, to obtain:

$$
\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}}-1=\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}}\frac{1+R_{t+1}^k}{1+R_{t+1}^e}\left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right].
$$

Let

$$
\tilde{u}_{t+1}\equiv\frac{1+R_{t+1}^k}{E\left(1+R_{t+1}^k|\Omega_t^\mu\right)},\ s_{t+1}\equiv\frac{E\left(1+R_{t+1}^k|\Omega_t^\mu\right)}{1+R_{t+1}^e}.
$$

Then, the non-negativity constraint on bank profits is:

$$
\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}}-1\leq\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}}\tilde{u}_{t+1}s_{t+1}\left[\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right],
\tag{2.12}
$$

From this we can see that $\bar{\omega}_{t+1}^N$ is a function of the capital to net worth ratio and $\left(1+R_{t+1}^e\right)/\left(1+R_{t+1}^k\right)$ only:

$$
\bar{\omega}_{t+1}^N=g\left(\frac{1+R_{t+1}^e}{1+R_{t+1}^k}\left(1-\frac{N_{t+1}}{Q_{\bar{K}',t}\bar{K}_{t+1}^N}\right)\right).
\tag{2.13}
$$

As noted above, competition implies that the loan contract is the best possible one, from the point of view of the entrepreneur. That is, it maximizes the entrepreneur's 'utility' subject to the zero profit constraint just stated. The entrepreneur's expected revenues over the period in which the standard debt contract applies is:

$$\begin{aligned} & E\left\{\int_{\bar{\omega}_{t+1}^N}^{\infty}\left[\left(1+R_{t+1}^k\right)\omega Q_{\bar{K}',t}\bar{K}_{t+1}^N-Z_{t+1}^N B_{t+1}^N\right]dF(\omega)|\Omega_t^{\mu}\right\} \\ = \; & E\left\{\int_{\bar{\omega}_{t+1}^N}^{\infty}\left[\omega-\bar{\omega}_{t+1}^N\right]dF(\omega)\left(1+R_{t+1}^k\right)|\Omega_t^{\mu}\right\}Q_{\bar{K}',t}\bar{K}_{t+1}^N. \end{aligned}$$

Note that

$$1=\int_0^{\infty}\omega dF(\omega)=\int_{\bar{\omega}_{t+1}^N}^{\infty}\omega dF(\omega)+G(\bar{\omega}_{t+1}^N),$$

so that the objective can be written:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^N)\right]\left(1+R_{t+1}^k\right)|\Omega_t^{\mu}\right\}Q_{\bar{K}',t}\bar{K}_{t+1}^N,$$

or, after dividing by $(1+R_{t+1}^e)N_{t+1}$ (which is constant across realizations of date $t+1$ uncertainty), and rewriting:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^N)\right]\tilde{u}_{t+1}|\Omega_t^{\mu}\right\}s_{t+1}\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}},\;\tilde{u}_{t+1}=\frac{1+R_{t+1}^k}{E\left(1+R_{t+1}^k|\Omega_t^{\mu}\right)},\;s_{t+1}=\frac{E\left(1+R_{t+1}^k|\Omega_t^{\mu}\right)}{1+R_{t+1}^e}, \tag{2.14}$$

where Ω_t^{μ} denotes all period t shocks. From this expression and the fact, $\Gamma'>0$, it is evident that the objective is decreasing in $\bar{\omega}_{t+1}^N$ for given $Q_{\bar{K}',t}\bar{K}_{t+1}^N/N_{t+1}$. This property of the objective was alluded to above.

The debt contract selects $Q_{\bar{K}',t}\bar{K}_{t+1}^N/N_{t+1}$ and $\bar{\omega}_{t+1}^N$ to optimize (2.14) subject to (2.12). It is convenient to denote:

$$k_{t+1}^N=\frac{Q_{\bar{K}',t}\bar{K}_{t+1}^N}{N_{t+1}}.$$

Writing the CSV problem in Lagrangian form,

$$\max_{\bar{\omega}^N,k^N}E\left\{\left[1-\Gamma(\bar{\omega}^N)\right]\tilde{u}_{t+1}s_{t+1}k^N+\lambda^N\left[k^N\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}^N)-\mu G(\bar{\omega}^N)\right)-k^N+1\right]|\Omega_t^{\mu}\right\}.$$

The single first order condition for k^N is:

$$E\left\{\left[1-\Gamma(\bar{\omega}_{t+1}^N)\right]\tilde{u}_{t+1}s_{t+1}+\lambda_{t+1}^N\left[\tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}_{t+1}^N)-\mu G(\bar{\omega}_{t+1}^N)\right)-1\right]|\Omega_t^{\mu}\right\}=0. \tag{2.15}$$

The first order conditions for $\bar{\omega}^N$ are, after dividing by $\tilde{u}_{t+1}s_{t+1}k_{t+1}^N$:

$$\Gamma'(\bar{\omega}_{t+1}^N) = \lambda_{t+1}^N \left[\Gamma'(\bar{\omega}_{t+1}^N) - \mu G'(\bar{\omega}_{t+1}^N)\right]. \tag{2.16}$$

Finally, there is the complementary slackness condition, $\lambda^N \left[k^N \tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}^N) - \mu G(\bar{\omega}^N)\right) - k^N + 1\right] = 0$. Assuming the constraint is binding, so that $\lambda^N > 0$, this reduces to:

$$k_{t+1}^N \tilde{u}_{t+1}s_{t+1}\left(\Gamma(\bar{\omega}_{t+1}^N) - \mu G(\bar{\omega}_{t+1}^N)\right) - k_{t+1}^N + 1 = 0. \tag{2.17}$$

It should be understood that λ_{t+1}^N in (2.15) is defined by (2.16). We can think of (2.15)-(2.17) as defining functions relating k_{t+1}^N and $\bar{\omega}_{t+1}^N$to s_{t+1}. Remember, k_{t+1}^N is not indexed by $\tilde{u}_{t+1}$, while $\bar{\omega}_{t+1}^N$ is. So, we think of $\bar{\omega}_{t+1}^N$ as a family of functions of s_{t+1}, each function being indexed by a different realization of $\tilde{u}_{t+1}$. Note that N_{t+1} does not appear in the equations that define k_{t+1}^N and $\bar{\omega}_{t+1}^N$. This establishes that the values of these variables in the CSV contract is the same for each value of N_{t+1}. For this reason, we can drop the superscript notation, N. That is, the functions we are concerned with are k_{t+1} and $\bar{\omega}_{t+1}$.

We find it convenient to drop time subscripts to keep the notation simple, and because it should entail no confusion. The equations that concern us are:

$$E\left\{[1 - \Gamma(\bar{\omega})]\tilde{u}s + \lambda\left[\tilde{u}s\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right) - 1\right]\right\} = 0, \tag{2.18}$$

$$\Gamma'(\bar{\omega}) = \lambda\left[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})\right], \tag{2.19}$$

$$k\tilde{u}s\left(\Gamma(\bar{\omega}) - \mu G(\bar{\omega})\right) - k + 1 = 0. \tag{2.20}$$

It is understood that the expectation operator is over different values of $\tilde{u}$, and k is constant across $\tilde{u}$ while λ and $\bar{\omega}$ vary with $\tilde{u}$. These three equations are used used to help characterize the equilibrium of the model.

2.4.4. Aggregating Across Entrepreneurs

We now discuss the evolution of the aggregate net worth of all entrepreneurs. In terms of the previous notation, if $f_{t+1}(N)$ is the density of entrepreneurs having net worth N_{t+1}, then aggregate net worth, $\bar{N}_{t+1}$, is:

$$\bar{N}_{t+1} = \int_0^\infty N f_{t+1}(N)dN.$$

We now discuss the law of motion of aggregate net worth. Suppose $\bar{N}_t$ is given. Let V_t^N denote the average of profits of N_t-type entrepreneurs, net of repayments to banks:

$$V_t^N = \left(1 + R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t^N - \Gamma(\bar{\omega}_t)\left(1 + R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t^N.$$

The aggregate capital stock is:

$$\bar{K}_t = \int_0^\infty f_t(N)\bar{K}_t^N dN$$

Given that R_t^k and $\bar{\omega}_t$ are independent of N_t, we have:

$$V_t \equiv \int_0^\infty f_t(N)V_t^N dN = \left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t - \Gamma(\bar{\omega}_t)\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t$$

Writing this out more fully:

$$\begin{aligned}
V_t &= \left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t - \left\{\left[1-F(\bar{\omega}_t)\right]\bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega)\right\}\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t \\
&= \left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t \\
&\quad - \left\{\left[1-F(\bar{\omega}_t)\right]\bar{\omega}_t + (1-\mu)\int_0^{\bar{\omega}_t} \omega dF(\omega) + \mu\int_0^{\bar{\omega}_t} \omega dF(\omega)\right\}\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t.
\end{aligned}$$

Notice that the first two terms in braces correspond to the net revenues of the bank, which must equal $(1+R_t^e)\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right)$. Substituting:

$$V_t = \left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t - \left\{1+R_t^e + \frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t}\right\}\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right). \tag{2.21}$$

Since entrepreneurs are selected randomly for death, the integral over entrepreneurs' net profits is just γV_t. So, the law of motion for $\bar{N}_t$ is:

$$\begin{aligned}
\bar{N}_{t+1} &= \gamma\left\{\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t - \left[1+R_t^e + \frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right) Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t}\right]\left(Q_{\bar{K}',t-1}\bar{K}_t - \bar{N}_t\right)\right\} \\
&\quad + W_t^e,
\end{aligned} \tag{2.22}$$

where W_t^e is the transfer payment to entrepreneurs. The $(1-\gamma)$ entrepreneurs who are selected for death, consume:

$$P_t C_t^e = \Theta(1-\gamma)V_t.$$

The 'external finance premium' is the ratio involving μ in square brackets above. It is the difference between the 'internal cost of funds', $1+R_t^e$, and the expected cost of borrowing to an entrepreneur. The reason for calling $1+R_t^e$ the internal cost of funds is that in principle one could imagine the entrepreneur using its net worth to acquire time deposits, instead of physical capital (the model does not formally allow this). In this sense, the cost of the entrepreneur's own funds, which do not involve any costly state verification, is $1+R_t^e$.

2.5. Banks

We assume that there is a continuum of identical, competitive banks. Each operates a technology to convert capital, K_t^b, labor, l_t^b, and excess reserves, E_t^b, into real deposit services, D_t/P_t. The production function is:

$$\frac{D_t}{P_t} = a^b x_t^b \left(\left(K_t^b \right)^{\alpha} \left(z_t l_t^b \right)^{1-\alpha} \right)^{\xi_t} \left(\frac{E_t^r}{P_t} \right)^{1-\xi_t} \tag{2.23}$$

Here a^b is a positive scalar, and $0 < \alpha < 1$. Also, x_t^b is a unit-mean technology shock that is specific to the banking sector. In addition, $\xi_t \in (0,1)$ is a shock to the relative value of excess reserves, E_t^r. The stochastic process governing these shocks will be discussed later. We include excess reserves as an input to the production of demand deposit services as a reduced form way to capture the precautionary motive of a bank concerned about the possibility of unexpected withdrawals.

We now discuss a typical bank's balance sheet. The bank's assets consist of cash reserves and loans. It obtains cash reserves from two sources. Households deposit A_t dollars and the monetary authority credits households' checking accounts with X_t dollars. Consequently, total time t cash reserves of the banking system equal $A_t + X_t$. Bank loans are extended to firms and other banks to cover their working capital needs, and to entrepreneurs to finance purchases of capital.

The bank has two types of liabilities: demand deposits, D_t, and time deposits, T_t. Demand deposits, which pay interest, R_{at}, are created for two reasons. First, there are the household deposits, $A_t + X_t$ mentioned above. We denote this by D_t^h. Second, working capital loans made by banks to firms and other banks are granted in the form of demand deposits. We denote firm and bank demand deposits by D_t^f. Total deposits, then, are:

$$D_t = D_t^h + D_t^f.$$

Time deposit liabilities are issued by the bank to finance the standard debt contracts offered to entrepreneurs and discussed in the previous section. Time and demand deposits differ in three respects. First, demand deposits yield transactions services, while time deposits do not. Second, time deposits have a longer maturity structure. Third, demand deposits are backed by working capital loans and reserves, while time deposits are backed by standard debt contracts to entrepreneurs.

We now discuss the demand deposit liabilities. We suppose that the interest on demand deposits that are created when firms and banks receive working capital loans, are paid to the recipient of the loans. Firms and banks just sit on these demand deposits. The wage bill isn't actually paid to workers until a settlement period that occurs after the goods market.

We denote the interest payment on working capital loans, net of interest on the associated demand deposits, by R_t. Since each borrower receives interest on the deposit associated with their loan, the gross interest payment on loans is $R_t + R_{at}$. Put differently, the spread between the interest on working capital loans and the interest on demand deposits is R_t.

The maturity of period t working capital loans and the associated demand deposit liabilities coincide. A period t working capital loan is extended just prior to production in period t, and then paid off after production. The household deposits funds into the bank just prior to production in period t and then liquidates the deposit after production.

We now discuss the time deposit liabilities. Unlike in the case of demand deposits, we assume that the cost of maintaining time deposit liabilities is zero. Competition among banks in the provision of time deposits and entrepreneurial loans drives the interest rate on time deposits to the return the bank earns (net of expenses, including monitoring costs) on the loans, R_t^e. The maturity structure of time deposits coincides with that of the standard debt contract, and differs from that of demand deposits and working capital loans. The maturity structure of the two types of assets can be seen in Figure 3. Time deposits and entrepreneurial loans are created at the end of a given period's goods market. This is the time when newly constructed capital is sold by capital producers to entrepreneurs. Time deposits and entrepreneurial loans pay off at the end of next period's goods market, when the entrepreneurs sell their undepreciated capital to capital producers (who use it as a raw material in the production of next period's capital). The payoff on the entrepreneurial loan coincides with the payoff on time deposits. Competition in the provision of time deposits guarantees that these payoffs coincide.

The maturity difference between demand and time deposits implies that the return on the latter in principle carries risks not present in the former. In the case of demand deposits, no shocks are realized between the creation of a deposit and its payoff. In the case of time deposits, there are shocks whose value is realized between creation and payoff (see Figure 3). Since time deposits finance assets with an uncertain payoff, someone has to bear the risk. We follow BGG in focusing on equilibria in which the entrepreneur bears all the risk. The ex post return on time deposits is know with certainty to the household at the time the deposit decision is made.

We now discuss the assets and liabilities of the bank in greater detail. We describe the banks' books at two points in time within the period: just before the goods market, when the market for working capital loans and demand deposits is open, and just after the goods market. At the latter point in time, the market for time deposits and entrepreneurial loans is open. Liabilities and assets just before the goods market are:

$$D_t + T_{t-1} = A_t + X_t + S_t^w + B_t, \tag{2.24}$$

where S_t^w denotes working capital loans. The monetary authority imposes a reserve requirement that banks must hold at least a fraction τ of their demand deposits in the form of currency. Consequently, nominal excess reserves, E_t^r, are given by

$$E_t^r = A_t + X_t - \tau_t D_t. \tag{2.25}$$

The bank's 'T' accounts are as follows:

Assets	Liabilities
Reserves	
A_t	D_t
X_t	
Short-term Working Capital Loans	
S_t^w	
Long-term, Entrepreneurial Loans	
B_t	T_{t-1}

After the goods market, demand deposits are liquidated, so that $D_t = 0$ and $A_t + X_t$ is returned to the households, so this no longer appears on the bank's balance sheet. Similarly, working capital loans, S_t^w, and 'old' entrepreneurial loans, B_t, are liquidated at the end of the goods market and also do not appear on the bank's balance sheet. At this point, the assets on the bank's balance sheet are the new entrepreneurial loans issued at the end of the goods market, B_{t+1}, and the bank liabilities are the new time deposits, T_t.

At the end of the goods market, the bank settles claims for transactions that occurred in the goods market and that arose from it's activities in the previous period's entrepreneurial loan and time deposit market. The bank's sources of funds at this time are: net interest from borrowers and $A_t + X_t$ of high-powered money (i.e., a mix of vault cash and claims on the central bank).[7] Working capital loans coming due at the end of the period pay R_t in interest and so the associated principal and interest is

$$(1 + R_t)S_t^w = (1 + R_t)\left(\psi_{l,t}W_t l_t + \psi_{k,t}P_t r_t^k K_t\right).$$

Loans to entrepreneurs coming due at the end of the period are the ones that were extended in the previous period, $Q_{\bar{K}',t-1}\bar{K}_t - N_t$, and they pay the interest rate from the previous period, after monitoring costs:

$$(1 + R_t^e)\left(Q_{\bar{K}',t-1}\bar{K}_t - N_t\right)$$

The bank's uses of funds are (i) interest and principle obligations on demand deposits and time deposits, $(1 + R_{at})D_t$ and $(1 + R_t^e)T_{t-1}$, respectively, and (ii) interest and principal

[7] Interest is not paid by the central bank on high-powered money.

expenses on working capital, i.e., capital and labor services. Interest and principal expenses on factor payments in the banking sector are handled in the same way as in the goods sector. In particular, banks must finance a fraction, $\psi_{k,t}$, of capital services and a fraction, $\psi_{l,t}$, of labor services, in advance, so that total factor costs as of the end of the period, are $(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b$. The bank's net source of funds, Π_t^b, is:

$$\begin{aligned}\Pi_t^b \;=\;& (A_t+X_t)+(1+R_t+R_{at})S_t^w-(1+R_{at})D_t \\ & -\left[(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b\right]-\left[(1+\psi_{l,t}R_t)\,W_t l_t^b\right] \\ & +\left[1+R_t^e+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t}{Q_{\bar{K}',t-1}\bar{K}_t-N_t}\right]B_t \\ & -\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\left(1+R_t^k\right)Q_{\bar{K}',t-1}\bar{K}_t-(1+R_t^e)\,T_{t-1} \\ & +T_t-B_{t+1}\end{aligned} \tag{2.26}$$

Because of competition, the bank takes all wages and prices and interest rates as given and beyond its control.

We now describe the bank's optimization problem. The bank pays Π_t^b to households in the form of dividends. It's objective is to maximize the present discounted value of these dividends. In period 0, its objective is:

$$E_0\sum_{t=0}^{\infty}\beta^t\lambda_t\Pi_t^b,$$

where λ_t is the multiplier on Π_t^b in the Lagrangian representation of the household's optimization problem. It takes as given its time deposit liabilities from the previous period, T_{-1}, and its entrepreneurial loans issued in the previous period, B_0. In addition, the bank takes all rates of return and λ_t as given. The bank optimizes its objective by choice of $\left\{S_t^w,\ B_{t+1},\ D_t,\ T_t,\ K_t^b,\ E_t^r; t\geq 0\right\}$, subject to (2.23)-(2.25).

In the previous section, we discussed the determination of the variables relating to entrepreneurial loans. There is no further need to discuss them here, and so we take those as given. To discuss the variables of concern here, we adopt a Lagrangian representation of the bank problem which uses a version of (2.26) that ignores variables pertaining to the entrepreneur. The Lagrangian representation of the problem that we work with is:

$$\begin{aligned}&\max_{A_t,S_t^w,K_t^b,l_t^b}\left\{R_tS_t^w-R_{at}\left(A_t+X_t\right)-R_t^bF_t-\left[(1+\psi_{k,t}R_t)\,P_t r_t^k K_t^b\right]-\left[(1+\psi_{l,t}R_t)\,W_t l_t^b\right]\right\} \\ &+\lambda_t^b\left[h(x_t^b,K_t^b,l_t^b,\frac{A_t+X_t+F_t-\tau_t\left(A_t+X_t+S_t^w\right)}{P_t},\xi_t,x_t^b,z_t)-\frac{A_t+X_t+S_t^w}{P_t}\right]\end{aligned}$$

where

$$
\begin{aligned}
h(x_t^b, K_t^b, l_t^b, e_t^r, \xi_t, x_t^b, z_t) &= a^b x_t^b \left(\left(K_t^b\right)^{\alpha} \left(z_t l_t^b\right)^{1-\alpha} \right)^{\xi_t} \left(e_t^r\right)^{1-\xi_t} \\
e_t^r &= \frac{E_t^r}{P_t} = \frac{A_t + X_t + F_t - \tau_t \left(A_t + X_t + S_t^w\right)}{P_t}
\end{aligned}
$$

Here, F_t is introduced to allow us to define a 'Federal Funds Rate', R_t^b, in the model. The quantity, F_t, corresponds to reserves borrowed in an interbank loan market. Note that borrowing F_t creates a net obligation of $R_t^b F_t$ at the end of the period. On the plus side, it adds to the bank's holdings of reserves. Of course, since our banks are formally identical, market clearing requires $F_t = 0$ in equilibrium. The banks first order necessary condition for optimality associated with F_t is:

$$
R_t^b = \frac{\lambda_t^b h_{e^r,t}}{P_t}.
$$

The first order conditions are, for A_t, S_t^w, K_t^b, l_t^b, respectively:

$$
\begin{aligned}
-R_{at} + \lambda_t^b \frac{1}{P_t} \left[\left(1 - \tau_t\right) h_{e^r,t} - 1 \right] &= 0 && (2.27) \\
R_t - \lambda_t^b \frac{1}{P_t} \left[\tau_t h_{e^r,t} + 1 \right] &= 0 && (2.28) \\
-\left(1 + \psi_{k,t} R_t\right) P_t r_t^k + \lambda_t^b h_{K^b,t} &= 0 && (2.29) \\
-\left(1 + \psi_{l,t} R_t\right) W_t + \lambda_t^b h_{l^b,t} &= 0 && (2.30)
\end{aligned}
$$

Substituting for λ_t^b in (2.29) and (2.30) from (2.28), we obtain:

$$
\left(1 + \psi_{k,t} R_t\right) r_t^k = \frac{R_t h_{K^b,t}}{1 + \tau_t h_{e^r,t}},
$$

and

$$
\left(1 + \psi_{l,t} R_t\right) \frac{W_t}{P_t} = \frac{R_t h_{l^b,t}}{1 + \tau_t h_{e^r,t}}.
$$

Similarly, after substituting out for the multiplier in the expression for R_t^b, we obtain:

$$
\begin{aligned}
R_t^b &= \frac{\lambda_t^b h_{e^r,t}}{P_t} = \frac{R_t h_{e^r,t}}{\tau_t h_{e^r,t} + 1} \\
&= R_{a,t} \frac{h_{e^r,t}}{\left(1 - \tau_t\right) h_{e^r,t} - 1}
\end{aligned}
$$

These are the first order conditions associated with the bank's choice of capital and labor. Each says that the bank attempts to equate the marginal product - in terms of extra loans - of an additional factor of production, with the associated marginal cost. The marginal product in producing loans must take into account two things: an increase in S^w requires an equal increase in deposits and an increase in deposits raises required reserves. The first raises loans by the marginal product of the factor in h, while the reserve implication works in the other direction.

Taking the ratio of (2.28) to (2.27), we obtain:

$$R_{at} = \frac{(1 - \tau_t)\, h_{e^r,t} - 1}{\tau_t h_{e^r,t} + 1} R_t. \tag{2.31}$$

This can be thought of as the first order condition associated with the bank's choice of A_t. The object multiplying R_t is the increase in S^w the bank can offer for one unit increase in A. The term on the right indicates the net interest earnings from those loans. The term on the left indicates the cost. Recall that R_t represents *net* interest on loans, because the actual interest is $R_t + R_{at}$, so that R_t represents the spread between the interest rate charged by banks on their loans and the cost to them of the underlying funds. Since loans are made in the form of deposits, and deposits earn R_{at} in interest, the net cost of a loan to a borrower is R_t.

The clearing condition in the market for working capital loans is:

$$S_t^w = \psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t^k K_t \tag{2.32}$$

Here, S_t^w represents the supply of loans, and the terms on the right of the equality in (2.32) represent total demand.

2.6. Households

There is a continuum of households, indexed by $j \in (0, 1)$. Households consume, save and supply a differentiated labor input. The sequence of decisions by the household during a period is as follows. First, it makes its consumption decision after the non-financial shocks are realized. In addition, it allocates its financial assets between currency and deposits. Second, it purchases securities whose payoffs are contingent upon whether it can reoptimize its wage decision. Third, it sets its wage rate after finding out whether or not it can reoptimize. Fourth, the current period monetary action is realized. Fifth, after the monetary action, and before the goods market, the household decides how much of its financial assets to hold in the form of currency and demand deposits. At this point, the time deposits purchased by the household in the previous period are fixed and beyond its control. Sixth, the household goes to the goods market, where labor services are supplied and goods are purchased. Seventh, after the goods market, the household settles claims arising from its goods market experience and makes its current period time deposit decision.

Since the uncertainty faced by the household over whether it can reoptimize its wage is idiosyncratic in nature, households work different amounts and earn different wage rates. So, in principle they are also heterogeneous with respect to consumption and asset holdings. A straightforward extension of arguments in Erceg, Henderson and Levin (2000) and Woodford (1996), establish that the existence of state contingent securities ensures that in equilibrium households are homogeneous with respect to consumption and asset holdings. Reflecting this result, our notation assumes that households are homogeneous with respect to consumption and asset holdings, and heterogeneous with respect to the wage rate that they earn and hours worked. The preferences of the j^{th} household are given by:

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left\{ u(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l} z(h_{j,t+l}) - \upsilon_{t+l} \frac{\left[\left(\frac{P_{t+l}C_{t+l}}{M_{t+l}} \right)^{\theta_{t+l}} \left(\frac{P_{t+l}C_{t+l}}{D_{t+l}^h} \right)^{1-\theta_{t+l}} \right]^{1-\sigma_q}}{1-\sigma_q} - H\left(\frac{M_{t+l}}{M_{t+l-1}}\right) \right\}, \tag{2.33}$$

where E_t^j is the expectation operator, conditional on aggregate and household j idiosyncratic information up to, and including, time $t-1$; C_t denotes time t consumption; h_{jt} denotes time t hours worked and ζ_t is a shock with mean unity to the preference for leisure. In order to help assure that our model has a balanced growth path, we specify that u is the natural logarithm. When $b > 0$, (2.33) allows for habit formation in consumption preferences. Various authors, such as Fuhrer (2000), and McCallum and Nelson (1998), have argued that this is important for understanding the monetary transmission mechanism. In addition, habit formation is useful for understanding other aspects of the economy, including the size of the premium on equity. The term in square brackets captures the notion that currency and demand deposits contribute to utility by providing transactions services. Those services are an increasing function of the level of consumption. Finally, H represents an adjustment costs in holdings of currency. We assume that $H' = 0$ along a steady state growth path, and $H'' > 0$ along such a path. The assumption on H' ensures that H does not enter the steady state of the model. Given our linearization strategy, the only free parameter here is H'' itself.

We now discuss the household's period t uses and sources of funds. Just before the goods market in period t, after the realization of all shocks, the household has M_t^b units of high powered money which it splits into currency, M_t, and deposits with the bank:

$$M_t^b - (M_t + A_t) \geq 0. \tag{2.34}$$

The household deposits A_t with the bank, in exchange for a demand deposit. Demand deposits pay the relatively low interest rate, R_{at}, but offer transactions services.

The central bank credits the household's bank deposit with X_t units of high powered money, which automatically augments the household's demand deposits. So, household demand deposits are D_t^h :

$$D_t^h = A_t + X_t.$$

As noted in the previous section, the household only receives interest on the non-wage component of its demand deposits, since the interest on the wage component is earned by intermediate good firms.

The household also can acquire a time deposit. This can be acquired at the end of the period t goods market and pays a rate of return, $1+R_{t+1}^e$, at the end of the period $t+1$ goods market. The rate of return, R_{t+1}^e, is known at the time that the time deposit is purchased. It is not contingent on the realization of any of the period $t+1$ shocks.

The household also uses its funds to pay for consumption goods, P_tC_t and to acquire high powered money, Q_{t+1}, for use in the following period. Additional sources of funds include profits from producers of capital, Π_t^k, from banks, Π_t^b, from intermediate good firms, $\int \Pi_t^j dj$, and $A_{j,t}$, the net payoff on the state contingent securities that the household purchases to insulate itself from uncertainty associated with being able to reoptimize its wage rate. Households also receive lump-sum transfers, $1-\Theta$, corresponding to the net worth of the $1-\gamma$ entrepreneurs which die in the current period. Finally, the households pay a lump-sum tax to finance the transfer payments made to the γ entrepreneurs that survive and to the $1-\gamma$ newly born entrepreneurs. These observations are summarized in the following asset accumulation equation:

$$\begin{aligned}
&\left[1+\left(1-\tau_t^D\right)R_{at}\right]\left(M_t^b - M_t + X_t\right) - T_t \qquad (2.35)\\
&-\left(1+\tau_t^c\right)P_tC_t + (1-\Theta)(1-\gamma)V_t - W_t^e + Lump_t\\
&+\left[1+\left(1-\tau_t^T\right)R_t^e\right]T_{t-1} + \left(1-\tau_t^l\right)W_{j,t}h_{j,t} + M_t + \Pi_t^b + \Pi_t^k + \int \Pi_t^f df + A_{j,t} - M_{t+1}^b \geq 0.
\end{aligned}$$

The household's problem is to maximize (2.33) subject to the timing constraints mentioned above, the various non-negativity constraints, and (2.35).

We consider the Lagrangian representation of the household problem, in which $\lambda_t \geq 0$ is the multiplier on (2.35). The consumption, M_t and wage decisions are taken before the realization of the financial market shocks. That is, these decisions are contingent on on Ω_t. The other decisions, M_{t+1}^b and T_t are taken after the realization of all shocks during the period, i.e., contingent on Ω_t^μ. The period t multipliers are functions of all the date t shocks. We now consider the first order conditions associated with C_t, M_{t+1}^b, M_t and T_t. The Lagrangian representation of the problem, ignoring constant terms in the asset evolution equation, is:

$$
\begin{aligned}
&E_0^j \sum_{t=0}^{\infty} \beta^t \{u(C_t - bC_{t-1}) - \zeta_t z(h_{j,t}) - \upsilon_t \frac{\left[P_t C_t \left(\frac{1}{M_t}\right)^{\theta_t} \left(\frac{1}{M_t^b - M_t + X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q}}{1-\sigma_q} \\
&+\lambda_t [\left[1 + \left(1 - \tau_t^D\right) R_{at}\right] \left(M_t^b - M_t\right) - T_t - \left(1 + \tau_t^c\right) P_t C_t \\
&+ \left[1 + \left(1 - \tau_t^T\right) R_t^e\right] T_{t-1} + \left(1 - \tau_t^l\right) W_{j,t} h_{j,t} + M_t - M_{t+1}^b]\}
\end{aligned}
$$

We now consider the various first order conditions associated with this maximization problem.

The first order condition with respect to T_t is:

$$
E\left\{-\lambda_t + \beta \lambda_{t+1} \left[1 + \left(1 - \tau_{t+1}^T\right) R_{t+1}^e\right] | \Omega_t^\mu\right\} = 0
$$

The first order condition with respect to M_t is:

$$
\begin{aligned}
E\{\upsilon_t \left[\left(\frac{P_t C_t}{M_t}\right)^{\theta_t} \left(\frac{P_t C_t}{M_t^b - M_t + X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q} &[\frac{\theta_t}{M_t} - \frac{(1-\theta_t)}{M_t^b - M_t + X_t}] \\
&- \lambda_t \left(1 - \tau_t^D\right) R_{at} | \Omega_t\} = 0
\end{aligned} \tag{2.36}
$$

The first order condition with respect to M_{t+1}^b is:

$$
\begin{aligned}
E\{\beta \upsilon_{t+1} \left(1 - \theta_{t+1}\right) \left[P_{t+1} C_{t+1} \left(\frac{1}{M_{t+1}}\right)^{\theta_{t+1}} \left(\frac{1}{M_{t+1}^b - M_{t+1} + X_{t+1}}\right)^{(1-\theta_{t+1})}\right]^{1-\sigma_q} &\frac{1}{M_{t+1}^b - M_{t+1} + X_{t+1}} \\
+\beta \lambda_{t+1} \left[1 + \left(1 - \tau_{t+1}^D\right) R_{a,t+1}\right] &- \lambda_t | \Omega_t^\mu\} = 0
\end{aligned}
$$

The first two terms on the left of the equality capture the discounted value of an extra unit of currency in base in the next period. The last term captures the cost, which is the multiplier on the current period budget constraint.

We now consider C_t. It is useful to define $u_{c,t}$ as the derivative of the present discounted value of utility with respect to C_t :

$$
E\left\{u_{c,t} - u'(C_t - bC_{t-1}) + b\beta u'(C_{t+1} - bC_t) | \Omega_t^\mu\right\} = 0.
$$

The first order condition associated with C_t is:

$$
E_t \left\{ u_{c,t} - \upsilon_t C_t^{-\sigma_q} \left[\left(\frac{P_t}{M_t}\right)^{\theta_t} \left(\frac{P_t}{M_t^b - M_t + X_t}\right)^{1-\theta_t}\right]^{1-\sigma_q} - \left(1 + \tau_t^c\right) P_t \lambda_t \right\} = 0.
$$

The wage rate set by the household that has the option to reoptimize in period t is $\tilde{W}_t$. The household takes into account that if it cannot reoptimize in period $t+1$, its wage rate then is

$$W_{t+1} = \pi_t \mu_{z,t+1} \tilde{W}_t.$$

Note the slight difference in timing between inflation and the technology shock. The former reflects that indexing is lagged. The latter reflects that indexing to the technology shock is contemporaneous.

The demand curve that the individual household faces is:

$$h_{t+j} = \left(\frac{\tilde{W}_{t+j}}{W_{t+j}} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+j} = \left(\frac{\tilde{W}_t \mu_{z,t+1} \times \cdots \times \mu_{z,t+l}}{w_{t+j} z_{t+j} P_t} X_{t,j} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+j}, \tag{2.37}$$

where $\tilde{W}_t$ denotes the nominal wage set by households that reoptimize in period t, and W_t denotes the nominal wage rate associated with aggregate, homogeneous labor, l_t. Also,

$$X_{t,l} = \frac{\pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+l-1}}{\pi_{t+1} \times \cdots \times \pi_{t+l}} = \frac{\pi_t}{\pi_{t+l}}.$$

The homogeneous labor is related to household labor by:

$$l = \left[\int_0^1 (h_j)^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \; 1 \leq \lambda_w < \infty.$$

The contractor that produces homogeneous labor is competitive in the relevant output market, where labor is sold for the wage rate, W_t, and in the input market. Optimization leads to the following restrictions:

$$W_t = \left[(1-\xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\pi_{t-1} \mu_{z,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{2.38}$$

The j^{th} household that reoptimizes its wage, $\tilde{W}_t$, does so to optimize (neglecting irrelevant terms in the household objective):

$$E_t \sum_{l=0}^{\infty} (\beta \xi_w)^{l-t} \{ -\zeta_{t+l} z(h_{j,t+l}) + \lambda_{t+l} (1-\tau_{t+l}^l) W_{j,t+l} h_{j,t+l} \},$$

where

$$z(h) = \psi_L \frac{h_t^{1+\sigma_L}}{1+\sigma_L}$$

The presence of ξ_w by the discount factor reflects that in optimizing its wage rate, the household is only concerned with the future states of the world in which it cannot reoptimize.

Linearizing the household's first order condition associated with the wage decision, as well as (2.38), and combining the result produces the following equilibrium relationship between the aggregate wage rate, inflation, employment, the technology shock, the labor supply shock and the labor income tax:

$$E_t\left\{\eta_0\hat{w}_{t-1}+\eta_1\hat{w}_t+\eta_2\hat{w}_{t+1}+\eta_{\bar{3}}\hat{\pi}_{t-1}+\eta_3\hat{\pi}_t+\eta_4\hat{\pi}_{t+1}+\eta_5\hat{l}_t+\eta_6\left[\hat{\lambda}_{z,t}-\frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l\right]+\eta_7\hat{\zeta}_t\right\}=0$$

where

$$\eta=\begin{pmatrix} b_w\xi_w \\ -b_w\left(1+\beta\xi_w^2\right)+\sigma_L\lambda_w \\ \beta\xi_w b_w \\ b_w\xi_w \\ -\xi_w b_w\left(1+\beta\right) \\ b_w\beta\xi_w \\ -\sigma_L\left(1-\lambda_w\right) \\ 1-\lambda_w \\ -\left(1-\lambda_w\right) \end{pmatrix}=\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_{\bar{3}} \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \end{pmatrix}.$$

2.7. Monetary Policy

We consider a representation of monetary policy in which base growth feeds back on the shocks. The law of motion for the base is:

$$M_{t+1}^b=M_t^b(1+x_t),$$

where x_t is the net growth rate of the monetary base. (Above, we have also used the notation, X_t, where $x_t=X_t/M_t^b$.) Monetary policy is characterized by a feedback from $\hat{x}_t$ $(=(x_t-x)/x)$ to an innovation in monetary policy and to the innovation in all the other shocks in the economy. Let the $p-$ dimensional vector summarizing these innovations be denoted $\hat{\varphi}_t$, and suppose that the first element in $\hat{\varphi}_t$ is the innovation to monetary policy. Then, monetary policy has the following representation:

$$\hat{x}_t=\sum_{i=1}^{p}x_{it},$$

where x_{it} is the component of money growth reflecting the i^{th} element in $\hat{\varphi}_t$. Also,

$$x_{it}=\rho_i x_{i,t-1}+\theta_i^0\hat{\varphi}_{it}+\theta_i^1\hat{\varphi}_{i,t-1}, \tag{2.39}$$

for $i=1,...,p$, with $\theta_1^0\equiv 1$.

2.8. Final Goods Market Clearing

We now develop the aggregate resource constraint for this economy, relating the use of final goods to the quantity of aggregate labor and capital. Our derivation takes into account that it is not just the aggregate quantity of factor inputs that matters, but also its distribution across sectors, and proceeds in the style of Tak Yun ( ).

Define Y^* as the unweighted integral of output of the intermediate good producers:

$$Y^* = \int_0^1 Y(f)df = \int_0^1 F(\epsilon, z, K(f), l(f))df,$$

where, assuming production is positive for each f,

$$F(\epsilon, z, K(f), l(f)) = \epsilon z^{1-\alpha} K(f)^{\alpha} l(f)^{1-\alpha} - z\phi.$$

Here, by $l(f)$ we mean homogeneous labor hired by the f^{th} intermediate good firm, $f \in (0, 1)$. Recall that all firms confront the same wage rate and rental rate on capital. As a result, they all have the same capital-labor ratio, $K(f)/l(f)$. Moreover, this ratio coincides with the ratio of the aggregate inputs:

$$\frac{K^f}{l^f},\ K^f = \int_0^1 K(f)df,\ l^f = \int_0^1 l(f)df,$$

where K^f and l^f are aggregate capital and labor used in the goods producing sector, respectively. Then, it is easy to see that $Y^* = F(\epsilon, z, K^f, l^f)$.

Unweighted integration of the demand curve for $Y(f)$, (2.1), yields

$$Y^* = Y P^{\frac{\lambda_f}{\lambda_f - 1}} (P^*)^{\frac{\lambda_f}{1-\lambda_f}}$$

where

$$P^* = \left[\int_0^1 P(f)^{\frac{\lambda_f}{1-\lambda_f}} df\right]^{\frac{1-\lambda_f}{\lambda_f}}.$$

Then,

$$Y = (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z^{1-\alpha} \epsilon (\nu K)^{\alpha} (\nu l)^{1-\alpha} - z\phi\right],\ p^* = \frac{P^*}{P},$$

where

$$K^f = \nu K,\ l^f = \nu l.$$

Note that l is the integral of all employment of the labor 'produced' by the representative labor contractor. It is not necessarily the simple sum over all the labor supplied by households. Let the unweighted integral of the differentiated labor supplied by households be denoted by L :

$$L = \int_0^1 h^j dj.$$

Evaluating the unweighted integral of the demand curve for differentiated household labor, (2.37), we obtain:

$$L = l \left(\frac{W}{W^*} \right)^{\frac{\lambda_w}{\lambda_w - 1}},$$

where

$$W^* = \left[\int_0^1 W_j^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

We conclude that the total output of final goods, Y, is related to total factor inputs by the following relationship:

$$Y = (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z^{1-\alpha} \epsilon \left(\nu K \right)^{\alpha} \left(\nu \left(w^* \right)^{\frac{\lambda_w - 1}{\lambda_w}} L \right)^{1-\alpha} - z\phi \right], \; w^* = \frac{W^*}{W}.$$

Note the presence in the last expression of two efficiency wedges, p^* and w^*. Productive efficiency and our symmetry assumptions imply that, ideally, all forms of specialized labor would be employed at the same rate, and that each intermediate good producer would use an equal amount of resources. In this case, $p^* = w^* = 1$. However, the presence of wage and price frictions implies that one or both of these conditions may not be satisfied. In this case, p^* and/or w^* are less than unity. In this sense, the standard sticky price framework that we adopt here has the potential to provide the 'theory of TFP' called for by, among others, Chari, Kehoe and McGrattan ( ). Unfortunately, the evidence so far is that the sticky price mechanism is unlikely to provide a basis for a quantitatively successful theory of TFP. This can be seen in two ways. Tak Yun ( ) showed that because we adopt assumptions which have the implication that $p^* = w^* = 1$ in steady state, it follows that, to a first approximation, this is true near steady state too.[8] Of course, the sort of shocks experienced in the Great

[8] The assumptions which have this implication concern the prices and wages set by firms and households which do not have the opportunity to reoptimize. The crucial assumption in the case of firms is that their price is indexed to past inflation. In the case of wages it is crucial that the wage be indexed to past inflation *and* that it be indexed to aggregate productivity. Any deviation from these assumptions, and there will be dispersion in wages and/or prices across agents in steady state. This will have numerous effects on the steady state. First, the expressions for aggregate inflation and wages change in basic ways, by including additional variables. Second, the efficiency expressions, p^* and w^*, will deviate from unity and be quided by their own laws of motion over time. These are substantial qualitative changes. We suspect that they do not represent substantial quantitative changes, however.

Depression are hardly local deviations from a steady state. Still, for plausible parameter values deviations must be truly enormous to produce much of a fall in TFP. Consider the following simple example. Suppose final goods use intermediate inputs from just two types of sectors, Y^1 and Y^2, according to the following production function:

$$Y = \left[\frac{1}{2}\left(Y^1\right)^{\frac{1}{\lambda_f}} + \frac{1}{2}\left(Y^2\right)^{\frac{1}{\lambda_f}}\right]^{\lambda_f}$$

A large number for λ_f is 1.4. This implies a markup of 40 percent. Consider two scenarios. In each case, the same amount of resources are used. In one, $Y^1 = Y^2 = 1$. In this case, obviously, $Y = 1$. In the other there is an enormous deviation from equality of inputs: $Y^1 = 0.5$ and $Y^2 = 1.5$. Then,

$$\begin{aligned} &= \left[\frac{1}{2}(1.5)^{\frac{1}{1.4}} + \frac{1}{2}(0.5)^{\frac{1}{1.4}}\right]^{1.4} \\ &= 0.962, \end{aligned}$$

implying only a 4 percent reduction in efficiency.

A more substantial drop in efficiency could be had by setting λ_f to a higher number, say 4. Of course, the monopolistic competition assumption would not be so plausible in this case, because it implies a markup of 300 percent. But, we could assume that intermediate good producers cannot charge a price above marginal cost because they are surrounded by a competitive fringe. In this case, $Y = 0.90$. It is not clear whether even this 10 percent drop in efficiency is enough, given the enormous misallocation of resources in the example. In addition, note that the swing in relative prices associated with such a large deviation from efficiency when substitutability is so low is quite large. In particular, $P_1/P_2 = (Y_2/Y_1)^{[(\lambda_f-1)/\lambda_f]}$, which is 0.44 in this case.

In any case, from here on we set $p^* = w^* = 1$, since this is correct to a first order approximation. To complete our discussion, final goods are allocated to monitoring for banks, utilization costs of capital, last meals of entrepreneurs slated for death, government consumption, household consumption and investment. So, the goods market clearing condition is:

$$\begin{aligned} &\mu \int_0^{\bar{\omega}_t} \omega dF(\omega)\left(1+R^k\right) Q_{\bar{K}',t-1}\bar{K} + a(u)\bar{K} + \Theta(1-\gamma)v_t z_t + G_t + C_t + I_t \quad (2.40) \\ \leq &\left[z^{1-\alpha}\epsilon\left(\nu K\right)^{\alpha}\left(\nu L\right)^{1-\alpha} - z\phi\right], \end{aligned}$$

Here, government consumption is modeled as in Christiano and Eichenbaum (1992):

$$G = zg,$$

where g is an exogenous process.

3. Model Calibration

The model parameters are listed in Table 1, and various properties of the moded's steady state are reported in Tables 2-4. In many cases, the corresponding sample averages for both US data from the 1920s and for the post war period are also reported. The parameters in Table 1 are grouped according to the sector to which they apply. We begin by discussing how the parameter values were selected. After reporting the parameter values we work with, we provide some indication about the resulting properties of the model. To a first approximation, the magnitudes in the model match those in the data reasonably well. The relative size of the banking sector, ratios such as consumption to output and various velocity measures roughly line up with their corresponding empirical counterparts.

3.1. Model Parameter Values

In selecting these parameter values, we were guided by two principles. First, for the analysis to be credible, we require that the degree of monetary non-neutrality in the model be empirically plausible. Because we have some confidence in estimates of the effects of monetary policy shocks in post-war data, we insist that the model be consistent with that evidence.[9] Our second guiding principle is that we want the model to be consistent with various standard ratios: capital output ratio, consumption output ratio, equity debt ratio, various velocity statistics, and so on. In one respect, we found that these two principles conflict. In particular, we found that to obtain a large liquidity effect, we required that the fraction of currency in the monetary base is higher than what is observed in the data. Because we assigned a higher weight to the first principle (and lack some confidence in the accuracy of our monetary data), we chose to go with the high currency to base ratio.

Our strategy for assigning values to the parameters requires numerically solving the model for alternative candidate parameter values. This requires first computing the model's nonstochastic steady state and then computing the model's approximate linear dynamics in a neighborhood about the steady state.[10] We found that, conditional on a specific set of values for the model parameters, computing the steady state is difficult. The reason is

[9]The evidence on the effects of monetary policy shocks that we have in mind requires identification assumptions. These are that monetary policy shocks have no contemporaneous impact on aggregate measures of the price level or economic activity. This assumption holds as an approximation in our model. After a monetary policy shock, output and employment change a small amount because the frequency of bankruptcy is affected by the shock, and this affects the amount of goods used and produced in monitoring bankrupt entrepreneurs.

[10]Our intention is to eventually obtain higher order approximations to the model solution, using perturbation methods. However, we have so far taken the first step in this direction, by obtaining the linear approximation.

that this involves solving a system of equations which, as far as we can determine, has little recursive structure. A more convenient computational strategy was found by specifying some of the economically endogenous variables to be exogenous for purposes of the steady state calculations. In particular, we set the steady state ratio of currency to monetary base, m, the steady state rental rate of capital, r^k, the steady state share of capital and labor in goods production, ν, and the steady share of government consumption of goods, G/Y. These were set to $m = 0.95$, $r^k = 0.045$, $\nu = 0.01$, $G/Y = 0.07$, respectively. The latter two values can be defended on the basis of the data for the 1920s (see Table 2). Each of the former two are probably a little high. The currency to base ratio was already mentioned. The value of r^k, conditional on the share in goods production of capital (see α in Table 1) implies a slightly low value for the capital output ratio (see Table 2). We nevertheless chose this value for r^k because a lower one generated an excessively high value for the debt to equity ratio. To make these four variables exogenous for purposes of computing the steady state required making four model parameters endogenous. For this purpose, we chose ψ_L, x^b, ξ and g. Details on how the steady state was computed appear in Appendix A below.

Consider the household sector first. The parameters, β, λ_w, σ_L and b were simply taken from ACEL. The values of σ_q and H'' were chosen to allow the model to produce a persistent liquidity effect after a policy shock to the monetary base. Numerical experiments suggest that setting $H'' > 0$ is crucial for this. A possible explanation is based on the sort of reasoning emphasized in the literature on limited participation models of money: $H'' > 0$ ensures that after an increase in the monetary base, the banking sector remains relatively liquid for several periods. Regarding the goods-producing sector, all but one of the parameters were taken from ACEL. The exception, ψ_k, was set to 0.7 in order to have greater symmetry with ψ_l (in ACEL, $\psi_k = 0$).

The Calvo price stickiness parameters, ξ_w and ξ_p imply that the amount of time between reoptimization for wages and prices is 1 year and 1/2 years, respectively. As noted in ACEL, these values are consistent with survey evidence on price frictions.

Our selection of parameter values for the entrepreneurial sector were based on the calibration discussion in BGG. Following them, we assume that the idiosyncratic shock to entrepreneurs, ω, has a log-normal distribution. We impose on our calibration that the number of bankruptcies corresponds roughly to the number observed in the data. In our calibration, $F(\bar{\omega})$ is 0.02, or 2 percent quarterly.[11] To understand how we were able to specify $F(\bar{\omega})$ exogenously, recall that the log-normal distribution has two parameters - the mean and variance of $\log \omega$. We set the mean of $\log \omega$ to zero. We are left with one degree of freedom, the variance of $\log \omega$. Conditional on the other parameters of the model, this can be set to

[11]BGG assert that the annual bankruptcy rate is 3 percent. The number we work with, 2 percent quarterly, is higher. We encountered numerical difficulties using smaller bankruptcy rates. We intend to study smaller values of $F(\bar{\omega})$ in the future.

ensure the exogenously set value of $F(\bar{\omega})$. The value of this variance is reported in Table 1.[12] As noted above, the two parameters of the banking sector were an output of the steady state calculations.

3.2. Steady State Properties of the Model

The implications of the model for various averages can be compared with the corresponding empirical quantities in Tables 2 - 4. For almost all cases, we have the empirical quantities that apply to the US economy in the 1920s. As a convenient benchmark, we also report the corresponding figures for the post-war US data.

There are five things worth noting about Table 2. First, as noted above, the capital output ratio in the model is a little low. Corresponding to this, the investment to output ratio is low, and the consumption to output ratio is high. Second, note that $N/(\bar{K}-N)$ is slightly above unity in the model's steady state. This corresponds well with the data if we follow BGG in identifying N with equity and $N-\bar{K}$ with debt. Third, the relative size of the banking sector, which is quite small, conforms roughly with the size of the actual banking sector. Fourth, although we have not obtained data on the fraction of GDP used up in bankruptcy costs, we suspect that the relatively low number of 0.84 percent is not be far from the mark. Finally, note that inflation in the 1920s is very low, by comparison with inflation in the post-war period. We nevertheless imposed a relatively high inflation rate on the model in order to keep away from the zero lower bound on the interest rate. Later, we will revisit the wisdom of this choice.

Table 3 reports the consolidated asset and liability accounts for our banks. Several things are worth noting here. First, in the model most demand deposits are created in the process of extending working capital loans. These deposits are what we call 'firm demand deposits', and they 47 times larger than the quantity of demand deposits created when households deposits their financial assets with banks (i.e., 'household demand deposits'). It is hard to say whether this matches data or not. As is typical in a discrete-time framework, the model does not restrict exactly where the deposits sit during the period. For example, if firms pay their variable input costs early in the period, then what we call 'firm demand deposits' are actually in the hands of households most of the time. We do not have data on the relative holdings of deposits by households and firms for the 1920s, but we do have such data for the post-war period. These data indicate household and firm holdings of demand deposits are a similar order of magnitude. Again, it is hard to know what to make of this, relative to our model.

Second, the results in the table suggest that the amount of bank reserves in our model is

[12] The variance reported by BGG, 0.28, is higher than ours. We intend to explore the reasons for this discrepancy.

too small. The second row of the table displays the ratio of reserves to a very narrow definition of bank assets: reserves plus working capital loans. Since working capital loans account for essentially all of bank demand deposits, and these are the only reservable liabilities of our banks, the entry corresponding to required reserves is basically our assumed reserve requirement. Note that the corresponding figure in the data is an order of magnitude higher. This suggests to us that the mismatch between reserves in our model and the reserves in the data does not necessarily reflect that reserves are too little in our model. More likely, we have not identified all the reservable liabilities of banks in the data. [We are currently investigating this further.]

Table 4 reports various monetary and interest rate statistics. The left set of columns shows that the basic orders of magnitude are right: base velocity and $M1$ velocity in the model and the data match up reasonably well with the data. The ratio of currency to demand deposits is also reasonable. However, the fraction of currency in the monetary base is high, for reasons noted above. The interest rate implications of the model could be improved [discussion to be continued. This needs to include a more careful discussion of the relative magnitude of].

4. Dynamic Properties of the Model

This section has two purposes. First, for our analysis to give the Friedman-Schwartz hypothesis a fair shot, it is necessary for the degree of non-neutrality of money in the model to be plausible. To assess this, we evaluate the model's ability to

4.1. Quantitative Importance of the Monetary Transmission Mechanism in the Model

This section reviews the dynamic response to shocks implied by the model. This will help to understand the simulations in the next section.

4.1.1. A Monetary Policy Shock

Figure 4 compares the effects of a monetary policy shock with the corresponding estimates (and plus/minus two standard error bands) reported in ACEL.[13] The specification of mon-

[13]The basic identification assumption in the ACEL analysis is that a monetary policy shock has no contemporaneous impact on the level of prices or measures of aggregate economic activity. This assumption holds as an approximation in our model. As we will see, there is a very small contemporaneous impact of a monetary policy shock on aggregate employment and output.

etary policy underlying the model results reported in Figure 4 is (2.39) with $i = 1$:

$$x_{1t} = \rho_1 x_{1,t-1} + \hat{\varphi}_{1t} + \theta_1^1 \hat{\varphi}_{1,t-1},\ \sigma_{\hat{\varphi}},$$

where $\sigma_{\hat{\varphi}}$ is the standard deviation of the policy shock. We use the parameter estimates reported in ACEL: $\rho_1 = 0.27$, $\theta_1^1 = 0$, $\sigma_{\hat{\varphi}} = 0.11$. To understand the magnitude of $\sigma_{\hat{\varphi}}$, recall from (2.39) that an innovation to x_{1t} is an innovation to $\hat{x}_t$, the percent change in the net growth rate of the base. Since the percent change in the monetary base is related to $\hat{x}_t$ by $(x/(1+x))\,\hat{x}_t$, it follows that a 0.11 shock to monetary policy corresponds to an immediate 0.11 percent shock to the monetary base. Given the specified value of ρ_1, this shock creates further increases in subsequent periods, with the base eventually being up permanently by 0.15 percent.[14]

Another way to understand the nature of the monetary policy shock is as follows. In the impact period, the monetary policy shock takes the form of an increase in the money growth rate, x_t, from its steady state value of 0.010 (4.1 percent per year) to 0.011 (4.5 percent per year). The growth rate then declines and is very nearly back to steady state within four quarters. With one caveat, this is ACEL's estimate of the nature of a monetary policy shock in the postwar period. The caveat is that ACEL measure the monetary policy shock in terms of its impact on $M2$, not the monetary base. [further discussion will appear in a later draft]

Consider first the model results, shown in the form of the solid line in Figure 4. The impact of the shock on the growth rate of $M1$ and on the growth rate of the base are exhibited in the bottom left graph. Note how the growth rate of $M1$ hardly responds in the impact period of a monetary policy shock. This reflects that $M1$ is dominated by demand deposits created in the process of extending working capital loans to firms. The latter are largely predetermined in the period of a monetary policy shock.[15] In subsequent periods, as working capital loans expand, $M1$ starts to grow. The fact that the impact on the model's

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[14]To see this, use the fact

$$\frac{M_{t+1}}{M_t} = 1 + x_t,$$

so that the percent change in the growth rate of the base, $d\log(M_{t+1}/M_t)$, is:

$$d\log\frac{M_{t+1}}{M_t} \simeq \frac{dx_t}{1+x} = \frac{x\hat{x}_t}{1+x},$$

where the identity, $x\hat{x}_t = dx_t$ has been used. The 0.15 percent figure in the text reflects our assumption, $x = 0.10$, so that $x/(1+x) = 0.0099$. Then, the percent change in the base from a one standard deviation innovation in policy is $100 \times 0.0099 \times 0.10 \simeq 0.11$. The eventual impact on the level of the base, in percent terms, is obtained from the fact that this is $0.11/(1-\rho_1)$.

[15]Actually, there is a tiny fall in $M1$. This reflects that there is a similarly small fall in working capital loans. This in turn reflects a slight decline in the labor for two reasons. First, the abundance of excess reserves allows banks to substitute away from labor to some extent.

monetary base is similar to the initial response of $M2$, in the data holds by construction of the monetary policy shock. In the periods after the shock, all three money growth figures are close to each other in that each lies inside the gray area.

Note that, with some small exceptions, the responses of the model closely resemble the ones estimated in the data. In particular, the interest rate drops substantially in the period of the shock and stays low for over a year. Output displays a hump-shape with peak response of about 0.2 percent occuring after about a year. The same is true for investment, consumption and hours worked. Inflation displays a very slow response to the monetary policy shock, with peak response occuring around 7 quarters after the shock. Interestingly, inflation does not display the dip that occurs briefly in the data after a positive monetary policy shock. This contrasts with the results in ACEL, where the inflation rate of the model follows the estimated inflation process closely, including the dip. The reason this happens in the ACEL model is that in that model the interest rate that enters marginal costs of price-setting firms, is the one that appears in the top right figure, and which drops so significantly in the aftermath of a positive monetary policy shock. In contrast, the federal funds rate in our model does not directly enter marginal costs. Instead, it is the loan rate on working capital loans, R_t, which enters. As it happens (see below), the fall in this interest rate after a positive monetary policy shock is very small.

There are two places where the model misses. First, the empirical evidence in Figure 4 suggests that real wages rise after a monetary policy shock, while the impact in the model is only slight. Second, velocity in the data displays a substantial drop, while we do not see this in the model's $M1$ velocity. Base velocity performs somewhat better in the impact period.[16] This discrepancy between base velocity and $M1$ velocity in the model in the impact period of a shock reflects the observations made above, that the base responds immediately to a shock, while $M1$ responds hardly at all.

Overall, the results in Figure 4 is consistent with the notion that the degree of non-neutrality in the model is empirically plausible. The variables described above as well as other variables in the model are displayed in Figure 5. Rates of return in that figure are reported at an annual rate, in percentage point terms (not basis points). Quantities like investment, i, consumption, c, the physical stock of capital, $kbar$, the real wage rate, w, and output are presented in percent deviations from their unshocked, steady state growth path.

Several things are worth noting in this figure. First, all but one of the interest rates react the way the Federal Funds rate reacts. Each drops by about 50 basis points. The exception

Second, the reduction in bankruptcies that the money injection causes results in a lower demand for goods to cover bankruptcy costs. We stress that both these effects are very small and, to a first approximation, are zero.

[16] We define base velocity as $Y_t P_t / M_{t+1}^b$, i.e., relative to the end of period base. This corresponds to the measurement in the data, where stocks like money are generally measured in end-of-period terms.

is the rate on working capital loans, R, which falls by less than one basis point. Second, the monetary injection has an interesting set of implications for entrepreneurs. It drives up the price of capital, q, which creates an immediate capital gain for owners of capital. This can be seen in the large initial rise in the rate of return to capital, R^k. The unexpected jump in R^k is the reason for the three percent jump in entrepreneurial net worth, n. The increase in purchases of capital spurs the rise in investment. At the same time, in spite of the rise in net worth, bank lending to entrepreneurs drops (a little) relative to total bank assets. This is because the prospective capital losses on capital as q returns to its steady state makes the return on capital after the initial period low. This fall in the return to capital exceeds the fall in the time deposit interest rate, and by itself would produce a fall in lending.[17] Finally, note the small rise in TFP.

4.1.2. A Shock to Aggregate Technology

Another measure of the importance of monetary policy is that, according to results in ACEL, monetary policy plays an important role in shaping the response of the economy to a technology shock. This is true in this model as well. If there is a positive technology shock, and there is no monetary policy response to that shock, then the employment and capital utilization actually fall in the wake of the shock. Output rises eventually, but only slowly, in response to the shock. Figure 6 shows the response of the economy to a technology shock, when there is monetary accommodation of the kind estimated in ACEL. The innovation to technology underlying the results in the figure is 0.12 percent. For the most part, the model comes reasonably close to the data. The variables in Figure 6, together with additional variables are presented in Figure 7. Note that the technology shock drives the bankruptcy rate down and net worth up, but borrowing by entrepeneurs (when expressed as a fraction of total bank assets) falls a small amount. Presumably, this reflects the same factor that we saw in the monetary shock: the technology shock triggers a transient jump in the relative price of capital. The expectation that the price will eventually return to normal triggers an expectation of capital losses, which accounts for the fall in the rate of return on capital. The monetary response is quite strong, and so it produces a fall in interest rates.

4.1.3. A Shock to the Wealth of Entrepreneurs

An important part of our analysis is to understand the role in the Great Depression of the loss of net worth due to the stock market collapse after 1929. We capture this collapse in a reduced form way as a drop in γ_t. By eliminating a random subset of entrepreneurs and

[17]BGG show that, in this environment, loans as a fraction of entrepreneurial net worth are an increasing function of the ratio of the return on captial to the interest rate on time deposits.

replacing them with newcomers who start out with a small amount of net worth, the drop in γ_t has the effect of destroying net worth. Figure 8 displays the response of the economy to this shock. We imagine the economy begins in a steady state where $\gamma_t = 0.97$. The shock then occurs, unexpectedly driving γ_t to 0.96, after which it gradually expected to return to steady state according to a scalar first order autoregression with root 0.9.

According to the results in Figure 8, the real value of net worth drops by about 7 percent after about a year. The rate of bankruptcy rises 50 percent from about 0.8 percent per quarter to a little over 1.2 percent in two years, before declining. This is a very strong response. For example, between 1929 and 1932 the number of bankruptcies also rises by 50 percent, but the fall in the value of the stock market over this period was ten times greater than the 7 percent fall in net worth in our model.[18]

The price of capital drops by 2 percent on impact and then returns back to steady state. Although the initial drop in the price of capital produces a capital loss for entrepreneurs, the prospective gradual rise in the price of capital creates anticipated capital gains. This accounts for the fact that the rate of return on capital is above steady state for a while after the shock. The drop in net worth inhibits entrepreneurs' ability to finance the purchase of new capital, and this is manifested in the fall in investment, which falls by as much as 10 percent after nearly three years. This is a very large amount. For example, between 1929 and 1932, US investment falls by 70 percent. Simple extrapolation from our model implies that with a stock market crash of the size observed, the fall in US investment would have been predicted to be around 100 percent.

A notable feature of the results is that loans to entrepreneurs actually rise a little, when expressed as a fraction of total bank assets, after the net worth shock. In steady state, they are 80 percent of total bank assets, and after a year or so, they stand a little below 81 percent of bank assets. The relative strength in entrepreneur lending reflects in part the relatively high return on capital which, other things the same, leads to an expansion of lending to entrepreneurs. Of course, although lending to entrepreneurs expands, it does not expand enough to undo the negative impact on investment of reduced investor net worth. The stength in entrepreneur lending is particularly interesting because, according to Cole and Ohanian (1999), loans as a fraction of output did not begin to drop much until 1933.

Given the large fall in investment, it is not surprising that output and labor fall too. In the case of output, this fall reaches a trough of over 1.5 percent after two years. In the case of labor, the fall is about 1.1 after 2 years. In contrast, consumption rises somewhat, putting it up by about 0.5 percent after two years. This rise in consumption presumably reflects in part a relative improvement in the wealth of households, who see the value of their money holdings rise with the fall in inflation. In addition, the drop in net worth brings with it a drop in interest rates, and this presumably encourages households to intertemporally

[18]The bankruptcy numbers were obtained from the NBER's historical data base.

substitute consumption towards the present.

4.1.4. A Shock to Demand for Reserves by Banks

We now consider the effect of a positive shock to bank demand for excess reserves. We capture this by a negative shock to ξ_t, which raises the power on excess reserves in the bank production function. We suppose the economy starts in a steady state with $\xi_t = 0.996$. The shock drives ξ_t down to 0.986, after which it slowly rises back to the steady state according to a first order autoregressive process with autoregressive coefficient 0.90. The economic consequences are exhibited in Figure 9. The effects of this shock are roughly what one might expect. Excess and total bank reserves increase. The federal funds rate increases. The quantity of M_1 drops by nearly 1 percent. Net worth and the price of investment-specific capital both fall, while bankruptcies rise. All interest rates rise. The economy lapses into recession, with output and hours falling by over 1 percent. Inflation falls, after an initial rise which is no doubt due to the sharp increase in the interest rate on working capital loans (see R).

4.1.5. A Shock to Demand for Currency versus Deposits by Households

The rise in the currency to deposit ratio during the Great Depression is often emphasized in analyses of that period. In our model, the currency to deposit ratio rises with a fall in θ_t.[19] The effects of this are displayed in Figure 10. In the calculations reported there, θ_t is at its steady state value of 0.75 initially, whereupon it unexpectedly falls to 0.7425. After this, it slowly rises back to its steady state value at the rate of a scalar first order autoregression with parameter 0.9.

Note how the shock leads to a rise in the currency to output ratio. In addition, it leads to a fall in $M1$, and in output and employment. The interest rate on time deposits rises. Somewhat puzzling to us is the fact that the fedderal funds rate and the interest rate on

[19] A rise in θ_t shifts the demand for currency down in our model. To establish this, we totally differentiated the first order condition for M_t with respect to M_t and θ_t, and evaluated the result in steady state. We found (ignoring the adjustment costs on changing currency holdings):

$$\frac{\hat{m}_t}{\hat{\theta}_t} = \frac{-\left[(1-\sigma_q)\left(\log(m) - \log(1-m+x)\right) + \frac{1+x}{m-\theta(1+x)}\right]\theta}{(1-\sigma_q)\left(\theta - (1-\theta)\frac{m}{1-m+x}\right) + \frac{\frac{\theta}{m} + \frac{1-\theta}{(1-m+x)^2}m}{\frac{\theta}{m} - \frac{1-\theta}{1-m+x}}} < 0,$$

for $1-\sigma_q > 0$, $m/(1-m+x) > 1$, and $(\theta/m) - (1-\theta)/(1-m+x) > 0$, all conditions satisfied in the model.

deposits actually fall. Presumably, this reflects the reduction in money demand induced by the fall in output. We still working to understand this better.

5. Analysis of the Great Depression

Here, we will report a simulation of the Great Depression. We will do this by choosing an appropriate sequence of shocks for γ_t, ξ_t and θ_t to obtain a time path of the major variables, that corresponds to what we saw in the 1930s.[20] In our baseline scenario we will model Federal Reserve Policy as following a constant growth rate rule for the monetary base. For the alternative scenario, we will adjust the monetary base in such a way that the quantity of $M1$ is prevented from falling. Our interpretation of the Friedman and Schwartz hypothesis is that the alternative scenario would have averted the worst of the Great Depression.

6. Conclusion

7. Appendix A: Nonstochastic Steady State for the Model

We now develop equations for the steady state of our benchmark model. For purposes of these calculations, the exogenously set variables are:

$$\tau^l,\ \tau^c,\ \beta,\ F(\bar{\omega}),\ \mu,\ x,\ \mu_z,\ \lambda_f,\ \lambda_w,\ \alpha,\ \psi_k,\ \psi_l,\ \delta,\ \upsilon,$$
$$\tau^k,\ \gamma,\ \tau,\ \tau^T,\ \tau^D,\ \sigma_L,\ \zeta,\ \sigma_q,\ \theta,\ \upsilon,\ w^e,\ \nu^l,\ \nu^k,\ m,\ \eta_g,\ r^k$$

The variables to be solved for are

$$q,\ \pi,\ R^e,\ R_a,\ h_{e^r},\ R,\ R^k,\ \bar{\omega},\ k,\ n,\ i,\ w,\ l,\ c,\ u_c^z,\ m^b,\ \lambda_z,\ \psi_L,\ e_z^r,\ e_v,\ a^b x^b,\ \xi,\ h_{K^b},\ y,\ g$$

The equations available for solving for these unknowns are summarized below. The first three variables are trivial functions of the structural parameters, and from here on we treat them as known. There remain 22 unknowns. Below, we have 22 equations that can be used to solve for them.

The algorithm proceeds as follows. Solve for R_a using (7.17); h_{e^r} using (7.12).

We now compute R to enforce (7.8). This equation is a nonlinear function of R. For a given R, evaluate (7.8) as follows. Solve for R^k using (7.4); solve for $\bar{\omega}$ using (7.5); solve for

[20] As emphasized by CKM and others, one feature of the Great Depression is the very large drop in TFP. Although we do have an endogenous theory of TFP in our model, it is likely to not be quantitatively large enough. We expect to be correcting for this problem with our model by also incorporating an exogenous drop in productivity.

k and n using (7.6) and (7.7); solve for i using (7.3); solve for w using (7.1); solve for l using (7.2); solve for c using (7.20); solve (7.22) and (7.23) for g and y; solve for u_c^z using (7.18); solve for m^b and λ_z using (7.15) and (7.16); solve (7.19) for ψ_L; solve for e_z^r using (7.14); solve ξ from (7.13); solve e_v from (7.11); solve $a^b x^b$ from (7.10); h_{K^b} from (7.9). Vary R until (7.8) is satisfied. In these calculations, all variables must be positive, and:

$$0 \leq m \leq 1 + x,\ 0 \leq \xi \leq 1,\ \lambda_z > 0,\ k > n > 0.$$

7.1. Firm Sector

From the firm sector, and the assumption that there are no price distortions in a steady state, we have

$$s = \frac{1}{\lambda_f}.$$

Also, evaluating (2.3) in steady state:

$$\frac{1}{\lambda_f} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r^k \left[1 + \psi_k R\right]\right)^{\alpha} \left(w \left[1 + \psi_l R\right]\right)^{1-\alpha}, \tag{7.1}$$

Combining (2.3) and (2.4):

$$\frac{r^k \left[1 + \psi_k R\right]}{w \left[1 + \psi_l R\right]} = \frac{\alpha}{1-\alpha} \frac{\mu_z l}{\bar{k}} \tag{7.2}$$

7.2. Capital Producers

From the capital producers,

$$\lambda_{zt} q_t F_{1,t} - \lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}} \lambda_{z,t+1} q_{t+1} F_{2,t+1} = 0$$

or, since $F_{1,t} = 1$ and $F_{2,t} = 0$,

$$q = 1.$$

Also,

$$\bar{k}_{t+1} = (1-\delta) \frac{1}{\mu_{z,t}} \bar{k}_t + \left[1 - S\left(\frac{i_t \mu_{z,t}}{i_{t-1}}\right)\right] i_t,$$

so that in steady state, when $S = 0$,

$$\frac{i}{\bar{k}} = 1 - \frac{1-\delta}{\mu_z}. \tag{7.3}$$

7.3. Entrepreneurs

From the entrepreneurs:

$$r^k = a'.$$

Also,

$$u = 1.$$

The after tax rate of return on capital, in steady state, is:

$$R^k = \left[(1-\tau^k)r^k + (1-\delta)\right]\pi + \tau^k\delta - 1 \tag{7.4}$$

Conditional on a value for R^k, R^e, the steady state value for $\bar{\omega}$ may be found using the following equation:

$$[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R^e} + \frac{1}{1-\mu\bar{\omega}h(\bar{\omega})}\left[\frac{1+R^k}{1+R^e}(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))-1\right] = 0, \tag{7.5}$$

where the hazard rate, h, is defined as follows:

$$h(\omega) = \frac{F'(\omega)}{1-F(\omega)}.$$

This equation has two additional parameters, the two parameters of the lognormal distribution, F. These two parameters, however, are pinned down by the assumption, $E\omega = 1$, and the fact that we specify $F(\bar{\omega})$ exogenously. With these conditions, the above equation forms a basis for computing $\bar{\omega}$. Note here that when $\mu = 0$,(7.5) reduces to $R^k = R^e$. Then, combining (7.4) with the first order condition for time deposits, we end up with the conclusion that r^k is determined as it is in the neoclassical growth model.

Conditional on $F(\bar{\omega})$ and $\bar{\omega}$, we may solve for k using (2.20):

$$\frac{\bar{k}}{n} = \frac{1}{1-\frac{1+R^k}{1+R^e}(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))}. \tag{7.6}$$

The law of motion for net worth implies the following relation in steady state:

$$n = \frac{\frac{\gamma}{\pi\mu_z}\left[R^k - R^e - \mu G(\bar{\omega})\left(1+R^k\right)\right]\bar{k} + w^e}{1-\gamma\left(\frac{1+R^e}{\pi}\right)\frac{1}{\mu_z}}. \tag{7.7}$$

7.4. Banks

The first order condition associated with the bank's capital decision is:

$$\left(1+\psi_k R\right) r^k = \frac{R h_{K^b}}{1+\tau h_{e^r}}. \tag{7.8}$$

The first order condition for labor is redundant given (7.1), (7.2), and (7.8), and so we do not list it here. In the preceding equations,

$$h_{K^b} = \alpha \xi a^b x^b \left(e_v\right)^{1-\xi} \left(\frac{\mu_z l}{k}\right)^{1-\alpha}, \tag{7.9}$$

$$h_{e^r} = \left(1-\xi\right) a^b x^b \left(e_v\right)^{-\xi}, \tag{7.10}$$

and

$$e_v = \frac{\left(1-\tau\right) m^b \left(1-m+x\right) - \tau\left(\psi_l w l + \frac{1}{\mu_z}\psi_k r^k \bar{k}\right)}{\left(\frac{1}{\mu_z}(1-\nu^k)\bar{k}\right)^{\alpha}\left((1-\nu^l)l\right)^{1-\alpha}}. \tag{7.11}$$

Another efficiency condition for the banks is (2.31). Rewriting that expression, we obtain:

$$1+\frac{R}{R_a} = h_{e^r}\left[\left(1-\tau\right)\frac{R}{R_a} - \tau\right] \tag{7.12}$$

Substituting out for $a^b x^b \left(e_v\right)^{-\xi}$ from (7.10) into the scaled production function, we obtain:

$$\frac{h_{e^r}}{\left(1-\xi\right)} e_z^r = m^b\left(1-m+x\right) + \psi_l w l + \psi_k r^k \frac{\bar{k}}{\mu_z}, \tag{7.13}$$

where

$$e_z^r = (1-\tau) m^b \left(1-m+x\right) - \tau\left(\psi_l w l + \psi_k r^k \frac{\bar{k}}{\mu_z}\right). \tag{7.14}$$

7.5. Households

The first order condition for T :

$$1+\left(1-\tau^T\right) R^e = \frac{\mu_z \pi}{\beta}$$

The first order condition for M :

$$\upsilon\left[c\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}\left[\frac{\theta}{m} - \frac{1-\theta}{1-m+x}\right]\left(m^b\right)^{\sigma_q-2} - \lambda_z\left(1-\tau^D\right) R_a = 0 \tag{7.15}$$

The first order condition for M^b

$$\upsilon (1-\theta) \left[c \left(\frac{1}{m} \right)^{\theta} \left(\frac{1}{1-m+x} \right)^{1-\theta} \right]^{1-\sigma_q} \left(\frac{1}{m^b} \right)^{2-\sigma_q} \left(\frac{1}{1-m+x} \right)$$
$$= \pi \lambda_z \left\{ \frac{\mu_z}{\beta} - \frac{\left[1 + \left(1 - \tau_t^D \right) R_a \right]}{\pi} \right\}$$

Under the ACEL specification of preferences, c in the previous two expressions are replaced by unity. The first order condition for consumption corresponds to:

$$u_c^z - (1+\tau^c) \lambda_z = \upsilon c^{-1} \left(m^b \right)^{\sigma_q - 1} \left[c \left(\frac{1}{m} \right)^{\theta} \left(\frac{1}{1-m+x} \right)^{1-\theta} \right]^{1-\sigma_q}, \tag{7.16}$$

Under the ACEL specification, the expression to the right of the equality in (7.16) is replaced by zero.

Taking the ratio of (7.15) and the first order conditions for m^b, and rearranging, we obtain:

$$\begin{aligned} R_a &= \frac{\frac{(1-m+x)}{m}\theta - (1-\theta)\left(\frac{\pi \mu_z}{\beta} - 1 \right)}{\frac{(1-m+x)}{m}\theta \quad (1-\tau^D)} \\ &= \left[1 - \frac{m}{1-m+x} \frac{(1-\theta)}{\theta} \right] \frac{1-\tau^T}{1-\tau^D} R^e \end{aligned} \tag{7.17}$$

The marginal utility of consumption is:

$$c u_c^z = \frac{\mu_{z,}}{\mu_z - b} - b\beta \frac{1}{\mu_z - b} = \frac{\mu_z - b\beta}{\mu_z - b} \tag{7.18}$$

The first order condition for households setting wages is:

$$w \frac{\lambda_z (1-\tau^l)}{\lambda_w} = \zeta \psi_L l^{\sigma_L} \tag{7.19}$$

7.6. Monetary Authority

$$\pi = \frac{(1+x)}{\mu_z}.$$

7.7. Resource Constraint

After substituting out for the fixed cost in the resource constraint using the restriction that firm profits are zero in steady state, and using $g = \eta_g y$, we obtain:

$$c = (1 - \eta_g) \left[\frac{1}{\lambda_f} \left(\frac{1}{\mu_z} \nu^k \bar{k} \right)^{\alpha} \left(\nu^l l \right)^{1-\alpha} - \mu G(\bar{\omega})(1 + R^k) \frac{k}{\mu_z \pi} \right] - i. \tag{7.20}$$

Here, we have made use of the facts,

$$y = \frac{1}{\lambda_f} \left(\frac{1}{\mu_z} \nu^k \bar{k} \right)^{\alpha} \left(\nu^l l \right)^{1-\alpha} - \mu G(\bar{\omega})(1 + R^k) \frac{k}{\mu_z \pi},$$

and $g = \eta_g y$, so that $c = (1 - \eta_g) y - i$.

We now develop the condition on ϕ to assure that intermediate good firm profits in steady state are zero. If we loosely write their production function as $F - \phi z$, then the total cost of labor and capital inputs to the firm are sF, where s is real marginal cost, or the (reciprocal of the) markup (at least, in steady state when aggregate price and the individual intermediate good firm prices coincide). We want sF to exhaust total revenues, $F - \phi z$, i.e., we want $sF = F - \phi z$, or, $\phi = F(1 - s)/z = (F/z)(1 - 1/\lambda_f)$, or

$$\phi = \left(\frac{z_{t-1} \nu^k K_t}{z_{t-1} z_t} \right)^{\alpha} \left(\nu^l l \right)^{1-\alpha} (1 - \frac{1}{\lambda_f}) = \left(\frac{\nu^k k}{\mu_z} \right)^{\alpha} \left(\nu^l l \right)^{1-\alpha} (1 - \frac{1}{\lambda_f}) \tag{7.21}$$

We obtain (7.20) by substituting from the last equation into the resource constraint:

$$y = \left(\frac{1}{\mu_z} \nu^k \bar{k} \right)^{\alpha} \left(\nu^l l \right)^{1-\alpha} - \phi - \mu G(\bar{\omega})(1 + R^k) \frac{k}{\mu_z \pi}, \tag{7.22}$$

We obtain g from output from:

$$g = \eta_g y. \tag{7.23}$$

8. Appendix B: Linearly Approximating the Model Dynamics

There are 24 endogenous variables whose values are determined at time t. We load them into a vector, z_t. The elements in this vector are reported in the following table. In addition, there is an indication about which shocks the variable depends on. If it depends on the realization of all period t shocks (i.e., the information set, Ω_t^{μ}, then we indicate a, for 'all'. If it depends only on the realization of the current period non-financial shocks, Ω_t, then we indicate p, for 'partial'. The table also indicates the information associated with each of the

24 equations used to solve the model. These equations are collected below from the preceding discussion. Note that the number of equations and elements in z_t is the same. Note also, in each case, the third and fourth columns always have the same entry. In several cases, z_t contains variables dated $t+1$. In the case of $\hat{\bar{k}}_{t+1}$, for example, the presence of a p in the third column indicates that $\hat{\bar{k}}_{t+1}$ is a function of the realization of the period t non-financial shocks, and is not a function of the realization of period t financial shocks, or later period shocks. In the case of $\hat{R}^e_{t+1}$, the presence of an a indicates that this variable is a function of all period t shocks, but not of any period $t+1$ shocks.

	z_t	information, z	information, equation
1	$\hat{\pi}_t$	p	p
2	$\hat{s}_t$	a	a
3	$\hat{r}^k_t$	a	a
4	$\hat{\imath}_t$	p	p
5	$\hat{u}_t$	p	p
6	$\hat{\bar{\omega}}_t$	a	a
7	$\hat{R}^k_t$	a	a
8	$\hat{n}_{t+1}$	a	a
9	$\hat{q}_t$	a	a
10	$\hat{\nu}^l_t$	a	a
11	$\hat{e}_{\nu,t}$	a	a
12	$\hat{m}^b_t$	a	a
13	$\hat{R}_t$	a	a
14	$\hat{u}^z_{c,t}$	a	a
15	$\hat{\lambda}_{z,t}$	a	a
16	$\hat{m}_t$	a	a
17	$\hat{R}_{a,t}$	a	a
18	$\hat{c}_t$	p	p
19	$\hat{w}_t$	p	p
20	$\hat{l}_t$	a	a
21	$\hat{\bar{k}}_{t+1}$	p	p
22	$\hat{R}^e_{t+1}$	a	a
23	$\hat{x}_t$	a	a

(8.1)

The last of these variables is money growth, $\hat{x}_t$. As we show below, this is simply a trivial function of the underlying shocks. In addition, recall (**??**), in which the 10^{th} and 11^{th} variables are the same. A combination of the efficiency conditions for labor and capital in the firm sector, equations (1) and (2) below, are redundant with the efficiency conditions for labor and capital in the banking sector, (11) and (12). We deleted equation (11) below from our system.

In fact, we have 25 equations and unknowns in our model. The system we work with is one dimension less because we set $\Theta \equiv 0$, so that $\hat{v}_t$ disappears from the system. When we want $\Theta > 0$, we can get our 25^{th} equation by linearizing (2.21), and $\hat{v}_t$ is then our 25^{th} variable.

8.1. Firms

The inflation equation, when there is indexing to lagged inflation, is:

$$(1)\ E\left[\hat{\pi}_t - \frac{1}{1+\beta}\hat{\pi}_{t-1} - \frac{\beta}{1+\beta}\hat{\pi}_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta)\,\xi_p}\left(\hat{s}_t + \hat{\lambda}_{f,t}\right)|\Omega_t\right] = 0$$

The linearized expression for marginal cost is:

$$(2)\ \alpha\hat{r}_t^k + \frac{\alpha\psi_k R}{1+\psi_k R}\hat{\psi}_{k,t} + (1-\alpha)\,\hat{w}_t + \frac{(1-\alpha)\,\psi_l R}{1+\psi_l R}\hat{\psi}_{l,t}$$
$$+\left[\frac{\alpha\psi_k R}{1+\psi_k R} + \frac{(1-\alpha)\,\psi_l R}{1+\psi_l R}\right]\hat{R}_t - \hat{\epsilon}_t - \hat{s}_t = 0$$

Another condition that marginal cost must satisfy is that it is equal to the marginal cost of one unit of capital services, divided by the marginal product of one unit of services. After linearization, this implies:

$$(3)\ \hat{r}_t^k + \frac{\psi_k R\left(\hat{\psi}_{k,t} + \hat{R}_t\right)}{1+\psi_k R} - \hat{\epsilon}_t - (1-\alpha)\left(\hat{\mu}_{z,t} + \hat{l}_t - \left[\hat{\bar{k}}_t + \hat{u}_t\right]\right) - \hat{s}_t = 0$$

8.2. Capital Producers

The 'Tobin's q' relation is:

$$(4)\ E\left\{\hat{q}_t - S''\mu_z^2(1+\beta)\hat{\imath}_t - S''\mu_z^2\hat{\mu}_{z,t} + S''\mu_z^2\hat{\imath}_{t-1} + \beta S''\mu_z^2\hat{\imath}_{t+1} + \beta S''\mu_z^2\hat{\mu}_{z,t+1}|\Omega_t\right\} = 0$$

8.3. Entrepreneurs

The variable utilization equation is

$$(5)\ E\left[\hat{r}_t^k - \sigma_a\hat{u}_t|\Omega_t\right] = 0,$$

where $\hat{r}_t^k$ denotes the rental rate on capital. The date t standard debt contract has two parameters, the amount borrowed and $\widehat{\bar{\omega}}_{t+1}$. The former is not a function of the period $t+1$ state of nature, and the latter is not. Two equations characterize the efficient contract. The first order condition associated with the quantity loaned by banks in period t in the optimal contract is:

$$
\begin{aligned}
&(6)\ E\{\lambda\left(\frac{R^k\hat{R}_{t+1}^k}{1+R^k}-\frac{R^e\hat{R}_{t+1}^e}{1+R^e}\right)\\
-\left[1-\Gamma(\bar{\omega})\right]\frac{1+R^k}{1+R^e}&\left[\frac{\Gamma''(\bar{\omega})\bar{\omega}}{\Gamma'(\bar{\omega})}-\frac{\lambda\left[\Gamma''(\bar{\omega})-\mu G''(\bar{\omega})\right]\bar{\omega}}{\Gamma'(\bar{\omega})}\right]\widehat{\bar{\omega}}_{t+1}|\Omega_t^\mu\} \quad = \quad 0.
\end{aligned}
$$

Note that this is not a function of the period $t+1$ uncertainty. Also, note that when $\mu=0$, so that $\lambda=1$, then this equation simply reduces to $E\left[\hat{R}_{t+1}^k|\Omega_t^\mu\right]=\hat{R}_{t+1}^e$. The linearized zero profit condition is:

$$
\begin{aligned}
(7)\quad &\left(\tfrac{\bar{k}}{n}-1\right)\tfrac{R^k}{1+R^k}\hat{R}_t^k-\left(\tfrac{\bar{k}}{n}-1\right)\tfrac{R^e}{1+R^e}\hat{R}_t^e+\left(\tfrac{\bar{k}}{n}-1\right)\tfrac{(\Gamma'(\bar{\omega})-\mu G'(\bar{\omega}))}{(\Gamma(\bar{\omega})-\mu G(\bar{\omega}))}\bar{\omega}\widehat{\bar{\omega}}_t\\
&-\left(\hat{q}_{t-1}+\hat{\bar{k}}_t-\hat{n}_t\right)= \qquad 0.
\end{aligned}
$$

The law of motion for net worth is:

$$
(8)\ -\hat{n}_{t+1}+a_0\hat{R}_t^k+a_1\hat{R}_t^e+a_2\hat{\bar{k}}_t+a_3\hat{w}_t^e+a_4\hat{\gamma}_t+a_5\hat{\pi}_t+a_6\hat{\mu}_{z,t}+a_7\hat{q}_{t-1}+a_8\widehat{\bar{\omega}}_t+a_9\hat{n}_t=0
$$

The definition of the rate of return on capital is:

$$
(9)\ \hat{R}_{t+1}^k-\frac{(1-\tau^k)r^k+(1-\delta)q}{R^kq}\pi\left[\frac{\left(1-\tau^k\right)r^k\hat{r}_{t+1}^k-\tau^kr^k\hat{\tau}_t^k+(1-\delta)q\hat{q}_{t+1}}{(1-\tau^k)r^k+(1-\delta)q}+\hat{\pi}_{t+1}-\hat{q}_t\right]-\frac{\delta\tau^k\hat{\tau}_t^k}{R^k}
$$

8.4. Banking Sector

In the equations for the banking sector, it is capital services, k_t, which appears, not the physical stock of capital, $\bar{k}_t$. The link between them is:

$$
\hat{k}_t=\hat{\bar{k}}_t+\hat{u}_t.
$$

An expression for the ratio of excess reserves to value added in the banking sector is:

$$
\begin{aligned}
&(10)\ -\hat{e}_{v,t}+n_\tau\hat{\tau}_t+n_{m^b}\hat{m}_t^b+n_m\hat{m}_t+n_x\hat{x}_t+n_{\psi_l}\hat{\psi}_{l,t}\\
&+n_{\psi k}\hat{\psi}_{k,t}+(n_k-d_k)\left[\hat{\bar{k}}_t+\hat{u}_t\right]+n_{r^k}\hat{r}_t^k+n_w\hat{w}_t\\
&+(n_l-d_l)\,\hat{l}_t+(n_{\mu_z}-d_{\mu_z})\,\hat{\mu}_{z,t}-d_{\nu^k}\hat{\nu}_t^k-d_{\nu^l}\hat{\nu}_t^l=0
\end{aligned}
$$

where m_t^b is the scaled monetary base, m_t is the currency-to-base ratio, x_t is the growth rate of the base

$$
\begin{aligned}
n_\tau &= \frac{-\tau m^b (1-m+x) - \tau\left(\psi_l wl + \frac{1}{\mu_z}\psi_k r^k k\right) - \tau\frac{1}{\mu_z}\psi_k r^k k}{n}, \\
n &= (1-\tau) m^b (1-m+x) - \tau\left(\psi_l wl + \frac{1}{\mu_z}\psi_k r^k k\right), \\
n_{m^b} &= (1-\tau) m^b (1-m+x)/n \\
n_m &= -(1-\tau) m^b m/n \\
n_x &= (1-\tau) m^b x/n \\
n_{\psi_l} &= n_w = n_l = -\tau\psi_l wl/n \\
n_{\psi_k} &= n_{r^k} = n_k = -\tau\frac{1}{\mu_z}\psi_k r^k k/n \\
n_{\mu_z} &= \tau\frac{1}{\mu_z}\psi_k r^k k/n
\end{aligned}
$$

and

$$
\begin{aligned}
d &= \left(\frac{1}{\mu_z}(1-\nu^k)k\right)^\alpha \left((1-\nu^l)l\right)^{1-\alpha} \\
d_{\mu_z} &= \frac{-\alpha\left(\frac{1}{\mu_z}(1-\nu^k)k\right)^\alpha \left((1-\nu^l)l\right)^{1-\alpha}}{\left(\frac{1}{\mu_z}(1-\nu^k)k\right)^\alpha \left((1-\nu^l)l\right)^{1-\alpha}} = -\alpha \\
d_k &= \alpha \\
d_{\nu^k} &= -\alpha\frac{\nu^k}{1-\nu^k} \\
d_l &= 1-\alpha \\
d_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l}
\end{aligned}
$$

The first order condition for capital in the banking sector is:

$$
\begin{aligned}
0 &= k_R\hat{R}_t + k_\xi\hat{\xi}_t - \hat{r}_t^k + k_x\hat{x}_t^b + k_e\hat{e}_{v,t} + k_\mu\hat{\mu}_{z,t} \\
&\quad + k_{\nu^l}\hat{\nu}_t^l + k_{\nu^k}\hat{\nu}_t^k + k_l\hat{l}_t + k_k\left[\hat{\bar{k}}_t + \hat{u}_t\right] + k_\tau\hat{\tau}_t + k_{\psi_k}\hat{\psi}_{k,t}
\end{aligned}
$$

$$
k_R = \left[1 - \frac{\psi_k R}{1+\psi_k R}\right],\ k_\xi = 1 - \log(e_v)\,\xi + \frac{\tau h_{e^r}\left[\frac{1}{1-\xi} + \log(e_v)\right]\xi}{1+\tau h_{e^r}}
$$

$$
\begin{aligned}
k_x &= \frac{1}{1+\tau h_{e^r}},\ k_e = 1-\xi+\frac{\tau h_{e^r}\xi}{1+\tau h_{e^r}},\ k_\mu = (1-\alpha) \\
k_{\nu^l} &= -(1-\alpha)\frac{\nu^l}{1-\nu^l},\ k_{\nu^k} = (1-\alpha)\frac{\nu^k}{1-\nu^k},\ k_l = (1-\alpha),\ k_k = -(1-\alpha) \\
k_\tau &= -\frac{\tau h_{e^r}}{1+\tau h_{e^r}},\ k_{\psi_k} = -\frac{\psi_k R}{1+\psi_k R}.
\end{aligned}
$$

The latter equation was deleted from our system, because it is redundant given the two firm Euler equations and the following equation.

The first order condition for labor in the banking sector is:

$$
\begin{aligned}
(11)\ 0 &= l_R\hat{R}_t + l_\xi\hat{\xi}_t - \hat{w}_t + l_x\hat{x}_t^b + l_e\hat{e}_{v,t} + l_\mu\hat{\mu}_{z,t} \\
&\quad + l_{\nu^l}\hat{\nu}_t^l + l_{\nu^k}\hat{\nu}_t^k + l_l\hat{l}_t + l_k\left[\hat{\bar{k}}_t + \hat{u}_t\right] + l_\tau\hat{\tau}_t + l_{\psi_l}\hat{\psi}_{l,t},
\end{aligned}
$$

where

$$
\begin{aligned}
l_i &= k_i \text{ for all } i,\ \text{except} \\
l_R &= \left[1-\frac{\psi_l R}{1+\psi_l R}\right],\ l_{\psi l} = -\frac{\psi_l R}{1+\psi_l R} \\
l_\mu &= k_\mu - 1,\ l_{\nu^l} = k_{\nu^l} + \frac{\nu^l}{1-\nu^l},\ l_l = k_l - 1, \\
l_{\nu^k} &= k_{\nu^k} - \frac{\nu^k}{1-\nu^k},\ l_k = k_k + 1.
\end{aligned}
$$

The production function for deposits is:

$$
\begin{aligned}
(12)\qquad & \hat{x}_t^b - \xi\hat{e}_{v,t} - \log(e_{v,t})\,\xi\hat{\xi}_t - \frac{\tau(m_1+m_2)}{(1-\tau)m_1-\tau m_2}\hat{\tau}_t \\
= & \left[\frac{m_1}{m_1+m_2} - \frac{(1-\tau)m_1}{(1-\tau)m_1-\tau m_2}\right]\left[\hat{m}_t^b + \frac{-m\hat{m}_t + x\hat{x}_t}{1-m+x}\right] \\
& + \left[\frac{m_2}{m_1+m_2} + \frac{\tau m_2}{(1-\tau)m_1-\tau m_2}\right] \\
& \times[\frac{\psi_l wl}{\psi_l wl+\psi_k r^k k/\mu_z}\left(\hat{\psi}_{l,t}+\hat{w}_t+\hat{l}_t\right) + \frac{\psi_k r^k k/\mu_z}{\psi_l wl+\psi_k r^k k/\mu_z}\left(\hat{\psi}_{k,t}+\hat{r}_t^k+\hat{k}_t-\hat{\mu}_{zt}\right)].
\end{aligned}
$$

The expression for $\hat{R}_{at}$ is:

$$(13)\ \hat{R}_{at} - \left[\frac{h_{e^r} - \tau h_{e^r}}{(1-\tau)\, h_{e^r} - 1} - \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right]\left[-\left(\frac{1}{1-\xi} + \log\left(e_v\right)\right)\xi\hat{\xi}_t + \hat{x}_t^b - \xi\hat{e}_{v,t}\right]$$
$$+\left[\frac{\tau h_{e^r}}{(1-\tau)\, h_{e^r} - 1} + \frac{\tau h_{e^r}}{\tau h_{e^r} + 1}\right]\hat{\tau}_t - \hat{R}_t = 0$$

8.5. Household Sector

The definition of u_c^z is:

$$(14)\ E\{u_c^z \hat{u}_{c,t}^z - \left[\frac{\mu_z}{c\left(\mu_z - b\right)} - \frac{\mu_z^2 c}{c^2\left(\mu_z - b\right)^2}\right]\hat{\mu}_{z,t} - b\beta\frac{\mu_z c}{c^2\left(\mu_z - b\right)^2}\hat{\mu}_{z,t+1}$$
$$+\frac{\mu_z^2 + \beta b^2}{c^2\left(\mu_z - b\right)^2}c\hat{c}_t - \frac{b\beta\mu_z}{c^2\left(\mu_z - b\right)^2}c\hat{c}_{t+1} - \frac{b\mu_z}{c^2\left(\mu_z - b\right)^2}c\hat{c}_{t-1}|\Omega_t^\mu\} \quad = \quad 0.$$

The household's first order condition for time deposits is:

$$(15)\ E\left\{-\hat{\lambda}_{z,t} + \hat{\lambda}_{z,t+1} - \hat{\mu}_{z,t+1} - \hat{\pi}_{t+1} - \frac{R^e \tau^T}{1 + \left(1 - \tau^T\right) R^e}\hat{\tau}_{t+1}^T + \frac{R^e\left(1 - \tau^T\right)}{1 + \left(1 - \tau^T\right) R^e}\hat{R}_{t+1}^e|\Omega_t^\mu\right\} = 0.$$

The first order condition for currency, M_t :

$$(16)\ E\{\hat{\upsilon}_t + \left(1 - \sigma_q\right)\hat{c}_t + \left[-\left(1 - \sigma_q\right)\left(\theta - (1-\theta)\frac{m}{1-m+x}\right) - \frac{\frac{\theta}{m} + \frac{1-\theta}{(1-m+x)^2}m}{\frac{\theta}{m} - \frac{1-\theta}{1-m+x}}\right]\hat{m}_t$$
$$-\left[\frac{\left(1-\sigma_q\right)(1-\theta)\, x}{1-m+x} - \frac{\frac{1-\theta}{(1-m+x)^2}x}{\frac{\theta}{m} - \frac{1-\theta}{1-m+x}}\right]\hat{x}_t$$
$$+\left[-\left(1-\sigma_q\right)\left(\log\left(m\right) - \log\left(1-m+x\right)\right) + \frac{1+x}{\theta\left(1+x\right) - m}\right]\theta\hat{\theta}_t$$
$$-H''\frac{\mu_z \pi}{m^b m}\left[\hat{\pi}_t + \hat{\mu}_{z,t} + \hat{m}_t^b + \hat{m}_t - \hat{m}_{t-1}^b - \hat{m}_{t-1}\right] + \beta H''\frac{\mu_z \pi}{m^b m}\left[\hat{\pi}_{t+1} + \hat{\mu}_{z,t+1} + \hat{m}_{t+1}^b + \hat{m}_{t+1} - \hat{m}_t^b - \hat{m}_t\right]$$
$$-\left(2 - \sigma_q\right)\hat{m}_t^b - \left[\hat{\lambda}_{z,t} + \frac{-\tau^D}{1-\tau^D}\hat{\tau}_t^D + \hat{R}_{a,t}\right]|\Omega_t\} =$$

The household's first order condition for currency, M_{t+1}^b, is:

$$(17)\ E\{\frac{\beta}{\pi\mu_z}\upsilon\left(1-\theta\right)\left[c\left(\frac{1}{m}\right)^{\theta}\right]^{1-\sigma_q}\left(\frac{1}{1-m+x}\right)^{(1-\theta)(1-\sigma_q)+1}\left(\frac{1}{m^b}\right)^{2-\sigma_q}$$

$$
\begin{aligned}
&\times\{\hat{\upsilon}_{t+1}-\frac{\theta\hat{\theta}_{t+1}}{1-\theta}+(1-\sigma_q)\hat{c}_{t+1}-(1-\sigma_q)\log(m)\,\theta\hat{\theta}_{t+1}-\theta(1-\sigma_q)\hat{m}_{t+1}\\
&-[(1-\theta)(1-\sigma_q)+1]\left(\frac{1}{1-m+x}\right)[x\hat{x}_{t+1}-m\hat{m}_{t+1}]\\
&+(1-\sigma_q)\log(1-m+x)\,\theta\hat{\theta}_{t+1}-(2-\sigma_q)\,\hat{m}^b_{t+1}\}\\
&+\frac{\beta}{\pi\mu_z}\lambda_z\left[1+\left(1-\tau^D\right)R_a\right]\hat{\lambda}_{z,t+1}\\
&+\frac{\beta}{\pi\mu_z}\lambda_z\left[\left(1-\tau^D\right)R_a\hat{R}_{a,t+1}-\tau^D R_a\hat{\tau}^D_{t+1}\right]-\lambda_z\left[\hat{\lambda}_{zt}+\hat{\pi}_{t+1}+\hat{\mu}_{z,t+1}\right]|\Omega^\mu_t\}\\
&\qquad\qquad\qquad\qquad\qquad = 0.
\end{aligned}
$$

The first order condition for consumption is:

$$
\begin{aligned}
&(18)\ E\{u^z_c\hat{u}^z_{c,t}-\upsilon c^{-\sigma_q}\left[\frac{1}{m^b}\left(\frac{1}{m}\right)^{\theta}\left(\frac{1}{1-m+x}\right)^{1-\theta}\right]^{1-\sigma_q}\\
&\times[\hat{\upsilon}_t-\sigma_q\hat{c}_t+(1-\sigma_q)\left(-\hat{m}^b_t-\theta_t\hat{m}_t-(1-\theta_t)\left(\frac{-m}{1-m+x}\hat{m}_t+\frac{x}{1-m+x}\hat{x}_t\right)\right)\\
&+(1-\sigma_q)\left[\log\left(\frac{1}{m}\right)-\log\left(\frac{1}{1-m+x}\right)\right]\theta\hat{\theta}_t]\\
&-(1+\tau^c)\,\lambda_z\left[\frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t+\hat{\lambda}_{z,t}\right]|\Omega_t\}=0
\end{aligned}
$$

The reduced form wage equation is:

$$
(19)\ E\left\{\eta_0\hat{w}_{t-1}+\eta_1\hat{w}_t+\eta_2\hat{w}_{t+1}+\eta_{\bar{3}}\hat{\pi}_{t-1}+\eta_3\hat{\pi}_t+\eta_4\hat{\pi}_{t+1}+\eta_5\hat{l}_t+\eta_6\left[\hat{\lambda}_{z,t}-\frac{\tau^l}{1-\tau^l}\hat{\tau}^l_t\right]+\eta_7\hat{\zeta}_t|\Omega_t\right\}=0
$$

where

$$
\eta=\left(\begin{array}{c}
b_w\xi_w\\
-b_w\left(1+\beta\xi_w^2\right)+\sigma_L\lambda_w\\
\beta\xi_w b_w\\
b_w\xi_w\\
-\xi_w b_w\left(1+\beta\right)\\
b_w\beta\xi_w\\
-\sigma_L\left(1-\lambda_w\right)\\
1-\lambda_w\\
-\left(1-\lambda_w\right)
\end{array}\right)=\left(\begin{array}{c}
\eta_0\\ \eta_1\\ \eta_2\\ \eta_{\bar{3}}\\ \eta_3\\ \eta_4\\ \eta_5\\ \eta_6\\ \eta_7
\end{array}\right).
$$

8.6. Aggregate Restrictions

The resource constraint is:

$$(20)\ 0 = \quad d_y \left[\frac{G'(\bar{\omega})}{G(\bar{\omega})} \bar{\omega} \widehat{\bar{\omega}}_t + \frac{R^k}{1+R^k} \hat{R}^k_t + \hat{q}_{t-1} + \widehat{\bar{k}}_t - \hat{\mu}_{z,t} - \hat{\pi}_t \right] + u_y \hat{u}_t + g_y \hat{g}_t + c_y \hat{c}_t + \bar{k}_y \frac{i}{\bar{k}} \hat{\imath}_t$$
$$+\Theta(1-\gamma) v_y \hat{v}_t - \alpha \left(\hat{u}_t - \hat{\mu}_{z,t} + \widehat{\bar{k}}_t + \hat{\nu}^k_t \right) - (1-\alpha) \left(\hat{l}_t + \hat{\nu}^l_t \right) - \hat{\epsilon}_t$$

where

$$\bar{k}_y = \frac{\bar{k}}{y + \phi + d},$$

and the object in square brackets corresponds to the resources used up in monitoring.

$$(21)\ \widehat{\bar{k}}_{t+1} - \frac{1-\delta}{\mu_z} \left(\widehat{\bar{k}}_t - \hat{\mu}_{z,t} \right) - \frac{i}{\bar{k}} \hat{\imath}_t = 0.$$

Monetary policy is represented by:

$$(22)\ \hat{m}^b_{t-1} + \frac{x}{1+x} \hat{x}_{t-1} - \hat{\pi}_t - \hat{\mu}_{z,t} - \hat{m}^b_t = 0$$

8.7. Monetary Policy

Monetary policy has the following representation:

$$(23)\ \hat{x}_t = \sum_{i=1}^{p} x_{it},$$

where the x_{it}'s are functions of the underlying shocks.

8.8. Collecting the Equations

We can write the 24 equations listed above in matrix form as follows:

$$\mathcal{E}_t[\alpha_0 z_{t+1} + \alpha_1 z_t + \alpha_2 z_{t-1} + \beta_0 s_{t+1} + \beta_1 s_t] = 0,$$

where z_t is defined above and $\mathcal{E}_t$ is the expectation operator which takes into account the information set associated with each equation. Also, s_t is constructed from the vector of shocks, Ψ_t, that impact on agents' environment, and it has the following representation:

$$s_t = P s_{t-1} + \tilde{\varepsilon}_t. \tag{8.2}$$

We now discuss the construction of the elements, s_t and P, of this time series representation.

There are $N = 20$ basic exogenous shocks, ς_t, in the model:

$$\begin{array}{l} \hat{\lambda}_{f,t},\ \hat{\tau}_t,\ \hat{\psi}_{l,t},\ \hat{\psi}_{k,t},\ \hat{\xi}_t,\ \hat{x}_t^b,\ \hat{\tau}_t^T,\ \hat{\theta}_t,\ \hat{\tau}_t^D,\ \hat{\tau}_t^l, \\ \hat{\tau}_t^k,\ \hat{\zeta}_t,\ \hat{g}_t,\ \hat{\upsilon}_t,\ \hat{w}_t^e,\ \hat{\mu}_{z,t},\ \hat{\gamma}_t,\ \hat{\epsilon}_t,\ \hat{x}_{pt},\ \hat{\tau}_t^C \end{array} \tag{8.3}$$

Here, λ_f is the steady state markup for intermediate good firms; τ is the reserve requirement for banks; ψ_l is the fraction of the wage bill that must be financed in advance; ψ_k is the fraction of the capital services bill that must be financed in advance; x_t is the growth rate of the monetary base; ξ_t is a shock influencing bank demand for reserves; x_t^b is a technology shock to the bank production function; τ_t^T is the tax rate on household earnings of interest on time deposits; θ_t is a shock to the relative preference for currency versus deposits; τ_t^D is the tax rate on household earnings of interest on deposits; τ_t^l is the tax rate on wage income; τ_t^k is the tax rate paid by entrepreneurs on their earnings of rent on capital services; ζ_t is a preference shock for household leisure; g_t is a shock to government consumption; $\hat{\upsilon}_t$ is a shock to the household demand for transactions services; $\hat{w}_t^e$ is a shock to the transfers received by entrepreneurs; $\hat{\mu}_{z,t}$ is a shock to the growth rate of technology; $\hat{\gamma}_t$ is a shock to the rate of survival of entrepreneurs; $\hat{\epsilon}_t$ is a stationary technology shock to intermediate good production; $\hat{x}_{pt}$ is the monetary policy shock; $\hat{\tau}_t^C$ is the tax on consumption.

In each case, we give the shock an ARMA(1,1) representation. In addition, we suppose that monetary policy corresponds to the innovation in a shock according to an ARMA(1,1), as in (2.39). Consider, for example, the first shock $\hat{\lambda}_{f,t}$. The following vector first order autoregression captures in its first row, the ARMA(1,1) representation of $\hat{\lambda}_{f,t}$ and in the third row the ARMA(1,1) representation of the response of monetary policy to the shock:

$$\begin{pmatrix} \hat{\lambda}_{f,t} \\ \epsilon_{f,t} \\ x_{f,t} \end{pmatrix} = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & \phi_f^1 & \phi_f^2 \end{bmatrix} \begin{pmatrix} \hat{\lambda}_{f,t-1} \\ \epsilon_{f,t-1} \\ x_{f,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{f,t} \\ \epsilon_{f,t} \\ \phi_f^0 \epsilon_{f,t} \end{pmatrix}.$$

There are 6 parameters associated with $\hat{\lambda}_{f,t} : \rho_f,\ \eta_f,\ \phi_f^2,\ \phi_f^1,\ \phi_f^0$ and the standard deviation of $\epsilon_{f,t}$, σ_f. The parameters ϕ_f^0 and σ_f are only needed when the model is simulated, such as for computing impulse response functions or obtaining second moments. It is not required for computing the model solution. In this way, there are 6 parameters associated with each of the first 18 shocks, and the 20^{th}. Since logically there is no monetary policy response to a monetary policy shock, there are only three parameters for that shock. So, the total number of parameters associated with the exogenous shocks is $19 \times 6 + 3 = 117$.

We now discuss the construction of (8.2) in detail. Define the $3N \times 1$ vector Ψ_t as follows:

$$\Psi_t = \begin{pmatrix} \Psi_{1,t} \\ \vdots \\ \Psi_{N,t} \end{pmatrix}.$$

Here, $\Psi_{i,t}$ is 3×1 for $i = 1, ..., N$:

$$\begin{aligned}
\Psi_{1,t} &= \begin{pmatrix} \hat{\lambda}_{f,t} \\ \epsilon_{f,t} \\ x_{f,t} \end{pmatrix}, \ \Psi_{2,t} = \begin{pmatrix} \hat{\tau}_t \\ \epsilon_{\tau,t} \\ x_{\tau,t} \end{pmatrix}, \ \Psi_{3,t} = \begin{pmatrix} \hat{\psi}_{l,t} \\ \epsilon_{l,t} \\ x_{l,t} \end{pmatrix}, \ \Psi_{4,t} = \begin{pmatrix} \hat{\psi}_{k,t} \\ \epsilon_{k,t} \\ x_{k,t} \end{pmatrix} \\
\Psi_{5,t} &= \begin{pmatrix} \hat{\xi}_t \\ \epsilon_{\xi,t} \\ x_{\xi,t} \end{pmatrix}, \ \Psi_{6,t} = \begin{pmatrix} \hat{x}^b_t \\ \epsilon_{b,t} \\ x_{b,t} \end{pmatrix}, \ \Psi_{7,t} = \begin{pmatrix} \hat{\tau}^T_t \\ \epsilon_{T,t} \\ x_{T,t} \end{pmatrix}, \ \Psi_{8,t} = \begin{pmatrix} \hat{\theta}_t \\ \epsilon_{\theta,t} \\ x_{\theta,t} \end{pmatrix}, \\
\Psi_{9,t} &= \begin{pmatrix} \hat{\tau}^D_t \\ \epsilon_{D,t} \\ x_{D,t} \end{pmatrix}, \ \Psi_{10,t} = \begin{pmatrix} \hat{\tau}^l_t \\ \epsilon_{\tau^l,t} \\ x_{\tau^l,t} \end{pmatrix}, \ \Psi_{11,t} = \begin{pmatrix} \hat{\tau}^k_t \\ \epsilon_{\tau^k,t} \\ x_{\tau^k,t} \end{pmatrix}, \ \Psi_{12,t} = \begin{pmatrix} \hat{\zeta}_t \\ \epsilon_{\zeta,t} \\ x_{\zeta,t} \end{pmatrix}, \\
\Psi_{13,t} &= \begin{pmatrix} \hat{g}_t \\ \epsilon_{g,t} \\ x_{g,t} \end{pmatrix}, \ \Psi_{14,t} = \begin{pmatrix} \hat{\upsilon}_t \\ \epsilon_{\upsilon,t} \\ x_{\upsilon,t} \end{pmatrix}, \ \Psi_{15,t} = \begin{pmatrix} \hat{w}^e_t \\ \epsilon_{w^e,t} \\ x_{w^e,t} \end{pmatrix}, \ \Psi_{16,t} = \begin{pmatrix} \hat{\mu}_{z,t} \\ \epsilon_{\mu_z,t} \\ x_{\mu_z,t} \end{pmatrix}, \\
\Psi_{17,t} &= \begin{pmatrix} \hat{\gamma}_t \\ \epsilon_{\gamma,t} \\ x_{\gamma,t} \end{pmatrix}, \ \Psi_{18,t} = \begin{pmatrix} \hat{\epsilon}_t \\ \epsilon_{\epsilon,t} \\ x_{\epsilon,t} \end{pmatrix}, \ \Psi_{19,t} = \begin{pmatrix} \hat{x}_{pt} \\ \epsilon_{p,t} \\ \hat{\tau}^k_{t-1} \end{pmatrix}, \ \Psi_{20,t} = \begin{pmatrix} \hat{\tau}^C_t \\ \epsilon_{\tau^C,t} \\ \hat{x}_{\tau^C,t} \end{pmatrix}
\end{aligned}$$

The non-financial market shocks are

$$\hat{\lambda}_{f,t}, \ \hat{\tau}_t, \ \hat{x}^b_t, \ \hat{\tau}^T_t, \ \hat{\tau}^D_t, \ \hat{\tau}^l_t, \ \hat{\tau}^k_t, \ \hat{\zeta}_t, \ \hat{g}_t, \ \hat{w}^e_t, \ \hat{\mu}_{z,t}, \ \hat{\epsilon}_t, \ \hat{\tau}^C_t$$

The financial market shocks are:

$$\hat{\psi}_{l,t}(7-9), \ \hat{\psi}_{k,t}(10-12), \ \hat{\xi}_t(13-15), \ \hat{\theta}_t(22-24), \ \hat{\upsilon}_t(40-42), \ \hat{\gamma}_t(49-51), \ \hat{x}_{pt}(55-56)$$

Numbers in parentheses correspond to the associated entries in Ψ_t.

The time series representation of Ψ_t is:

$$\Psi_t = \rho \Psi_{t-1} + \varepsilon^{\Psi}_t,$$

where ρ is a $3N \times 3N$ matrix. With one exception, it is block diagonal in a way that is conformable with the partitioning of Ψ_t. Each block is 3×3. Thus, with one exception, ρ

has the following structure:

$$\rho = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \rho_N \end{bmatrix},$$

with the partitioning being conformable with the partitioning of Ψ_t. The exception is the 31^{st} entry in the 57^{th} row of ρ, which is unity. For example,

$$\rho_1 = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & \phi_f^2 & \phi_f^1 \end{bmatrix}, \; \rho_{19} = \begin{bmatrix} \rho_f & \eta_f & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In general, ρ_i is 3×3 for $i = 1, ..., 18$, with zeros in the middle row and in the 1,3 and 3,1 elements. Similarly, we partition

$$\varepsilon_t^{\Psi} = \begin{bmatrix} \varepsilon_{1t} \\ \vdots \\ \varepsilon_{20t} \end{bmatrix},$$

where ε_{it} is 3×1 for $i = 1, ..., 20$, and the last element of $\varepsilon_{19,t}$ is zero. The first two entries of ε_{it} are equal and represent the innovation in the associated exogenous shock variable. The last entry is proportional to the second, where the factor of proportionality characterizes the contemporaneous response of monetary policy to the shock.

We now discuss the relation between s_t and Ψ_t. In the 'standard case' we assume that the information set in each equation is Ω_t^{μ}. In this case,

$$s_t = \theta_t, \;\; P = \rho, \;\; \tilde{\varepsilon}_t = \varepsilon_t^{\Psi}.$$

If any one of the information sets in any one of the equations contains less information than Ω_t^{μ}, then s_t is constructed slightly differently:

$$s_t = \begin{pmatrix} \Psi_t \\ \Psi_{t-1} \end{pmatrix}, \; P = \begin{bmatrix} \rho & 0 \\ I & 0 \end{bmatrix}, \; \tilde{\varepsilon}_t = \begin{pmatrix} \varepsilon_t^{\Psi} \\ 0 \end{pmatrix}. \tag{8.4}$$

The matrices, β_0 and β_1 provided to the computational algorithm are the ones that are suitable for the standard case. If the algorithm detects that some information sets are small, then it makes appropriate adjustments to the β's.

Monetary policy is a function of Ψ_t, according to equation (24) and (2.39):

$$\hat{x}_t = \left[\sum_{i=1, \text{ i}\neq 19}^{20} (0,0,1)\Psi_{it} \right] + (1,0,0)\Psi_{19,t}.$$

A solution to the model is a set of matrices, A and B, in:

$$z_t = Az_{t-1} + Bs_t,$$

where B is restricted to be consistent with our information set assumptions. The computation of the A and B matrices is discussed in Christiano ( )[21].

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Table 1: Model Parameters (Time unit of Model: quarterly)		
Panel A: Household Sector		
β	Discount rate	$1.03^{-0.25}$
ψ_L	Weight on Disutility of Labor	153.76
σ_L	Curvature on Disutility of Labor	1.00
υ	Weight on Utility of Money	2e-008
σ_q	Curvature on Utility of money	-10.00
θ	Power on Currency in Utility of money	0.75
H''	Curvature on Currency Adjustment Cost	500.00
b	Habit persistence parameter	0.63
ξ_w	Fraction of households that cannot reoptimize wage within a quarter	0.70
λ_w	Steady state markup, suppliers of labor	1.05
Panel B: Goods Producing Sector		
μ_z	Growth Rate of Technology (APR)	1.50
S''	Curvature on Investment Adjustment Cost	7.69
σ_a	Curvature on capital utilization cost function	0.01
ξ_p	Fraction of intermediate good firms that cannot reoptimize price within a quarter	0.50
ψ_k	Fraction of capital rental costs that must be financed	0.70
ψ_l	Fraction of wage bill that must be financed	1.00
δ	Depreciation rate on capital.	0.02
α	Share of income going to labor	0.36
λ_f	Steady state markup, intermediate good firms	1.20
Panel C: Entrepreneurs		
γ	Percent of Entrepreneurs Who Survive From One Quarter to the Next	97.00
μ	Fraction of Realized Profits Lost in Bankruptcy	0.120
$F(\bar{\omega})$	Percent of Businesses that go into Bankruptcy in a Quarter	0.80
$Var(log(\omega))$	Variance of (Normally distributed) log of idiosyncratic productivity parameter	0.08
Panel D: Banking Sector		
ξ	Power on Excess Reserves in Deposit Services Technology	0.9960
x^b	Constant In Front of Deposit Services Technology	82.4696
Panel E: Policy		
τ	Bank Reserve Requirement	0.100
τ^c	Tax Rate on Consumption	0.00
τ^k	Tax Rate on Capital Income	0.29
τ^l	Tax Rate on Labor Income	0.04
x	Growth Rate of Monetary Base (APR)	4.060

Table 2: Steady State Properties of the Model, Versus US Data			
Variable	Model	US, 1921-29	US, 1964-2001
$\frac{k}{y}$	8.35	10.8[1]	9.79
$\frac{i}{y}$	0.20	0.24	0.25
$\frac{c}{y}$	0.73	0.67	0.57
$\frac{g}{y}$	0.07	0.07	0.19
r^k	0.043		
$\frac{N}{K-N}$ ('Equity to Debt')	1.029	1-1.25[2]	1-1.25[2]
$\frac{W^e}{py}$	0.057		
Percent of Goods Output Lost to Bankruptcy	0.365%		
Percent of Aggregate Labor and Capital in Banking	1.00%	1%[3]	2.5%[5]
Inflation (APR)	2.52%	-0.6%[4]	4.27%[6]

Note: [1]End of 1929 stock of capital, divided by 1929 GNP, obtained from CKM. [2]Masoulis (1988) reports that the debt to equity ratio for US corporations averaged 0.5 - 0.75 in the period 1937-1984. [3]Share of value-added in the banking sector, according to Kuznets (1941), 1919-1938. [4]Average annual inflation, measured using the GNP deflator, over the period 1922-1929. [5]Based on analysis of data on the finance, insurance and real estate sectors [6] Average annual inflation measured using GNP deflator.

Table 3: Consolidated Banking Sector Balance Sheet, Model versus US Data							
Variable	Model	1921-1929	1995-2001	Variable	Model	1921-1929	1995-2001
Assets (Fraction of Annual GNP)	1.269	0.722	0.604	Liabilities (Fraction of Annual GNP)	1.269		0.604
Total Reserves	0.103	0.152	0.081	Total Demand Deposits	1.000	1.0	1.0
◦ Required Reserves	0.100	0.118	0.052	◦ Firm Demand Deposits	0.897		0.523
◦ Excess Reserves	0.003	0.034	0.029	◦ Household Demand Deposits	0.103		0.477
Working Capital Loans	0.897	0.848	0.919				
◦ Capital Rental Expenses	0.254						
◦ Wage Bill Expenses	0.643						
Entrepreneurial Loans	0.803	0.525	0.828	Time Deposits	0.803	0.525	0.828

Notes on Table 3: Total assets consists of reserves plus working capital loans plus loans to entrepreneurs. The first line shows the ratio of these to annual goods output. With the exception of the bottom row of numbers, remaining entries in the table are expressed as a fraction of bank reserves plus working capital loans. The bottom row of numbers is expressed as a fraction of total assets.

Data for the period 1995-2001: we define working-capital loans as total demand deposits minus total reserves. This number is the same order of magnitude as the sum of short-term bank loans with maturity 24 months or less (taken from the Fed's 'Banking and Monetary Statistics') and commerical paper (Table L101 in Flow of Funds) Long-term entrepreneurial loans are defined as the total liabilities of the non-financial business sector (non-farm non-financial corporate business plus non-farm non-corporate business plus farm business) net of municipal securities, trade payables, taxes payables, 'miscellaneous liabilites' and the working capital loans. Source: With exception of required and excess reserves, the source is the Federal Reserves' Flow of Funds' data. Required and excess reserves are obtained from Federal Reserve Bank of St. Louis.

Data for the period 1921-1929: we define working-capital loans as total demand deposits minus total reserves for all banks. Entrepreneurial loans are constructed on the basis of all bank loans minus working capital loans plus outstanding bonds issued by all industries. Source: Banking and Monetary Statistics, Board of Governorns, September 1943, and NBER Historical Database.

Table 4: Money and Interest Rates, Model versus US Data							
Money	Model	1921-1929	1964-2002	Interest Rates (APR)	Model	1921-1929	1964-2002
Monetary Base Velocity	10.29	12	16.6	Demand Deposits	0.44		3.21
M1 Velocity	4.01	3.5	6.5	Time Deposits	7.18		6.96
				Rate of Return on Capital	9.47		17.33
Currency / Demand Deposits	0.29	0.2	0.3	Entrepeneurial Standard Debt Contract	7.85	5.74	8.95
Currency / Monetary Base	0.75	0.55	0.73	Interest Rate on Working Capital Loans	4.66	4.72	7.10
Curr. / Household D. Deposit	2.81			Federal Funds Rate	5.12	3.90	6.86

Notes to Table 4:

Data for 1921-1929: (1) 'Federal Funds Rate' is the average of Bankers' Acceptances Rate. (2) Interest rate on working capital loans is the commerical paper rate. (3) Rate on loans to entrepreneurs is the average between AAA and BAA corporate bonds. (4) Rate on time deposits is available only from 1933 onwards. Reported data in Board of Governors (1943) only cite the administrative rate (maximum rate) set by the Fed. The average of this rate was 2.7% over the period 1933-41. (5) There are no data available on the rate paid on demand deposits (to our knowledge).

Data for 1964-2002:

(1) The Federal Funds Rate is over the period 1964.3-2002.3. Source: Federal Reserve Board of Governors. (2) The rate on demand deposits is the 'Money Zero Maturity Own Rate' (1964.3-2002.3). Source: Federal Reserve Bank of Saint Louis. (3) The rate on loans to entrepreneurs is the average between AAA and BAA corporate bonds (1964.3-2002.3). Source: Federal Reserve Board of Governors. (4) The rate on time deposit is the rate on 3-month CDs (1964.3-2002.3). Source: Federal Reserve Board of Governors. (5) The rate of return on capital is the rate of profit on stockholders' equity for the manufacturing sector (1980.1-2001.4). Source: Bureau of the Census (2002), Table I. (6) The rate on Working Capital Loans is the rate on Commercial paper (dealer-placed unsecured short-term negotiable promissory note issued by companies with Aa bond ratings and sold to investors). Average over 1971.2-2002.3. Source: Federal Reserve Board of Governors. (7) The Currency to M1 ratio is an average over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis. (8) The Currency to Monetary Base ratio is the average over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis. (9) The Monetary Base and M1 velocities are averages over 1964.3-2002.3 (currency includes dollars held abroad). Source: Federal Reserve Bank of Saint Louis.

Figure 1: Timing in the Model

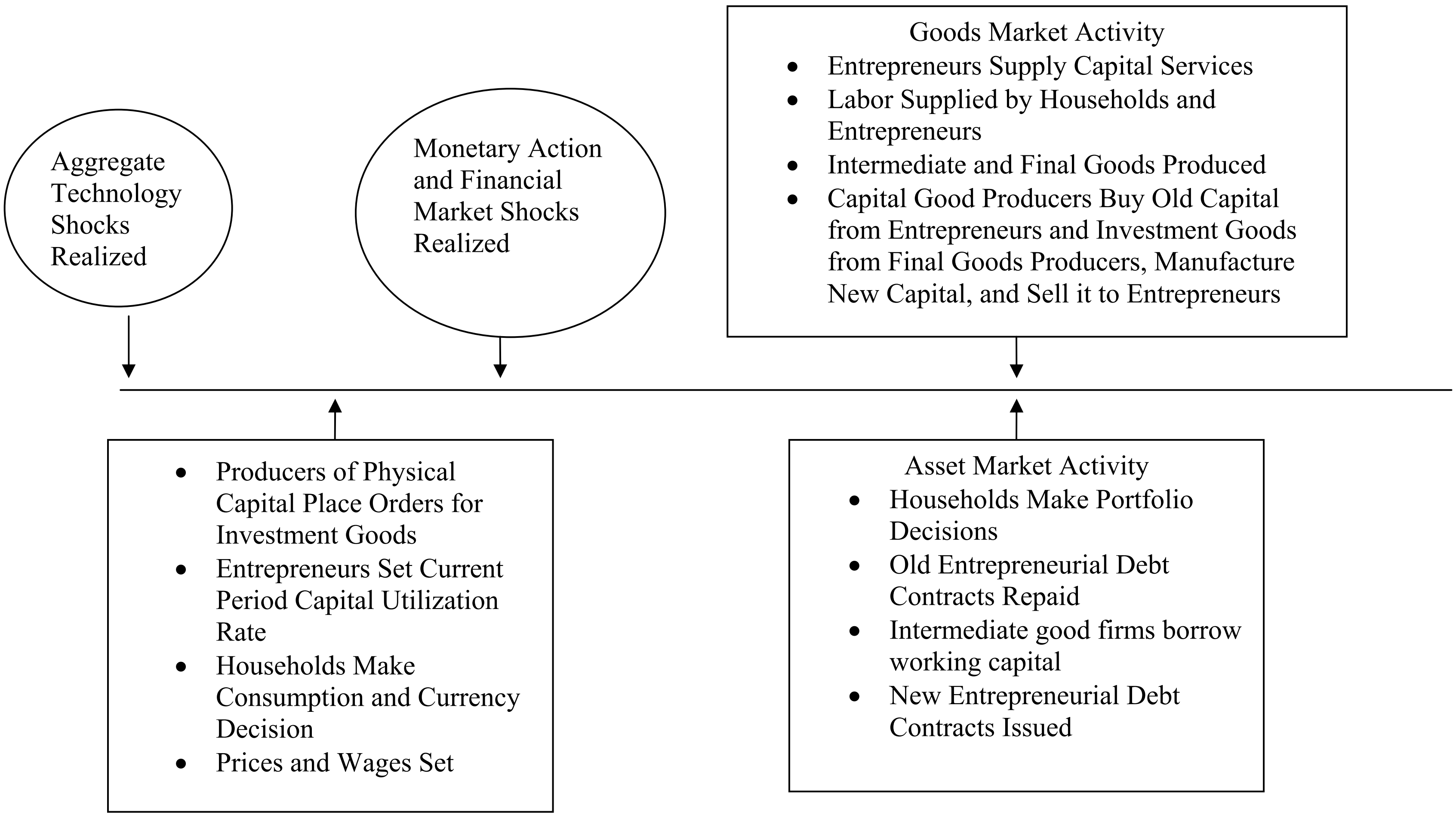
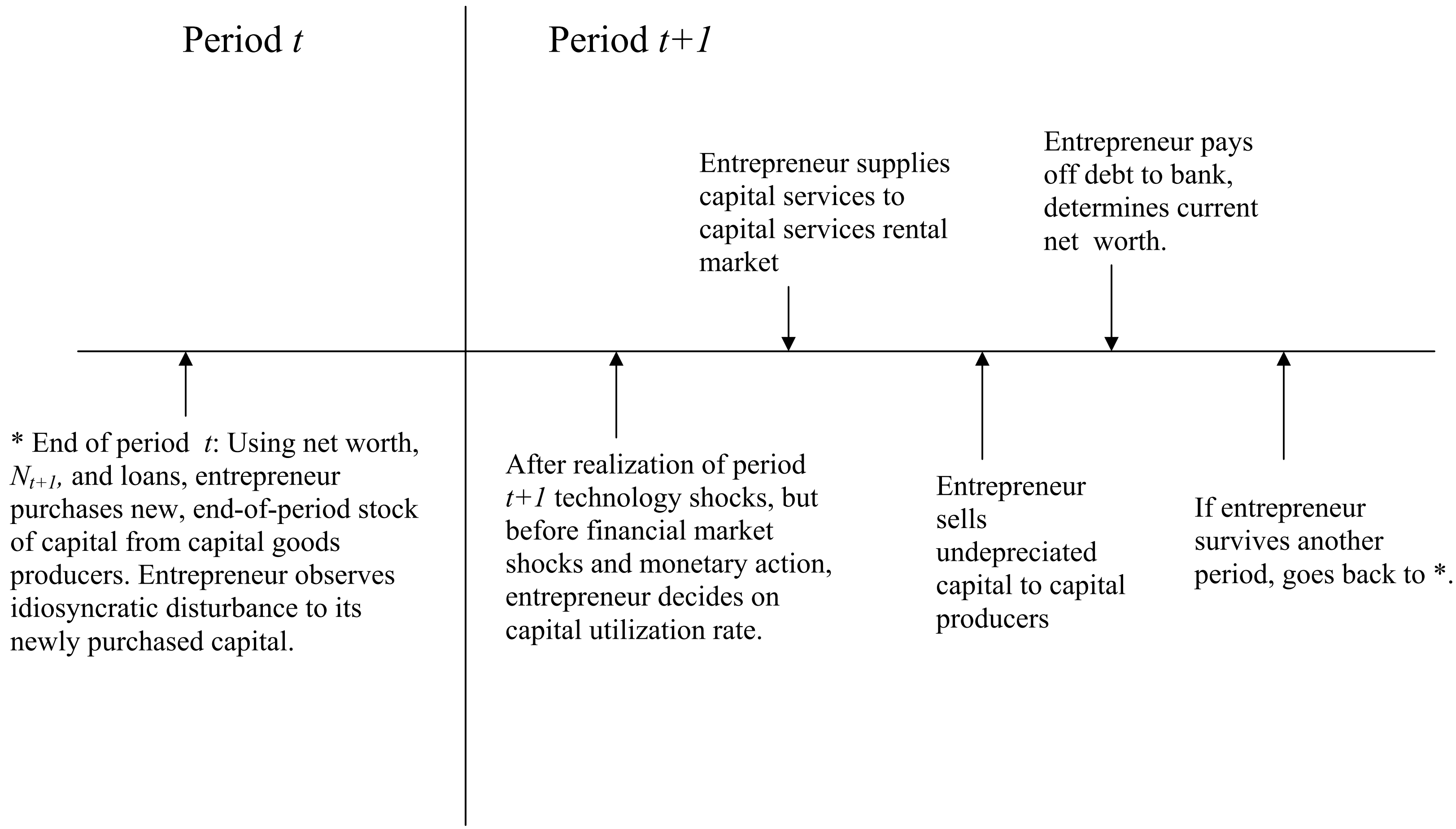


FIGURE 2: A Day in the Life of an Entrepreneur



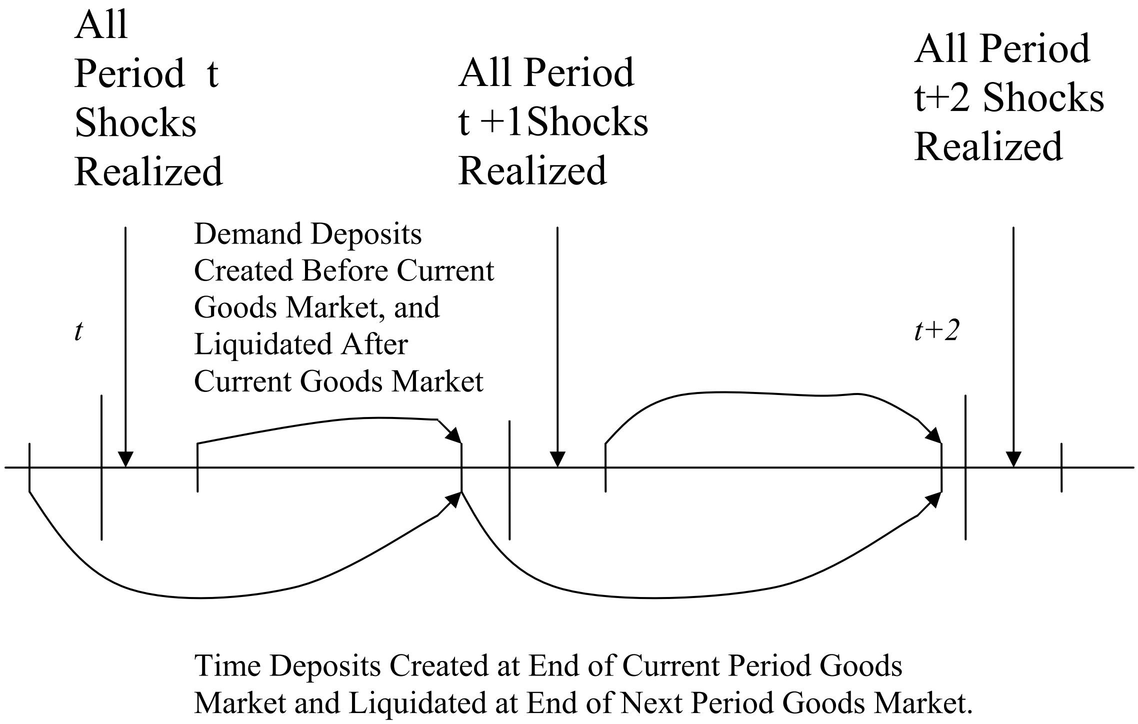


Figure 3: Maturity Structure of Time and Demand Deposits

Figure 4: Response, Policy Shock to Base (VAR: +, Model: Solid)

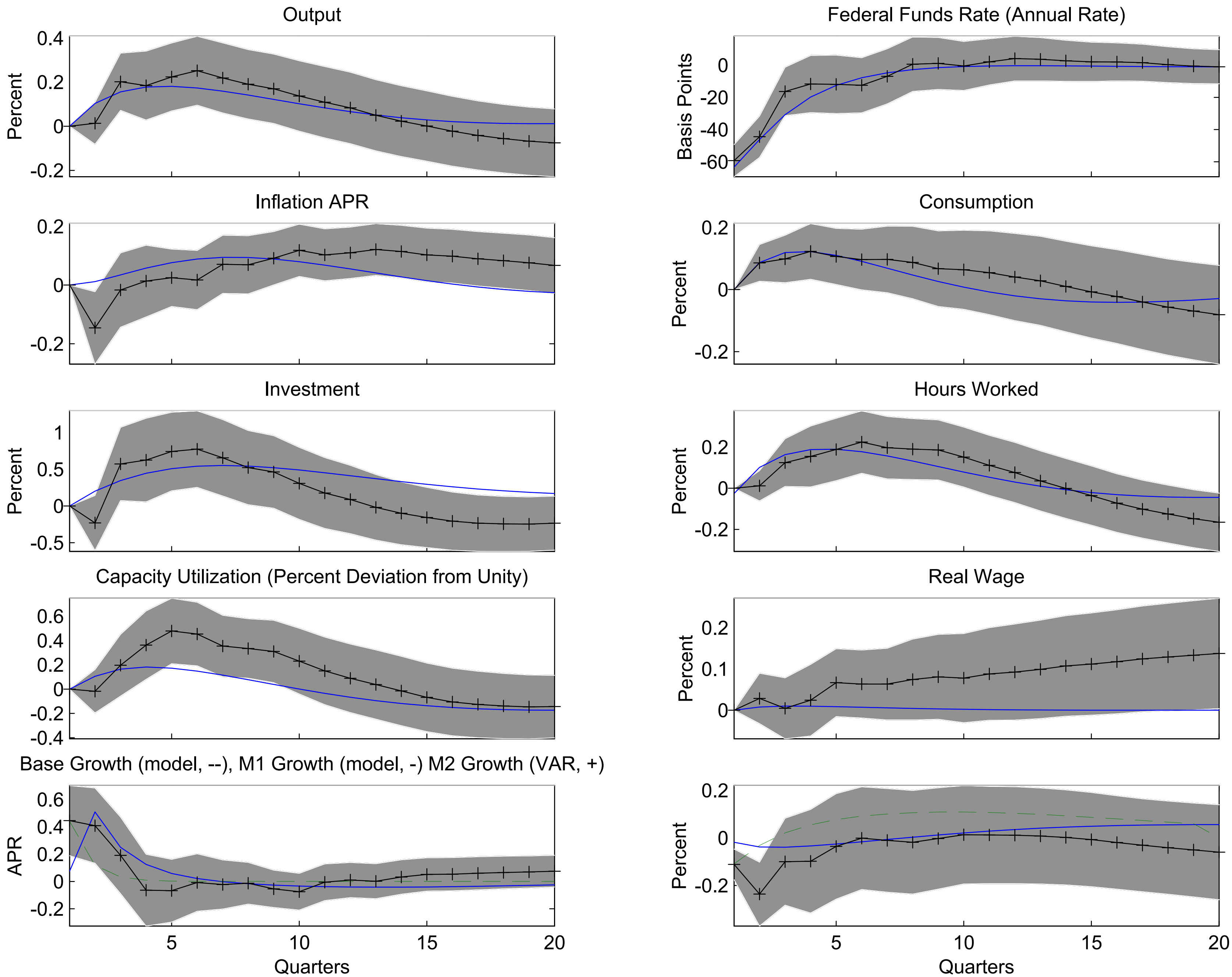


Figure 5: Monetary policy shock

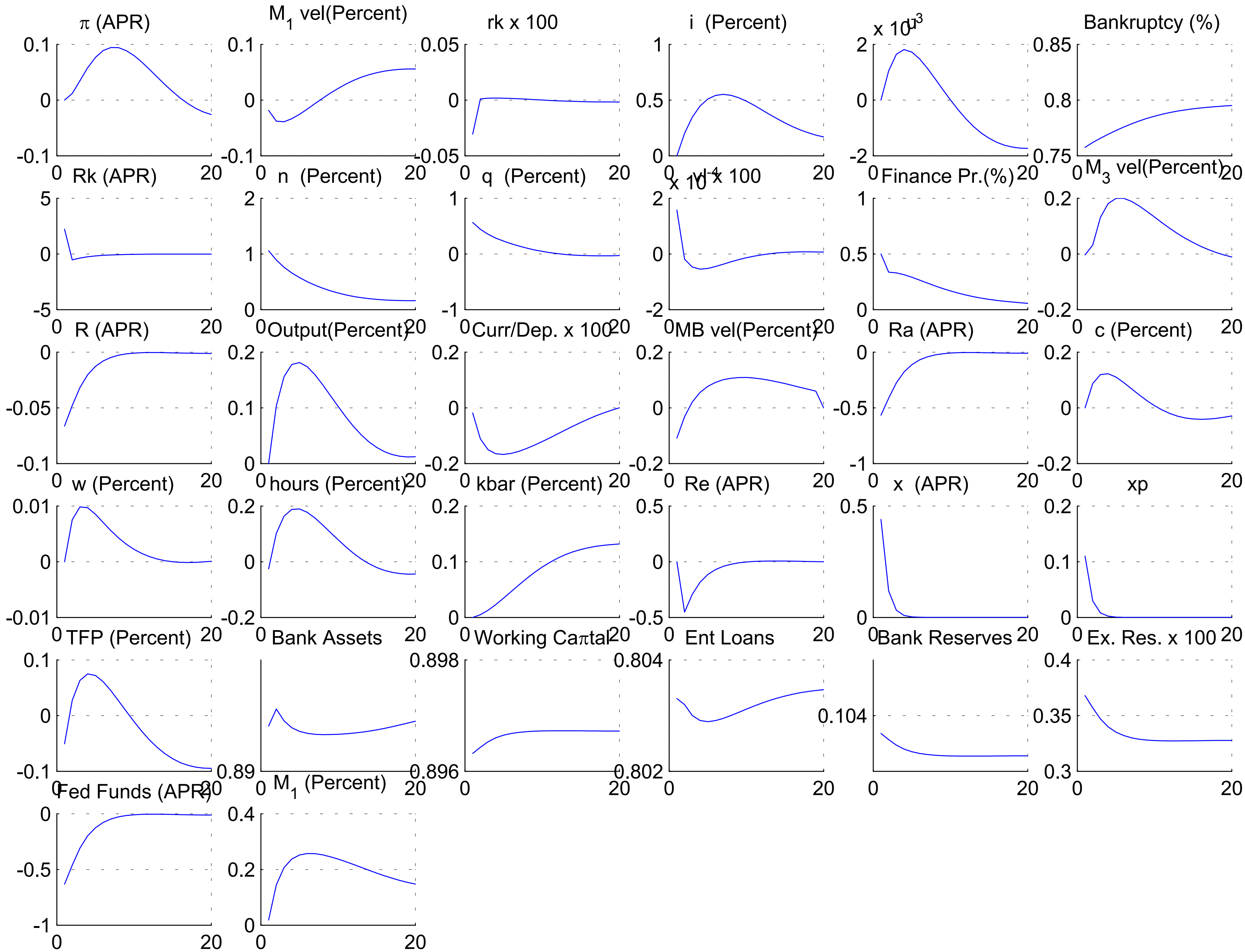


Figure 6: Response to Permanent Technology Shock (VAR: +, Model: Solid)

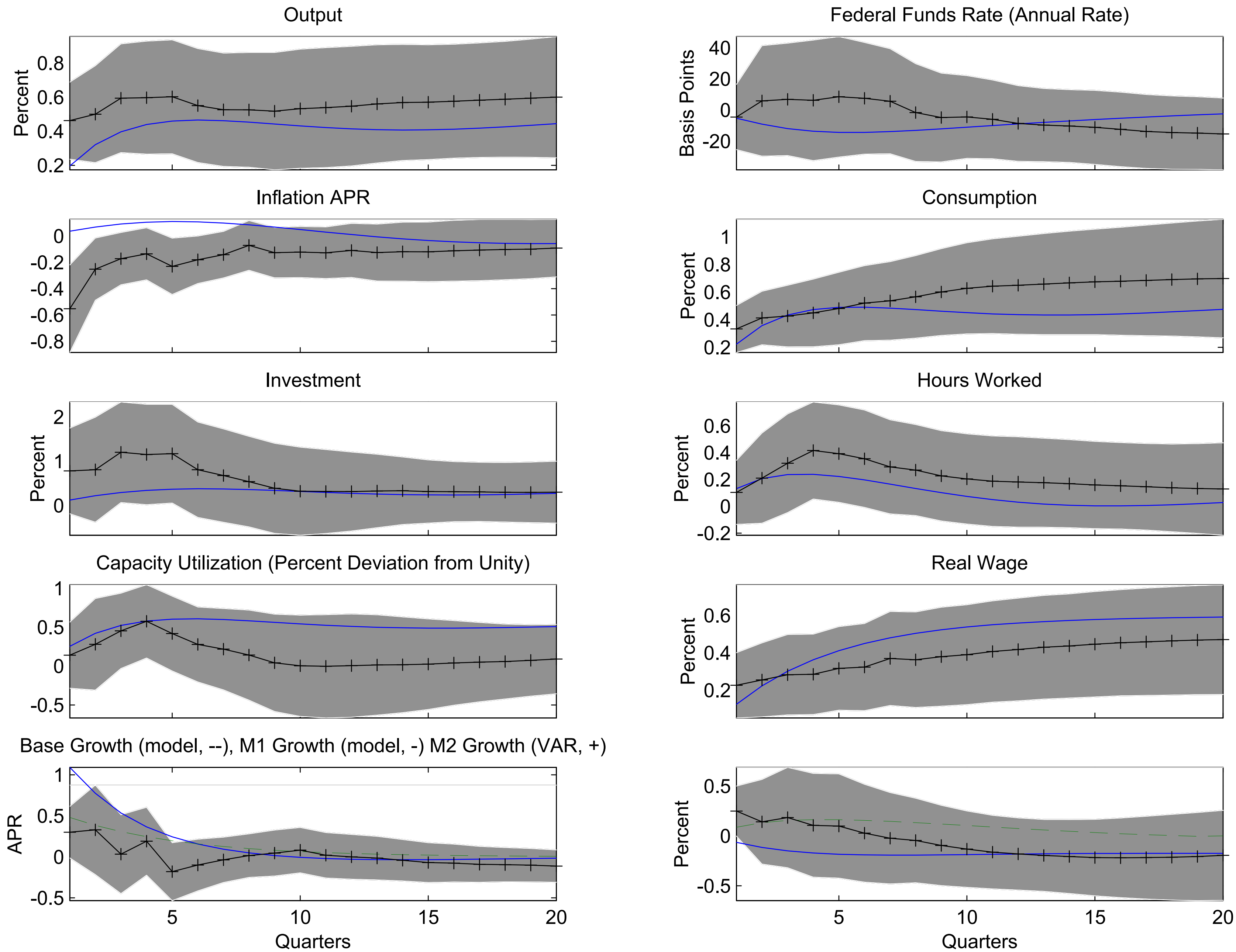


Figure 7: Permanent shock to technology

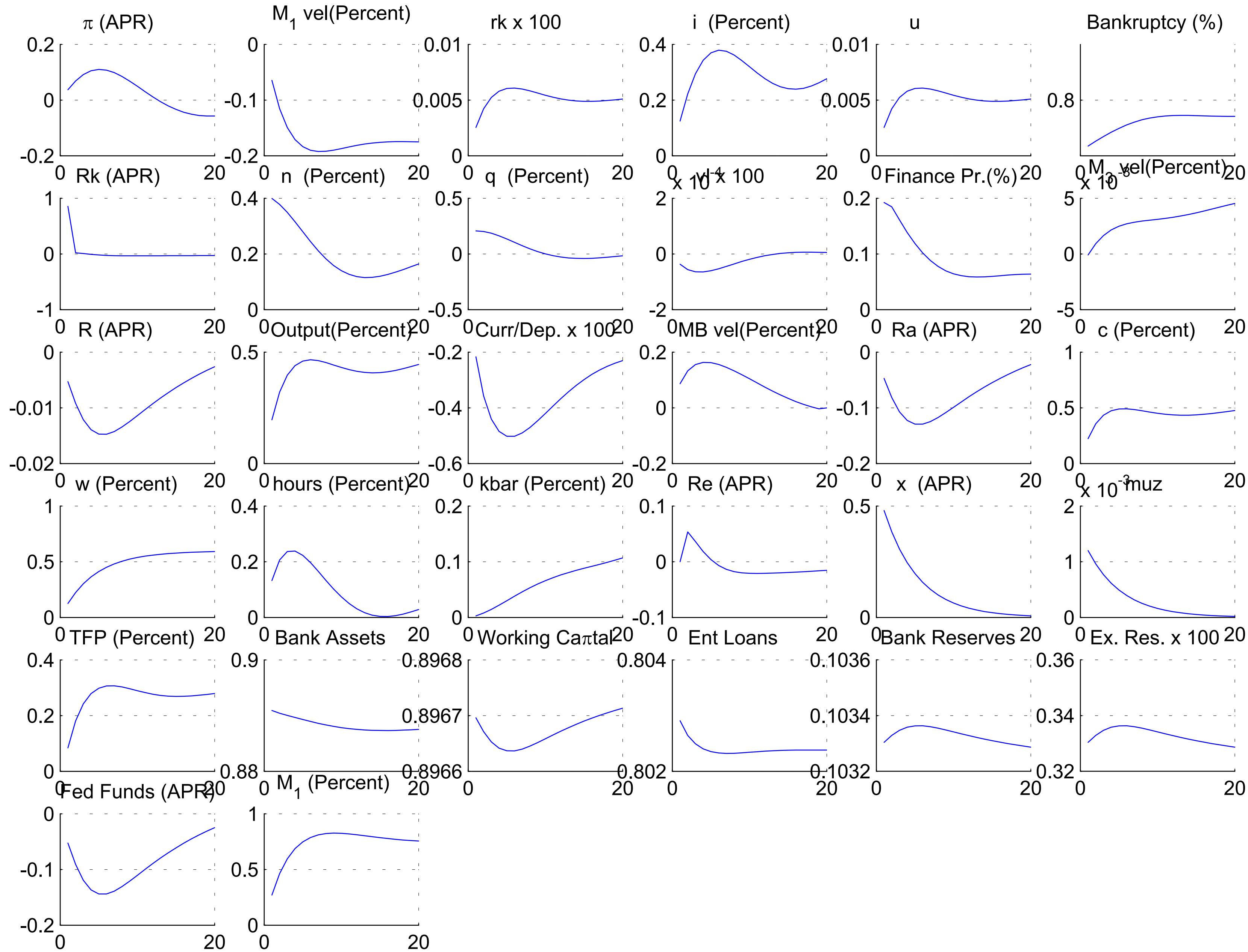


Figure 8: Shock to entrepreneurial net worth

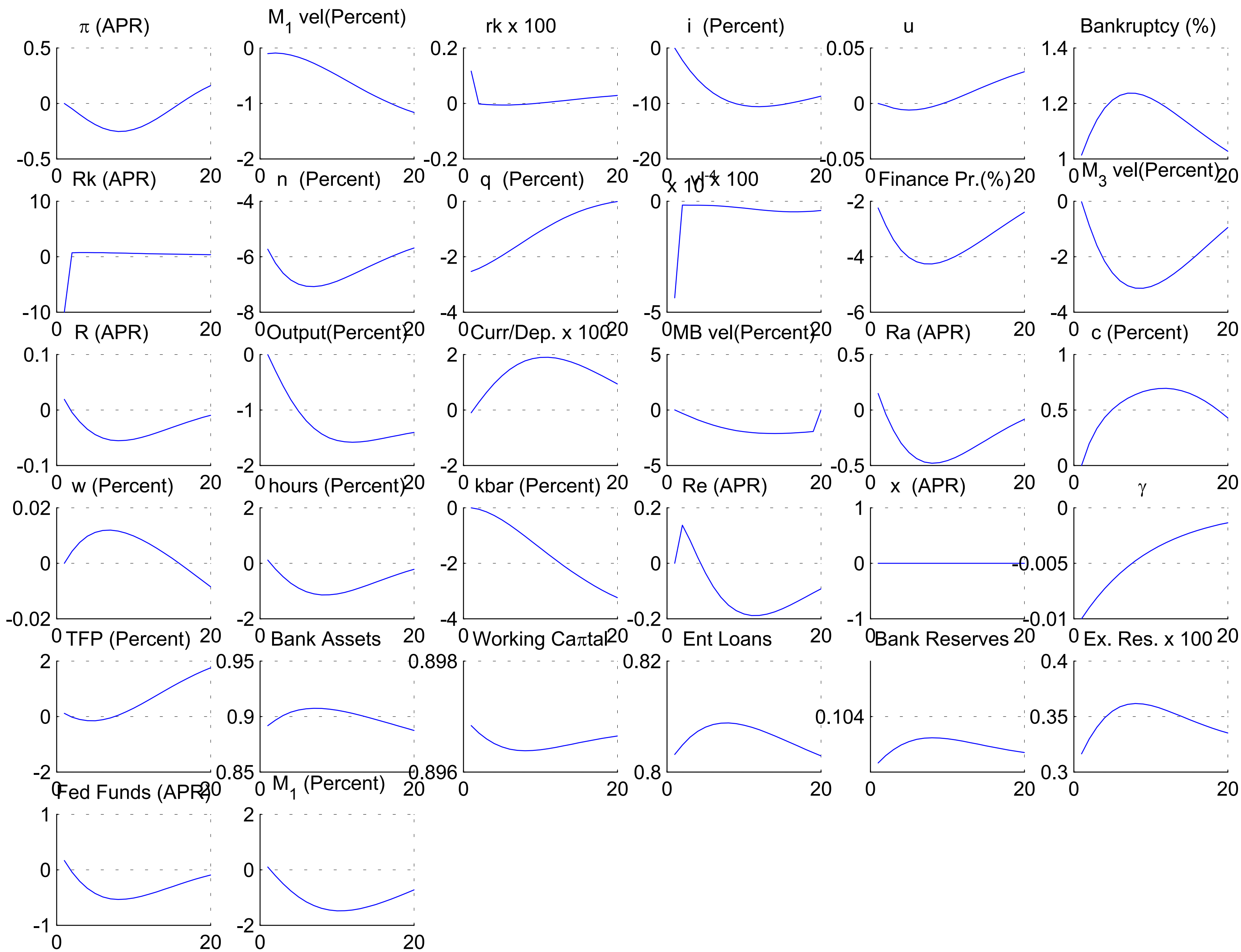


Figure 9: Shock to demand for reserves by banks

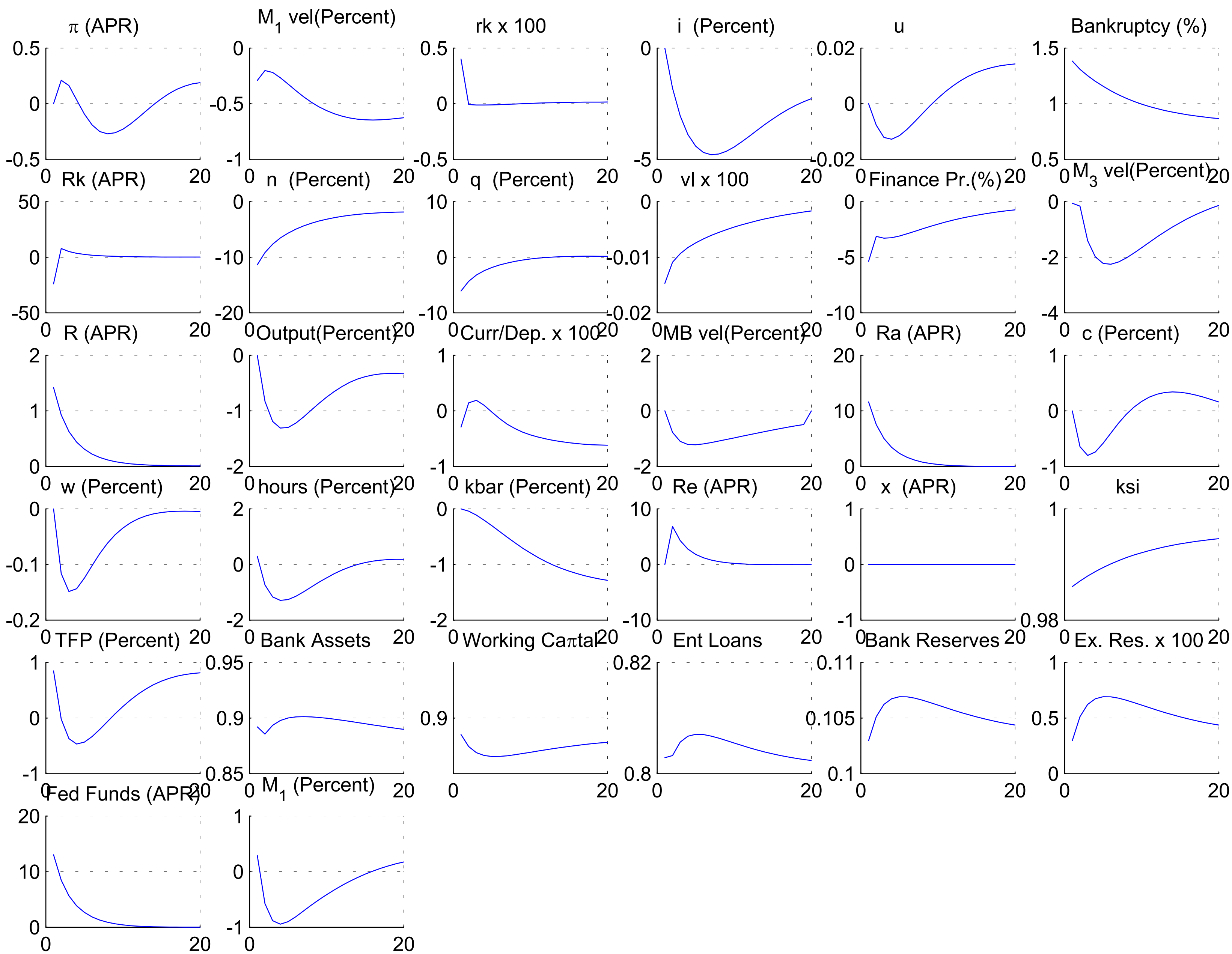


Figure 10: Shock to preference for currency versus deposits