Directed Search in the Interbank Money Market*

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Very preliminary and incomplete

Abstract

We present a model of monetary policy implementation that closely resembles the corridor system adopted by many central banks. First banks bid for reserves at a liquidity auction, then they trade on an OTC market, and finally they can access the central bank standing facilities. The model is tractable as it uses directed search by banks. In addition to price dynamics, the model provides insight on the nature of market volume, liquidity and volatility which features largely absent from the canonical models of monetary policy implementation in the tradition of Poole (1968). As there is an active role for interbank trading in the model, it provides a framework for discussing a number of money market features highlighted by the financial crisis and the recent period of unconventional monetary policies, including counterparty risk. Moreover, we find that the model fits well with a number of stylized empirical facts with respect to money market dynamics during normal times, as well as during the recent period of unconventional monetary policies. We consider different matching protocols, and we find that the data is best explained by a matching protocol which is not the most efficient. We conclude that frictions prevent the market from adopting better matching rules.

1 Introduction

This paper studies a model of the interbank market when the central bank remunerates excess reserves and offers to lend reserves at a penalty rate. This way to implement

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monetary policy is known as a corridor, or channel, system. In a corridor system, the central bank offers a lending and a deposit facility where banks can borrow or deposit reserves. Arbitrage implies that the interbank market rate lies within the band – or the corridor – defined by the deposit rate, a.k.a. the floor rate, and the lending rate. Knowingly or not, recently many central banks, including the Fed, have adopted new rules that brought their implementation framework closer to a corridor system. In this paper we study the behavior of commercial banks in the interbank market when the central bank operates a corridor system.

Historically, the US implemented monetary policy by specifying a target for the rate on unsecured overnight reserves loans between banks, while guiding this rate through a combination of open market operations. The target rate in turn influenced other interest rates and hence financing conditions in the wider economy. The stance of monetary policy was loosen (tighten) by lowering (increasing) the target for the overnight rate. Those banks who could not find a counterparty could borrow from the Fed by accessing a "discount window". Banks hardly used it, however, as the costs attached to borrowing at the discount window made it a last resort option. The crisis has changed the Fed modus operandi in drastic ways: It offers to lend reserves at a rate above the market rate (with no other strings attached), its different interventions flushed the market with reserves, and it started remunerating excess reserves in 2010. As a result, the Fed moved its monetary policy implementation framework closer to a corridor system.[1]

The decision of the Fed and many central banks to adopt a new framework for the implementation of monetary policy forces us to think about its impact on banks' behavior in the interbank market. In particular, can the midpoint of the corridor be seen as the policy rate, or will it fail to be the best indicator of the general level of short-term rates?[2] How does the interbank market work when it is flushed with reserves? And how does counterparty risk affects interbank lending in a corridor system? To tackle this and other issues, we present four stylized facts related to the functioning of the interbank market in a corridor system, using data from six jurisdictions that use a corridor system broadly defined. Then we present a model of the interbank money

[1] Attempts at taxonomies are available in Borio and Disyatat (2010) and Ishi, Fujita and Stone (2011).

[2] Several members of the Federal Open Market Committee (FOMC) have "raised the possibility that the federal funds rate might not, in the future, be the best indicator of the general level of short-term interest rates, and supported further staff study of potential alternative approaches to implementing monetary policy in the longer term and of possible new tools to improve control over short-term interest rates" (FOMC 2013).

market that is able to match these facts.

The four stylized facts are the following. First, the overnight rate tends to hover around the mid-point of the corridor when there is no liquidity surplus or deficit, while large liquidity surplus drives the rate to the floor of the corridor. Second, the overnight rate *volatility* is a decreasing function of excess reserves. Third, the aggregate market volume is a decreasing function of excess reserves. Finally, the overnight rate is an increasing function of counterparty risk. *[Include stylized facts about access to CB facilities.]*

Standard economic models used to study the interbank market and monetary policy implementation have difficulties in matching those facts. In particular, models of banks' reserve management in the tradition of Poole (1968), e.g., Woodford (2002), Bindseil (2004), Whitesell (2006) and Ennis and Keister (2011), focus solely on price (i.e. overnight rate) dynamics and consequently, have little to say about quantity and liquidity dynamics. In these models, a representative bank trades reserves in a frictionless market before it receives a liquidity shock. If following the shock, the bank ends up with a negative reserves balance, then it accesses the lending facility. Otherwise, it can earn the interest paid on excess reserves. The interbank rate is then equal to an average of the facilities rates, weighted by the probability to access each facility. In the basic model, there is just one representative bank, so there is no trading in the interbank market in equilibrium.[3]

We modify the basic model in two important ways: First, we assume that banks receive a liquidity shock before the interbank market opens. Second, we assume that banks trade bilaterally in an over-the-counter (OTC) market and bargain over the lending rates.

We consider two matching functions in the bilateral trading stage: Perfect pairing or random pairing with directed search. We see these two matching functions as proxies for the role of brokers in the interbank market. Under perfect pairing, banks are matched in a way that allows them to perfectly insure against the liquidity shock. In this sense, brokers would match banks efficiently. However, this model fails along several dimensions to explain the stylized facts, and we can rule out that efficient brokers are operating in the interbank market.

Using random pairing with directed search, banks can choose whether they want

[3]It is of course possible to extend Poole's model to assume banks are heterogeneous. However, this would imply that the interbank market trading is only due to shocks, and not to the level of aggregate excess reserves.

to borrow or lend reserves. Then banks on either side of the market are matched randomly. We see this matching function as one with no brokers, or when brokers are not very efficient. The equilibrium decision to become a borrower or a lender implies an interbank market tightness: if there are very few lenders, the market for reserves will be very tight and some borrowers will have to access the lending facility. Remarkably, the unique equilibrium is one where banks that are short of reserves, relative to the reserve requirement, become borrowers while banks that are long become lenders. Hence, excess reserves have a direct impact on the interbank market tightness. We find that this model can match all the stylized facts. We then enrich this model by introducing counterparty risk. We show that counterparty risk can decrease the volume of the interbank market but increases the overnight rate.

The model is remarkably tractable and we are also able to compute banks' willingness to pay for liquidity at a central bank liquidity auction. While we do not model the details of the auction, to our knowledge, this is the first model that offers a notion of willingness to bid for reserves, while taking into account the option to trade these reserves on the interbank market. Finally our model can be used to analyze market volume and other statistics as a function of the width of the corridor. In particular, we find that the volume of trades on the interbank market is not a function of the size of the corridor, except when there is counterparty risk.

In terms of the literature on monetary policy implementation, we already mentioned Poole (1968). The corridor system has also been studied in Berentsen and Monnet (2008) although from the point of view of the transmission mechanism of monetary policy rather than on the functioning of the interbank market. Afonso and Lagos (2012) may be the paper closest to ours, and we review it in depth in Bech and Monnet (2013). Here, let us just mention that they present a continuous time model to explain the intraday pattern of reserves holdings in the federal funds market, which shows that reserve holdings across banks tend to narrow as the day advances (see Ashcraft and Duffie, 2007). In their model, banks also trade in an OTC market and bargain over the terms of trade. However, their OTC market consists of several rounds of random bilateral meetings. In their model, the aggregate market volume is a function of the volatility of the payment shock and the number of trading rounds but does not depend on the amount of excess reserves. Finally, they do not model default risk.

We structured the paper has follows. Section 2 presents the four stylized facts. The basic environment is in Section 3. We then study the equilibrium for the two matching functions in Section 4 and we introduce counterparty risk in Section 5. We conclude in

Section 6.

2 Stylized Facts

This section is mainly based on Bech and Monnet (2013) where more details are available. We document four stylized facts related to "standard" and "unconventional" monetary policies, defined as a large amount of excess reserves being available in the market. There, we focus on six markets in the developed world: the *federal funds* market for the US dollar, the *Eonia* market for the euro, the *call loan* market for the yen, the *Sonia* market for pound sterling and the overnight markets for the Canadian and Australian dollars.[4]

The six panels of Figure 1 show the average overnight rate in each market - along with the prevailing rates at which banks could deposit or borrow funds from the central bank. The substantial cuts in policy rates that followed the onset of the financial crisis are evident across all markets and six years later the overnight rates remain significantly below those of mid-2007. With the exception of Australia, the overnight rates in all markets have flirted with the zero lower bound.

In our sample, the Federal Reserve, the European Central Bank (ECB), the Bank of England, the Bank of Japan and the Bank of Canada have all increased reserves substantially following the crisis, as a result of either liquidity backstops, asset purchases or very long term refinancing operations (see Figure 2). In contrast, the Reserve Bank of Australia (RBA) has by and large dealt with the crisis by relying on conventional policy measures. Therefore, we see Australia as our "control" market and the remainder as our "treatment" markets. The four stylized facts are the following.

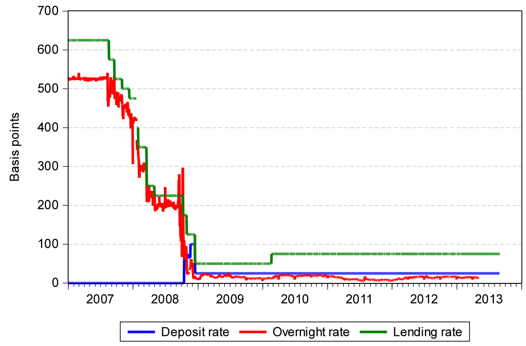
1. While in normal times, the overnight rate tend to hover around the mid-point of the corridor, large liquidity surplus drive the rate to the floor of the corridor.

2. The overnight rate volatility is a decreasing function of excess reserves, as it is larger in normal times than with a large liquidity surplus.

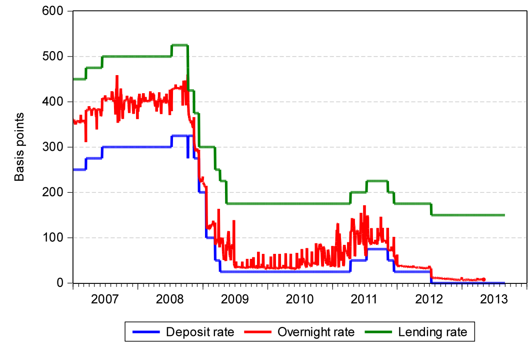
3. The aggregate market volume is a decreasing function of excess reserves.

4. The overnight rate is an increasing function of counterparty risk.

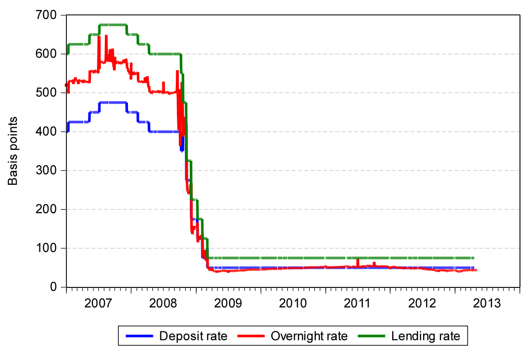
[4] In Bech and Monnet (2013) we also review two models of interbank markets, the now standard model of Poole (1968) and the new framework of Afonso and Lagos (2012). We find that these models fit a subset of the stylized facts. We refer the reader to our companion paper for the complete analysis.



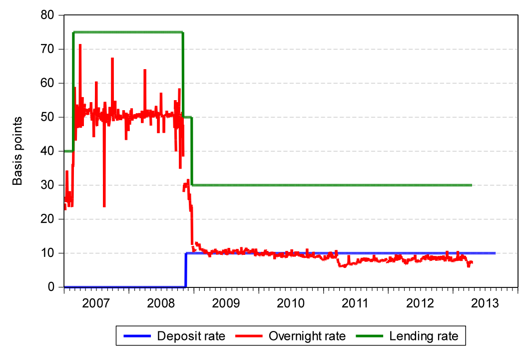
(a) USD - Federal funds rate



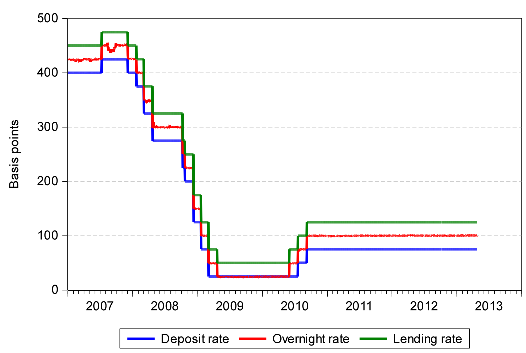
(b) EUR - Eonia Rate



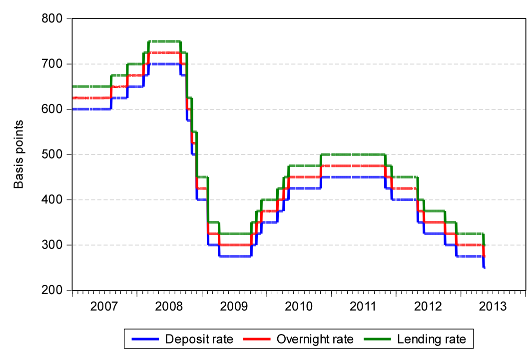
(c) GBP - Sonia rate



(d) JPY - Call rate



(e) CAD - money market financing rate



(f) AUD - Cash rate

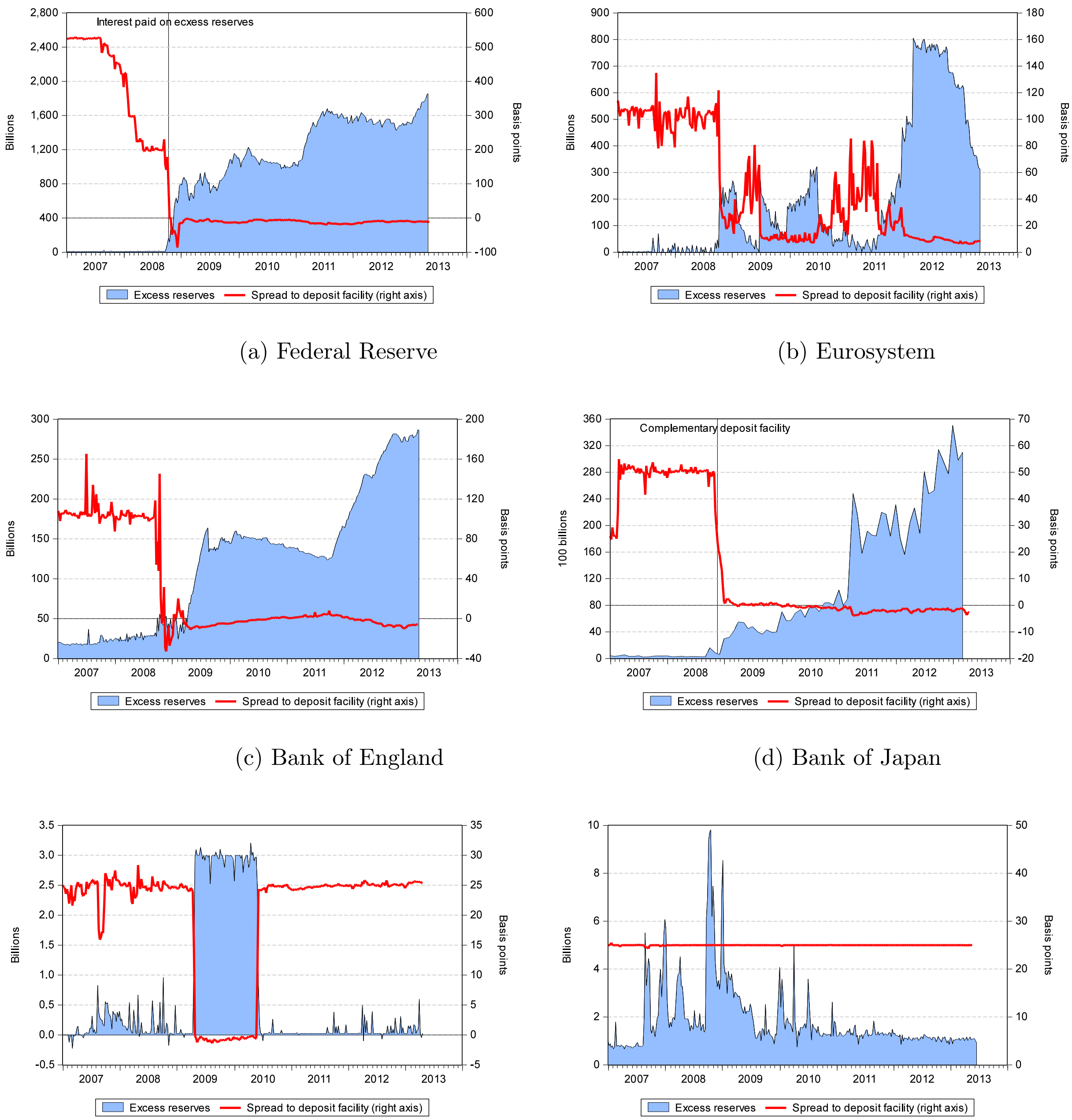
Sources: Datastream and national data.

Figure 1: Overnight and standing facilities rates

Stylized fact 1: Overnight rate is decreasing in excess reserves

Since, late 2008 all six central banks have remunerated excess reserves and hence have been operating using a corridor system.[5] In addition to the amount of excess reserves,

[5]The Federal Reserve began to pay pay interest on depository institutions' required and excess reserve balances on 9 Octobter 2008. The Bank of Japan began to pay interest on current account balances and special reserve account balances on 16 November 2008.



(a) Federal Reserve

(b) Eurosystem

(c) Bank of England

(d) Bank of Japan

(e) Bank of Canada

(f) Bank of Japan

Sources: Datastream and national data.

Figure 2: Excess reserves and the overnight rate.

the panels of Figure 2 plots the spread between the overnight rate and the central bank deposits rate. Within our treatment group this spread narrows as the amount of excess reserves increases. The substantial increase in excess reserves drove the overnight interbank rates towards the rate at which the central bank remunerates reserves, i.e., the floor of the corridor. Hence, these central banks abandoned their usual practice of keeping the overnight rate close to the midpoint of the corridor spanned by the standing facilities rates. In contrast, Australia saw much smaller shifts in excess reserves and the RBA has been able to keep the cash rate at the center of the 50 basis point wide corridor. In some markets, the average overnight rate has even moved below the deposit rate, i.e., a negative spread to the deposit rate. This, somewhat surprising outcome, is due to a combination of market segmentation and limits to arbitrage.[6]

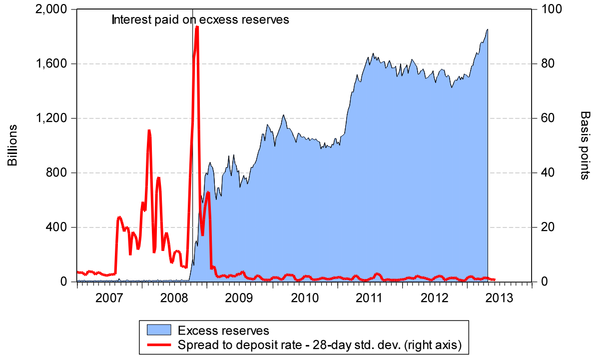
Stylized fact 2: Overnight rate volatility is decreasing in excess reserves

The massive expansion of excess reserves have also tended to reduce the volatility of the overnight rates relative to normal times. The panels in Figure 3 plot the amount of excess reserves together with the 28-day rolling standard deviation of the spread between the overnight rate and the central bank deposit rate. In addition, the panel also includes two measures of the intraday dispersion for the federal funds rate. Everyday a number of brokers in the federal funds market submit to the Federal Reserve Bank of New York (FRBNY) the amount of transactions they have brokered at different rates. Based on this data, the FRBNY determines not only the average federal funds rate plotted in Figure 1 but also the standard deviation of rates as well as the low and the high rate of the day. Eyeballing the panels suggest that for central banks in the treatment group overnight rate volatility decreased as central bank balance sheet expanded while volatility in the Australian overnight market has remained fairly constant.

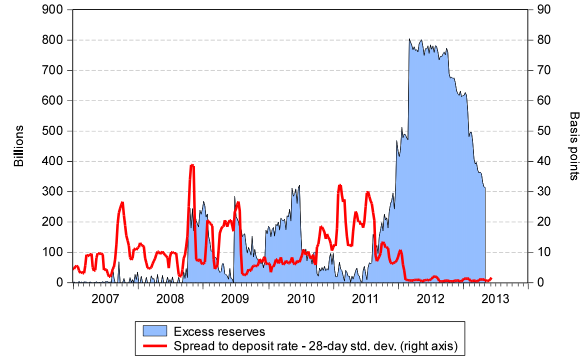
Stylized fact 3: The market volume is decreasing in excess reserves

Figure 4 plots interbank market volume together with the amount of excess reserves. Unfortunately, volume information is only publicly available for the euro (Eonia) and the sterling Sonia markets. To get a clearer picture on the impact of reserves on market

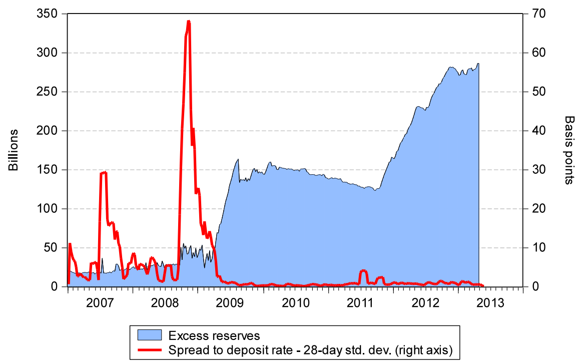
[6] See Bech and Klee (2011) or Chiu and Monnet (2013).



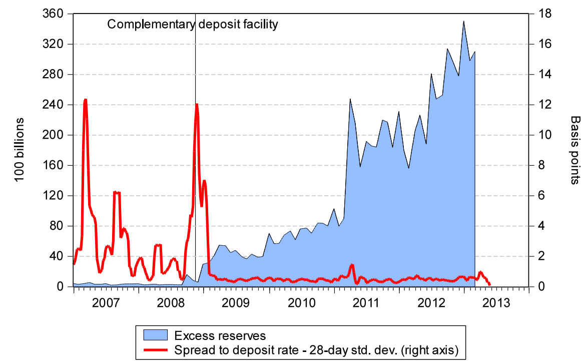
(a) Federal Reserve



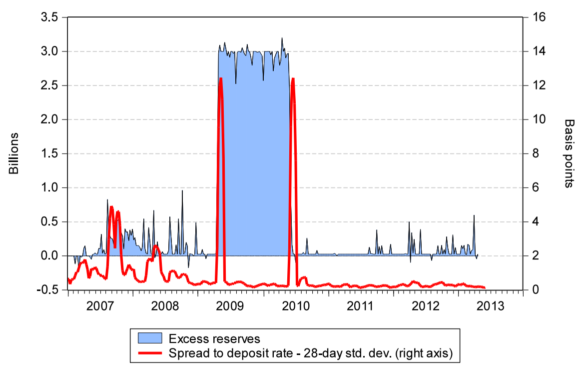
(b) Eurosystem



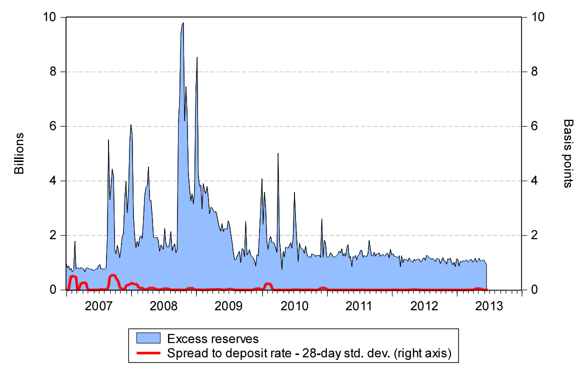
(c) Bank of England



(d) Bank of Japan



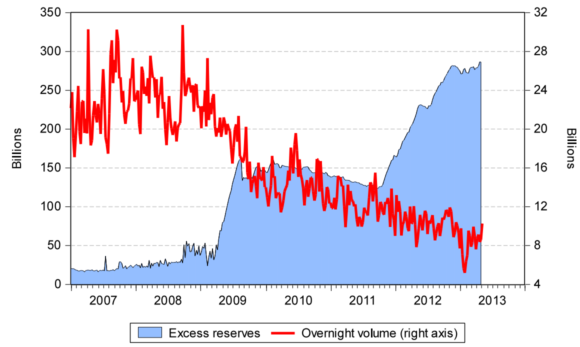
(e) Bank of Canada



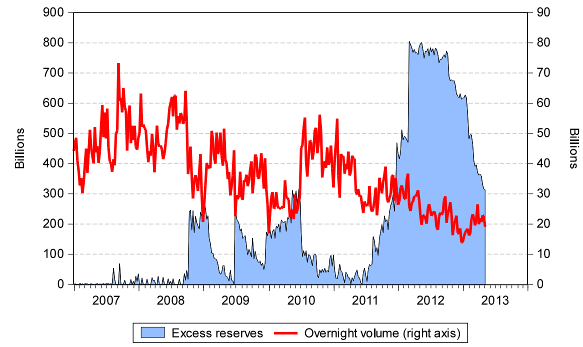
(f) Bank of Japan

Sources: weekly data from Bloomberg, Datastream, and national data.

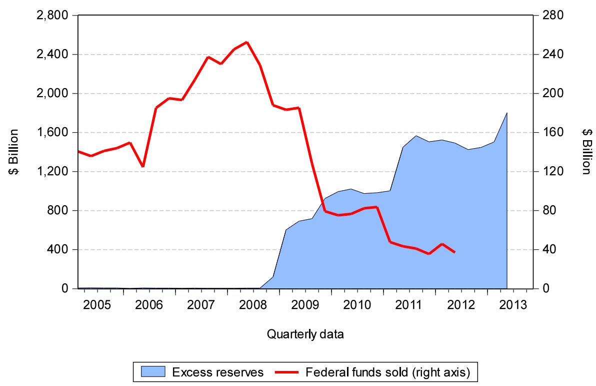
Figure 3: Excess reserves and overnight rate volatility.



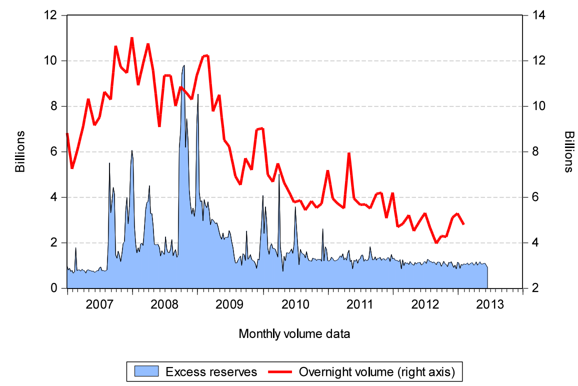
(a) Bank of England



(b) Eurosystem



(c) Federal Reserve



(d) Australia

Sources: Bloomberg, Datastream, and national data. Weekly data unless otherwise stated.

Figure 4: Volume.

volume, we also show quarterly market volume for the federal funds market and monthly information for the Australian overnight market.[7] The volume has fallen with excess reserves across all markets. For the Eonia and the federal funds markets, the movement in volume broadly correlates with the movement in excess reserves. For the Sonia market the relationship in less pronounced but volumes have fallen significantly. For the Aussie dollar market volume has fallen despite excess reserves being fairly constant over the period.

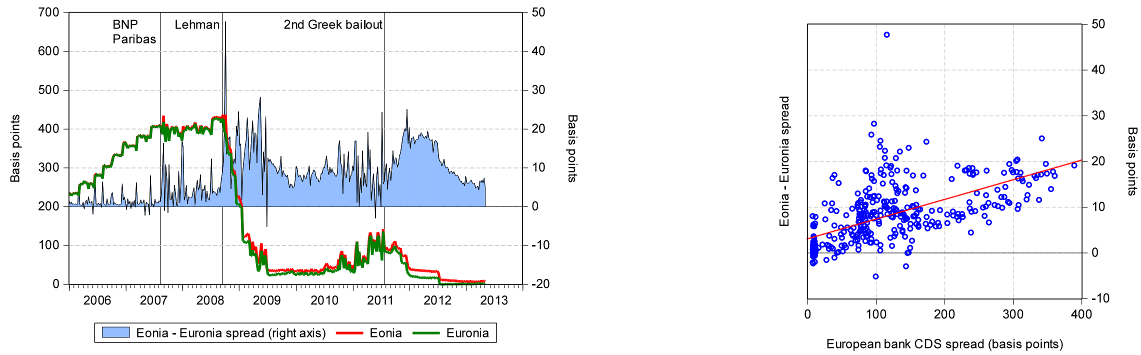
[7]The data for the federal funds market is from Kreicher, McCauley and McGuire (2013) and the data from the Australian markets was provided by the Reserve Bank of Australia.

Stylized fact 4: Credit risk increases the overnight rate

The widening of interest rate spreads during the recent financial crisis represented both deteriorating liquidity and greater credit risk. Some papers seek to disentangle the two effects.[8] Unfortunately, good measures of the impact of credit risk on the overnight rates are hard to find. One example - used by market analysts - is the spread between Eonia and the Euronia rate (see e.g. Marraffino and Fransolet, 2012).[9] The number of participants is smaller in the Euronia market and they are generally regarded as having been - on average - of greater credit worthiness than the participants in the Eonia market. Consequently, the spread between the two rates is thought to reflect primarily differences in credit risk. The left hand panel of Figure 5 plots the Eonia and the Euronia rates as well as their spread since 2006. Prior to the financial crisis, the difference was small, on average a couple of basis points. However, the spread moved up in August 2007 and climbed further in the aftermath of the Lehman bankruptcy in September 2008. It has remained elevated ever since and has generally followed the ebbs and flows in terms of the perceived health of the European banks. The right hand panel of Figure 5 shows a scatter plot of a credit default swap index for European banks and the Eonia - Euronia spread and illustrates this point.

[8] e.g. Michaud and Upper (2008), Taylor and Williams (2008, 2009), McAndrews et al (2008), Christensen et al (2009), Heider et al (2009), Schwarz (2010), Angelini et al (2011)

[9] The Eonia rate is a weighted average of all overnight unsecured lending transactions in the interbank market, undertaken by a panel of banks in the European Union and European Free Trade Association (EFTA) countries. In contrast, the Euronia rate is the weighted average rate of all unsecured euro overnight cash transactions brokered in London by contributing Wholesale Markets Brokers' Association (WMBA) member firms.



(a) Eonia-Euronia level and spread

(b) Correlation with banks CDS

Sources: Bloomberg, Datastream, and national data.

Figure 5: Credit risk and overnight rate

3 The Environment

We consider the following model of bank's reserves management. There are two periods and a unit measure of banks. All banks are risk neutral and they do not discount the future. Banks must hold required reserves $\bar{m}$ at the end of the economy (a.k.a. the maintenance period). If they fall short of holding $\bar{m}$, banks can borrow reserves at the central bank, at a cost i_ℓ. If they hold more than $\bar{m}$, then banks can earn a per-unit interest rate of i_d. Banks are trying to maximize their profit given the have to hold reserves.

Banks can do two things to manage their reserves. First, they can access a central market for reserves where they can acquire reserves by producing a numeraire at unit cost (this is akin to accessing a central bank auction). In this market, the total supply of reserves m is decided by the central bank. If $\bar{m} > m$ then there is a structural liquidity deficit and if $\bar{m} < m$ then there is a structural liquidity surplus.

Once they exit the reserves market, and as in Poole (1968), banks receive shocks v symmetrically distributed over on $[-\bar{v}, \bar{v}]$ according to a c.d.f. $F(.)$ where $\bar{v}$ is possibly infinite. However, contrary to the Poole's model, banks can access an interbank market for reserves, where banks trade bilaterally and over-the-counter.

In the sequel, we consider two matching functions in the OTC market: (1) directed search with random pairing, and (2) directed search with perfect pairing. We explain this taxonomy precisely below, but let us say that these matching functions could be seen as a short hand for the work of brokers: In the absence of brokers, banks meet at random. Otherwise, banks can contact their broker either as a lender or a borrower and depending on the quality of their broker, they can be matched more or less well.[10]

We can compute the rates and trades in each match irrespective of the matching technology. Once two banks are matched, the borrower holding m_b reserves and the lender holding m_ℓ bargain over the quantity of reserves to be lent $q(m_b, m_\ell)$ and the terms of trade $i_m(m_b, m_\ell)$.

Banks who did not find a trading partner, or banks who still have too little or too much reserves will access the central bank's facility where they can borrow reserves at rate i_ℓ or deposit reserves and earn rate i_d. Finally, we assume that required reserves are remunerated at rate i_d (the same rate as excess reserves).

4 Equilibrium

We solve the model backward in several steps. First we compute the rates and volumes in the OTC interbank market. We can then compute the payoff of each bank at the end of the economy depending on their history of trade. Second, given those payoffs, we can solve for the decision of banks to become borrowers or lenders, and therefore, the interbank market tightness. Finally, we can then find a bank's willingness to pay for reserves at the time of the central bank's reserves auction.

4.1 OTC market rates and volumes

We assume that banks use Nash bargaining and have the same bargaining power. Since banks are risk neutral, we assume they equate their reserves holdings so that[11]

$$q(m_b, m_\ell) = \frac{m_\ell - m_b}{2}. \tag{1}$$

To be consistent with our labels, it should be that $m_\ell > m_b$ so that a lender actually does lend to the borrower (and not the reverse). As we will show, this will always be the case in the equilibrium with directed search. Given this quantity, banks bargain over the interest rate i_m.

Since they are risk neutral, the assumption of equal bargaining weights implies that

[10] We refer the reader to Bech and Monnet (2013) for the analysis with random matching.

[11] Afonso and Lagos (2012) uses the same assumption.

a borrower banks holding m_b and a lender bank holding m_ℓ will equate their surplus from trade. In the Appendix, we show that the OTC rates as a function of (m_ℓ, m_b) are defined as follows

$$i_m = \begin{cases} i_d & \text{if } m_\ell > m_b > \bar{m} \\ \frac{m_\ell - \bar{m}}{m_\ell - m_b} i_d + \frac{\bar{m} - m_b}{m_\ell - m_b} i_\ell & \text{if } m_b < \bar{m} < m_\ell \\ i_\ell & \text{if } m_b < m_\ell < \bar{m} \end{cases} \tag{2}$$

In words, when both banks have enough reserves to satisfy the reserves requirement, they trade at the deposit rate. On the other hand, if both banks have too few to satisfy their requirement, then they trade at the lending rate. Finally, if one of them can satisfy the reserve requirement and the other cannot, then they trade at a rate which is a weighted average of the corridor rates, where the weights are given by the relative position of each bank: if the lender has a lot of excess reserves, then the rate will tend to the deposit rate, and otherwise, the rate will tend to the lending rate.

We can use (1) and (2) to obtain the payoff of the borrower and the lender banks. Denoting the payoff of a borrower bank holding m_b and meeting a lender bank holding m_ℓ by $P^b(m_\ell, m_b)$, we have

$$P^b(m_\ell, m_b) = \begin{cases} (1+i_d)m_b & \text{if } m_\ell > m_b > \bar{m} \\ (1+i_d)m_b - \frac{\bar{m} - m_b}{2}(i_\ell - i_d) & \text{if } m_b < \bar{m} < m_\ell \text{ and } m_b + m_\ell > 2\bar{m} \\ (1+i_\ell)(m_b - \bar{m}) + (1+i_d)\bar{m} + \frac{m_\ell - \bar{m}}{2}(i_\ell - i_d) & \text{if } m_b < \bar{m} < m_\ell \text{ and } m_b + m_\ell < 2\bar{m} \\ (1+i_\ell)(m_b - \bar{m}) + (1+i_d)\bar{m} & \text{if } m_b < m_\ell < \bar{m} \end{cases} \tag{3}$$

and the payoff of the lender bank is

$$P^\ell(m_\ell, m_b) = \begin{cases} (1+i_d)m_\ell & \text{if } m_\ell > m_b > \bar{m} \\ (1+i_d)m_\ell + \frac{\bar{m} - m_b}{2}(i_\ell - i_d) & \text{if } m_b < \bar{m} < m_\ell \text{ and } m_b + m_\ell > 2\bar{m} \\ (1+i_d)m_\ell + \frac{m_\ell - \bar{m}}{2}(i_\ell - i_d) & \text{if } m_b < \bar{m} < m_\ell \text{ and } m_b + m_\ell < 2\bar{m} \\ (1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} & \text{if } m_b < m_\ell < \bar{m} \end{cases} \tag{4}$$

Both payoffs are quite intuitive. If both banks in the pair have enough reserves to satisfy the reserve requirement, then they both get the interest on reserves. If the borrower is missing some reserves but the lender can compensate what he misses, then both the borrower and the lender get the interest rate on reserves. Since the borrower does not have to access the lending facility for the missing amount $\bar{m} - m_b$, his surplus from trade is $(\bar{m} - m_b)(i_\ell - i_m)$ which is split equally between the borrower and the lender. The same is true when the lender does not have enough to satisfy both banks' reserve requirements. However, now the lender is making a surplus from lending reserves to the borrower of $(m_\ell - \bar{m})(i_m - i_d)$ and it is split equally with the borrower. Finally, when the pair does not have enough reserves to satisfy either of the reserve requirements, then there is no surplus from trade and both banks access the lending facility to borrow the missing reserves, while they receive the deposit rate on their required reserves.

Equipped with these payoffs, we can now determine the endogenous interbank market tightness as a function of the aggregate excess liquidity.

4.2 Directed search with perfect pairing

As a useful benchmark, we first consider the case where banks are perfectly paired. By perfect pairing, we mean the following. Suppose the monetary policy stance is to supply m units of reserves in the central market. In equilibrium, all banks exit the central stage with m units of reserves. Then they receive some liquidity shocks. With perfect pairing, the matching technology will pair a bank holding $m + v$ with a bank holding $m_2 = m - v$ such that both banks exit the OTC market with exactly m units of reserves, *as if* they did not receive any shocks.

Notice a very important assumption regarding the matching technology: it is not a function of a bank's shock but only of its reserves holdings. Therefore if a bank holds $\tilde{m} \neq m$ at the start of the OTC market and receives no shock, then it is matched with a bank that, after the shock, holds $m - \tilde{m}$. This is crucial to understand the marginal value of reserves in the central market.

Why is this a useful benchmark? With no further shock, all banks exit the central market with m units of reserves, and they either all access the lending facility if $m < \bar{m}$, or the deposit facility if $m > \bar{m}$. Therefore the case of perfect pairing mimics a central market where banks could just trade their shocks away. However, as we will see below, there are important differences between perfect pairing and a central market. Most importantly, while there would be a unique rate on the market, there will be some rate dispersion with perfect pairing that is originating from the bargaining protocol.

4.2.1 Equilibrium rates

For now, we will assume and later verify that all banks enter the OTC market with the same amount of reserves, m. A bank who receives a positive shock $v > 0$ is matched with the bank who received shock $-v$. Therefore the bank with the positive shock lends v to the other at the rates described in (2). Following trade, all banks exit the OTC market with m units of balances. We can then rewrite the rates and the payoff of the lender and borrower banks, making use of the fact that $m_b = m - v$ while $m_\ell = m + v$. Arranging the expression, banks trade reserves at the rate

$$i_m(v) = \begin{cases} i_d & \text{if } v < m - \bar{m} \\ \frac{i_\ell + i_d}{2} + \frac{\bar{m} - m}{2v}(i_\ell - i_d) & \text{if } v > m - \bar{m} > -v \\ i_\ell & \text{if } v < \bar{m} - m \end{cases} \tag{5}$$

where $v > 0$ is the lender's shock. This function has a natural interpretation, e.g., when $v < \bar{m} - m$ the borrower has enough to cover the reserve requirement $\bar{m}$ and he does not need to borrow, which forces the OTC rate to i_d.

In particular, it is clear that *all* rates will be equal to the mid-point of the corridor if the central bank has a neutral stance, i.e. $m = \bar{m}$. Also, notice that perfect pairing implies that there are only two possible rates depending on the aggregate liquidity supply. In a liquidity surplus, $m > \bar{m}$ and

$$i_m(v) = \begin{cases} i_d & \text{if } v < m - \bar{m} \\ \frac{i_\ell + i_d}{2} + \frac{\bar{m} - m}{2v}(i_\ell - i_d) & \text{if } v > m - \bar{m} \end{cases} \tag{6}$$

while in a liquidity deficit, $m < \bar{m}$ and

$$i_m(v) = \begin{cases} \frac{i_\ell + i_d}{2} + \frac{\bar{m} - m}{2v}(i_\ell - i_d) & \text{if } v > \bar{m} - m \\ i_\ell & \text{if } v < \bar{m} - m \end{cases} \tag{7}$$

In particular, if $v < \bar{m} - m$ then the lender has too few reserves to satisfy his requirement. As a result he will have to borrow at the lending facility and he will only be willing to lend to the borrower at rate i_ℓ. Figure 6 illustrates $i_m(v)$ as a function of the liquidity surplus/deficit. Banks have the same amount of reserves m before they are hit by the liquidity shock. Since they are perfectly paired, a bank with the shock $-v < 0$ (on the $x-$axis) is matched with a bank with shock $v > 0$ (on the y-axis). If the resulting reserves for the pair $(m-v, m+v)$ falls in the green region, then the rate is i_d if $m - \bar{m} > 0$ and i_ℓ otherwise, as both banks have enough to satisfy their reserves requirement in the first case, while neither does in the second. If the reserves for the pair falls in the red region, then banks trade at the midpoint of the corridor plus a term to share the gains from trade. We can write the payoff of a bank with liquidity shock

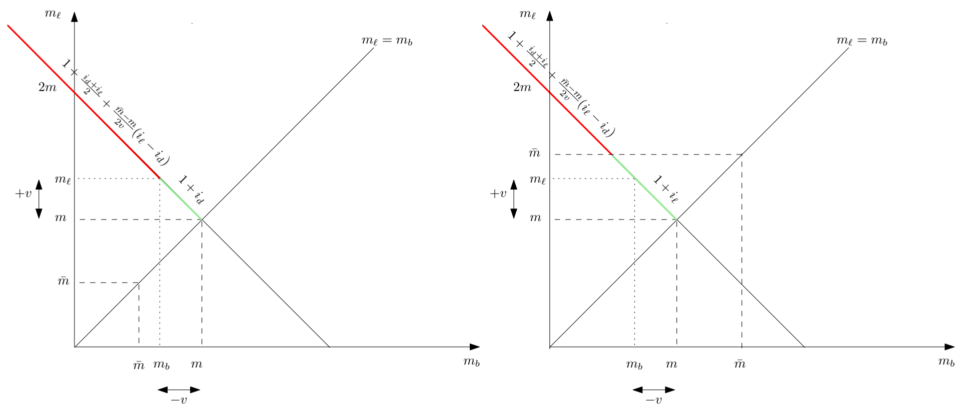


Figure 6: Interest rates - left: $\bar{m} - m < 0$, right: $\bar{m} - m > 0$

$v \in [-\bar{v}, \bar{v}]$, when the central bank policy stance is m, as

$$P(m, v) = (1 + i_d)\bar{m} + (m - \bar{m})[\mathbb{I}_{\{m > \bar{m}\}}(1 + i_d) + \mathbb{I}_{\{m < \bar{m}\}}(1 + i_\ell)] + (1 + i_m(|v|))v \quad (8)$$

The first term is the interest rate paid on banks' required reserves. Absent any OTC market, the second term captures the benefits/cost of accessing the central bank facilities to cover excess reserves $m - \bar{m}$. Finally, the last term captures the gain from trading the liquidity shock v on the OTC market.

We can now compute the willingness to pay for reserves in the central market. All banks face the same uncertainty and they will therefore behave in the same way in this market. Since the supply of reserves is m, they will all exit the central market with m units of reserves. However, to compute their marginal value at m, we need to compute their marginal value at any other level $\tilde{m}$. Notice that a bank who exits the central market with $\tilde{m} \neq m$ (before the shock hits) and is then hit by a shock of size v has a payoff

$$P(\tilde{m}, v) = P(m, v + \tilde{m} - m).$$

Indeed, it is as if the bank had exited the central stage with holding m and received a shock $v + \tilde{m} - m$.[12] Therefore, the value of exiting the centralized stage with $\tilde{m}$ units

[12] Our assumption regarding the matching technology plays a very important role here.

of reserves when the central bank supplies m is simply

$$W(\tilde{m}) = \int_{-\bar{v}}^{\bar{v}} P(m, v + \tilde{m} - m) dF(v)$$

Equipped with this expression, we can then compute $W'(\tilde{m})$, which we do in the Appendix. In a symmetric equilibrium, all banks exit the central stage with m units of reserves and their willingness to pay for reserves at $m > \bar{m}$ is simply

$$W'(m) = (1+i_d)\left[F(m-\bar{m}) - F(\bar{m}-m)\right] + \left[1 + \frac{i_\ell + i_d}{2}\right]\left[1 - F(\bar{m}-m) + F(\bar{m}-m)\right]$$

This is rather intuitive: Starting from a situation of liquidity surplus, banks exit the central market with more reserves than they need to satisfy their reserves requirements $\bar{m}$. If banks receive a relatively small shock $v \in [\bar{m}-m; m-\bar{m}]$ they know they will be matched with a bank who also received a shock of the same magnitude but of opposite sign and they still both have enough reserves to satisfy their reserves requirements. Hence, they will trade at the floor rate. If however, banks receive a large shock $v \in [-\bar{v}, \bar{m}-m] \cup [m-\bar{m}, \bar{v}]$ they are paired with a bank who also received a large shock and one of them cannot satisfy the reserves requirement. In this case they trade at the mid-point of the corridor, sharing the surplus from trade. A similar expression results in the case with liquidity deficit, $m < \bar{m}$. Using symmetry of the shock distribution, we can simplify both expressions for the rates to

$$W'(m) = (1+i_d)F(m-\bar{m}) + (1+i_\ell)\left[1 - F(m-\bar{m})\right].$$

Therefore, and maybe surprisingly, the banks' willingness to pay for reserves in the "auction" market in the model with perfect pairing is the same as the banks' willingness to pay in the Poole model. This may be surprising as there is no market in the Poole's model where banks can trade their shocks away. However, looking at (8) notice that banks do not expect any gains *on average* from participating in the OTC market as the shock averages to zero. Therefore, they value reserves in the auction market as if there was no OTC market.

4.2.2 OTC volume, weighted average rate, and volatility

We can now compute some OTC market statistics, when the matching functions pairs banks perfectly. We compute the market volume, average weighted rates and the rate

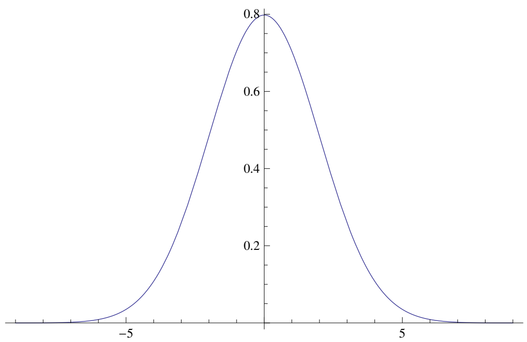


Figure 7: Aggregate volume with perfect pairing.

volatility.

OTC market volume Ignoring those trades with relatively low shocks, that do not generate any surplus, the market volume is simply[13]

$$Q(\bar{m}-m)=\int_{|\bar{m}-m|}^{\bar{v}} v dF(v)$$

which is a decreasing function of the central bank's liquidity stance. Figure (7) plots the aggregate volume as a function of excess reserves, $m-\bar{m}$ on the $x-$axis for normally distributed shocks with standard deviation of 2.

OTC rate Then the weighted average rate is simply

$$\bar{i_m}(\bar{m}-m)=\frac{1}{Q(\bar{m}-m)}\int_{|\bar{m}-m|}^{\bar{v}} i_m(v) v dF(v)$$

and using (6)-(7) we obtain

$$\bar{i_m}(\bar{m}-m)=\frac{i_\ell+i_d}{2}+\frac{\bar{m}-m}{Q(\bar{m}-m)}\frac{(i_\ell-i_d)}{2}\left[1-F(|\bar{m}-m|)\right].$$

Hence, the average rate is the mid-point of the corridor, plus the average gains of trading in the OTC market. In particular, under a neutral liquidity provision $m=\bar{m}$ the average rate is at the mid-point of the corridor, it is below whenever there is an aggregate liquidity surplus $m>\bar{m}$, and it is above if there is a liquidity deficit. Figure

[13]If we were to consider those trades, the market volume would be $\int_0^{\bar{v}} v dF(v)$, a constant independent of the size of the liquidity surplus.

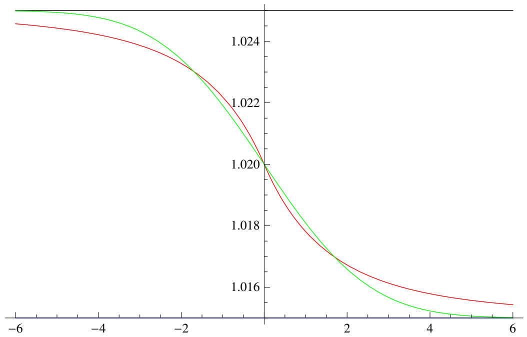


Figure 8: OTC weighted rate (red: perfect pairing, green: Poole 1968)

8 plots the weighted rate as a function of the liquidity surplus, $m - \bar{m}$. The OTC rate is steeper than the Poole rate (or the auction rate) for a simple reason: the OTC rate gives more weight to the corridor mid-point as this is the prevalent rate when there is relatively large shock. For example, the Poole rate gives more weight to i_d as m increases, while the OTC rates gives more weight to $(i_\ell + i_d)/2$, which explains why the red curve is above the green curve when m is large. Symmetrically, this explains why the order is reversed when m is small, as the Poole rate now gives more weight to i_ℓ while the OTC rates gives more weight to $(i_\ell + i_d)/2$.

Volatility Finally, we compute the rate volatility, as

$$\sigma_w^2(i_m) = \int_{|\bar{m}-m|}^{\bar{v}} \frac{v}{Q(\bar{m}-m)} [i_m(v) - \bar{i_m}]^2 dF(v)$$

and using the expression for the rates and the weighted rate, we obtain

$$\sigma_w^2(i_m) = \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} \left[\int_{|\bar{m}-m|}^{\bar{v}} \frac{1}{v} dF(v) - \frac{[1 - F(|\bar{m}-m|)]^2}{Q(\bar{m}-m)} \right]$$

Notice that the variability is M-shaped, as it is zero whenever $\bar{m} = m$, or when $|\bar{m} - m|$ is large. Also, it is symmetric around $\bar{m} - m = 0$, as the expression only depends on the absolute value $|\bar{m} - m|$. Furthermore, as

$$Q(\bar{m}-m) \geq |\bar{m}-m| (1 - F(|\bar{m}-m|)$$

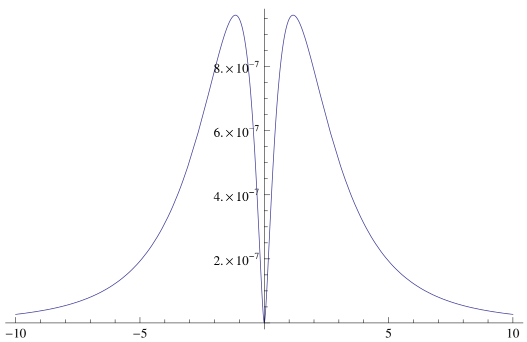


Figure 9: Volatility OTC rate

we can also bound $\sigma_w^2(i_m)$ as follows

$$\sigma_w^2(i_m) \leq \frac{(i_\ell - i_d)^2}{16}.$$

Hence the rates are not going to vary much in this model, even if the shocks are very volatile. For example, if the corridor width is 100 basis points, then the volatility of the rate is at most $\frac{1}{16}$ basis points. Figure shows the volatility of the OTC market rate. Clearly, the volatility of the model with perfect pairing is at odd with the data. We now move to analyzing the model with directed search and imperfect pairing.

4.3 Directed search with imperfect pairing

Now, suppose that the matching function is imperfect, and it just matches a would-be borrower to a would-be lender. In other words, banks decide whether they prefer to lend or to borrow and a bank who wishes to borrow is matched with a bank who wishes to lend. The matching is not purely random, in the sense that if a bank wants to borrow, then it meets a lender bank; however, which lender bank, will be random. More precisely, suppose there is a measure n of borrowers and a measure $1-n$ of lenders. The probability that a borrower meets a lender is then $\theta(n) = \min\{1; \frac{1-n}{n}\}$ and the probability that a lender meets a borrower is $\frac{n}{1-n}\theta(n) = \min\{1; \frac{n}{1-n}\}$. In other words, $\theta(n)$ denotes the interbank market tightness.

Now, we want to solve for the choice of each bank to become a borrower or a lender given the above equilibrium OTC rates. We denote by $V^b(\tilde{m})$ the expected value of being a borrower for a bank holding $\tilde{m}$ units of reserves and by $V^\ell(\tilde{m})$ the expected value of being a lender for the same bank, as it enters the OTC market for reserves.

Once we will have defined those values we will show that there is a threshold $\hat{m}$

such that $V^b(\hat{m}) = V^\ell(\hat{m})$ and for which $V^b(\tilde{m}) > V^\ell(\tilde{m})$ for all $\tilde{m} < \hat{m}$. Therefore all banks with reserves $\tilde{m} < \hat{m}$ will choose to become borrowers and all banks with reserves $\tilde{m} > \hat{m}$ will prefer to become lenders. To simplify matter further, we will assume (and later verify) that all banks exits the ("auction"-like) centralized reserves market with the same amount of reserves balances, equal to the aggregate supply of reserves: m. Therefore, the position of each bank can be summarized by m as well as its idiosyncratic shock v.

In the Appendix, we show the following results.

Proposition 1. *All banks with reserves below $\bar{m}$ choose to become borrowers, while banks with reserves above $\bar{m}$ choose to become lenders. The number of borrowers is $n = F(\bar{m} - m)$. The marginal value of reserves is*

$$\begin{aligned}
\frac{\partial V^b(\tilde{m})}{\partial \tilde{m}} &= 1 + i_\ell - \theta(n)\frac{(i_\ell - i_d)}{2}\frac{1 - F(2\bar{m} - m - \tilde{m})}{1 - F(\bar{m} - m)} \text{ for any } \tilde{m} < \bar{m} \\
\frac{\partial V^\ell(\tilde{m})}{\partial \tilde{m}} &= 1 + i_d + \frac{n}{1-n}\theta(n)\frac{(i_\ell - i_d)}{2}\frac{F(2\bar{m} - m - \tilde{m})}{F(\bar{m} - m)} \text{ for any } \tilde{m} \geq \bar{m}
\end{aligned}$$

It is rather intuitive that the threshold level of reserves to become a borrower or a lender is $\bar{m}$, the amount of required reserves. Indeed, suppose that all banks but one (bank i, say) choose the threshold $\bar{m}$. Suppose bank i has more reserves than $\bar{m}$ ($m_i > \bar{m}$, i.e. it receives a shock $v_i > \bar{m} - m$). If bank i chooses to become borrower, then it will meet a lender bank that has more that $\bar{m}$ (as all banks, but bank i chose to be lenders only if they have more than $\bar{m}$). We know these two banks will trade at the rate $i_m = i_d$. In this case the payoff of bank i is just $(1 + i_d)m_i$, so that bank i makes no gain out of choosing to become borrower. If bank i chooses to become a lender, then it will meet a borrower with some reserves $m_b < \bar{m}$. Looking at (4), the payoff of bank i is always higher than if it chooses to become a borrower. Therefore, given all banks choose a threshold $\bar{m}$, bank i will also choose the threshold $\bar{m}$. The same argument applies when bank i has few reserves, or $m_i < \bar{m}$. Hence, we have shown that the threshold $\bar{m}$ is an equilibrium. However, is there another equilibrium threshold?

Now, suppose that all banks, but bank i, choose a threshold $\hat{m} > \bar{m}$. The same argument as above shows that if bank i has $m_i > \hat{m}$ then it will choose to be a lender. Now, suppose $\hat{m} > m_i > \bar{m}$. In this position, other banks choose to become borrowers. What should bank i do? If it chooses to become a borrower, then it will get payoff $(1 + i_d)m_i$. If it chooses to become a lender, then bank i will meet a borrower with

$m_b < \bar{m} < m_i$ with probability $F(\bar{m} - m)/F(\hat{m} - m)$. In this case bank i gets a higher payoff than $m_i(1 + i_d)$, as shown by (4). In case bank i meets a borrower with $\bar{m} < m_b < \hat{m}$ then bank i's payoff is again $m_i(1 + i_d)$. Therefore, bank i has a higher payoff from becoming a lender whenever $\hat{m} > m_i > \bar{m}$, although all other banks in this position choose to be borrower. Therefore, $\hat{m} > \bar{m}$ is not an equilibrium. A similar argument applies when $\hat{m} < \bar{m}$. Hence the unique equilibrium is one where the threshold is $\bar{m}$.

It then easily follows that the measure of borrowers is given by the measure of banks receiving a low shock, i.e. v such that $m + v < \bar{m}$. Hence, the measure of borrowers is $n = F(\bar{m} - m)$. Finally, the marginal values of reserves are easily interpreted. The marginal value of holding an additional unit of reserves for a borrower is the rate that it won't have to pay at the lending facility $1 + i_\ell$, minus the expected benefit of acquiring this unit in the OTC market. The expected benefits naturally depends on the market tightness $\theta(n)$, as well as the probability to meet a lender who has enough reserves so that both banks can satisfy their reserve requirements. This is similar for lenders.

Relative to the Poole (1968) model where banks cannot access any OTC market, it is useful to notice that the OTC market is making reserves less valuable for borrowers and more valuable for lenders. The reason is that the OTC market is one way for banks to smooth their reserves holdings which is not present in the standard Poole model.

Proposition 1 is crucial in understanding the bidding behavior of banks in the centralized auction like market. In particular, banks will tend to be more responsive to a rate change as they can "insure" themselves somewhat on the OTC market. We study this next.

4.3.1 Central – "Auction"-like – market rate

In the very first period of the economy, banks have the opportunity to purchase reserves at the central bank. We assume that the central bank supplies m units of reserves. If $m = \bar{m}$ we say that the central bank has a neutral liquidity policy, while if $m > \bar{m}$ the central bank has an aggregate liquidity surplus policy. Given our previous results we can compute the willingness to pay of each bank when the central bank supplies m units of reserves. Since all banks face the same type of uncertainty, they all have the same value for reserves and so we assume that they behave in the same way.

To find each bank's willingness to pay, we need to define the value of exiting this centralized stage with reserves m. This is

$$
\begin{aligned}
W(m) &= \int_{-\bar{v}}^{\bar{v}} \max\left[V^b(m+v), V^\ell(m+v)\right] dF(v) \\
&= \int_{-\bar{v}}^{\bar{m}-m} V^b(m+v)dF(v) + \int_{\bar{m}-m}^{\bar{v}} V^\ell(m+v)dF(v)
\end{aligned}
$$

as banks will choose to become borrowers when they receive a sufficiently low shock and lenders when they receive a high enough shock. The interest rate on the auction market will be given by the marginal value of reserves, or $W'(m)$. Using Leibniz' rule as well as (22) and (25) we find that $W'(m)$ is

$$
\begin{aligned}
W'(m) &= F(\bar{m}-m)(1+i_\ell) + (1-F(\bar{m}-m))(1+i_d) \\
&\quad -\theta(n)(i_\ell - i_d)\left[\int_{-\bar{v}}^{\bar{m}-m} \left[1-F(2(\bar{m}-m)-v)\right] \frac{dF(v)}{1-F(\bar{m}-m)}\right] \\
&\quad +\frac{n}{1-n}\theta(n)(i_\ell - i_d)\int_{\bar{m}-m}^{\bar{v}} F(2(\bar{m}-m)-v)\frac{dF(v)}{F(\bar{m}-m)}
\end{aligned} \tag{9}
$$

The first term is the same as in Poole (1968). The second term, however, is new and refers to the likelihood of being able to share one's balances on the OTC market. To understand the second term, notice that $1-F(2(\bar{m}-m)-v)$ is the probability that, when a bank holds $m+v$ reserves, this bank meets another bank holding enough reserves for them both to meet the reserve requirements. Therefore, $1-F(2(\bar{m}-m)-v)$ is the probability that a borrower bank holding $m+v<\bar{m}$ reserves and in an OTC meeting meets the reserve requirements at the end of the day. So the first term in bracket is the probability that a bank gets a sufficiently high shock v that, by borrowing on the interbank market, would still meet its reserve requirements: This reduces the bank's willingness to pay for extra reserves by the cost of reserves in such a meeting, or $i_\ell - i_d$. Similarly, when a bank turns out to be a lender, it might have to lend the extra reserves to the borrower making a gain of $i_\ell - i_d$ on it then. This is the third term in the above expression. Notice that in the case of neutral liquidity stance (and symmetric liquidity shock), where $\bar{m}=m$, these two extra terms (due to the OTC market) vanish, and we are back to the basic Poole's model. Using $n=F(\bar{m}-m)$, we can simplify (9) as

$$
W'(m) = n(1+i_\ell) + (1-n)(1+i_d) - \frac{\theta(n)}{1-n}(i_\ell - i_d)\left[n - \int_{-\bar{v}}^{\bar{v}} F(2(\bar{m}-m)-v)dF(v)\right]
$$

Figure 10 below plots the rates for the Poole's model in green and the one for our

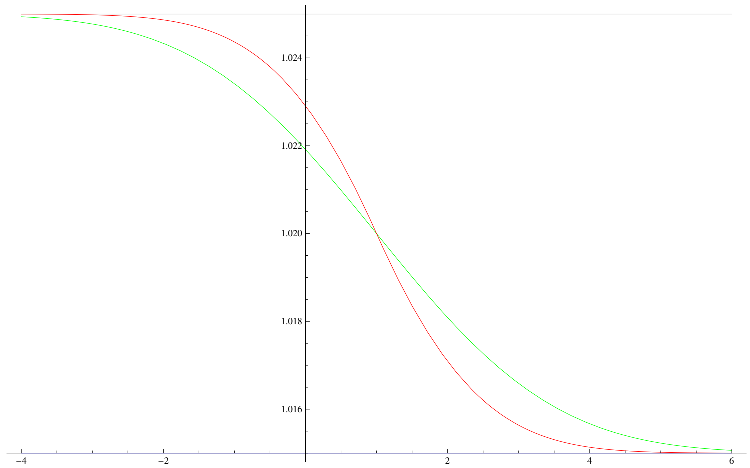


Figure 10: Willingness to pay for reserves (Green: Poole 1968's model; Red: this model).

benchmark model in red, for required reserves of $\bar{m} = 1$ and aggregate reserves of m varying from -5 to $+5$, where v is distributed according to a Normal with mean 0 and variance 2. We used $1 + i_\ell = 1.025$ and $1 + i_d = 1.015$.

4.3.2 OTC volume, weighted average rate, and rate volatility

Market volume Using the model, we can now describe how the aggregate liquidity deficit $\bar{m} - m > 0$ impacts the volume of trade on the OTC interbank market. The aggregate trade volume is given by the total size of trades $\tilde{Q}$ times the number of matches. Given our matching function we obtain that the aggregate trade volume is $Q(\bar{m} - m) = \min\{n, 1-n\}\tilde{Q}(\bar{m} - m)$ where $\tilde{Q}$ is the average trade size *when a trade occurs*,

$$\tilde{Q}(\bar{m} - m) = \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} q(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b)$$

where F_ℓ is the distribution of the lenders' reserves in the OTC market and, similarly, F_b is the distribution of the borrowers' reserves on the OTC market. Defining Q, we have already taken into account that, in equilibrium, borrowers always hold less than $\bar{m}$ units of reserves, while lenders always hold more than $\bar{m}$. Also, we know that in equilibrium borrowers hold $m_b = m + v_b$ for some $v_b \leq \bar{m} - m$ while lenders hold $m_\ell = m + v_\ell$ for some $v_\ell \geq \bar{m} - m$. Using (1) we have the following result,

Proposition 2. *Suppose v has a zero mean and is distributed symmetrically around its mean. Then $Q(\bar{m} - m) \geq 0$ for all values of $\bar{m} - m$. Also the aggregate trading volume $Q(\bar{m} - m)$ is single-picked and attain its unique maximum at $\bar{m} = m$. Also $\lim_{\bar{m}-m \to \bar{v}} Q(\bar{m} - m) = \lim_{\bar{m}-m \to -\bar{v}} Q(\bar{m} - m) = 0$, i.e. aggregate volume decreases to zero as the liquidity deficit or surplus become large.*

Under the assumption of Proposition , we can simplify Q as

$$Q(x) = \frac{\min\{F(x), 1 - F(x)\}}{2F(x)[1 - F(x)]} \int_{-\bar{v}}^{x} (-v) dF(v), \tag{10}$$

where x measures the liquidity deficit. In general, we can show that $\lim_{\bar{m}-m \to \bar{v}} Q(\bar{m} - m) = \lim_{\bar{m}-m \to -\bar{v}} Q(\bar{m} - m) = 0$. Figure 11 plots the aggregate volume on the interbank market as a function of x, for the same parameters as before.

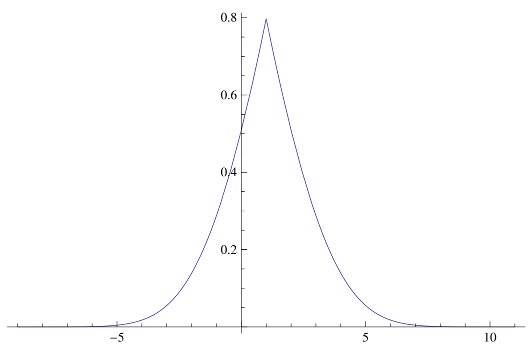


Figure 11: Volume on the interbank market as a function of $\bar{m} - m$.

Average rate Once we have the interbank market volume, it is straightforward to compute the weighted average rate in the OTC market. This is $\bar{i}_m$ which satisfies

$$\bar{i}_m = \min\{F(\bar{m} - m), 1 - F(\bar{m} - m)\} \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q(m_b, m_\ell)}{Q(\bar{m} - m)} i_m(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b)$$

which we can simplify as

$$\bar{i}_m = \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q(m_b, m_\ell)}{\tilde{Q}(\bar{m} - m)} i_m(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b)$$

and using (1), (10) combined with (2) as well as $E[v]=0$, we can simplify this expression further to obtain (the details are in the Appendix):

$$\bar{i_m} = (1-F(\bar{m}-m))i_\ell + F(\bar{m}-m)i_d + \frac{(\bar{m}-m)}{\tilde{Q}(\bar{m}-m)}\frac{(i_\ell - i_d)}{2}$$

The last component of $\bar{i_m}$ is the average gains from trading in the OTC market. The first and second components do not have the "usual" Poole's weights. To understand why, notice that the weights have a natural interpretation. The first weight $1-F(\bar{m}-m)$ is half of the average liquidity shock of a borrower relative to the aggregate trading volume, and the reservation value on these shocks is i_ℓ. Also, $F(\bar{m}-m)$ is half of the average liquidity shock of a lender relative to the aggregate trading volume, and the reservation value on these trades is i_d. [14] Therefore $\bar{i_m}$ is the average of the facility rates weighted by the relative size of the borrowers' and lenders' shocks and adjusted for the gains from trade. With neutral liquidity conditions, $\bar{m}=m$, the interbank rate is at the mid-corridor point. As a function of the liquidity deficit x, the interbank rate is

$$\bar{i_m}(x) = (1-F(x))i_\ell + F(x)i_d + \frac{x}{\tilde{Q}(x)}\frac{(i_\ell - i_d)}{2}$$

and it's derivative

$$\bar{i_m}'(x) = f(x)(i_d - i_\ell) + \frac{(i_\ell - i_d)}{2}\frac{\tilde{Q}(x) - x\tilde{Q}'(x)}{\tilde{Q}(x)^2}$$

We cannot sign $\bar{i}'_m(x)$ as the first term is always negative. On the one hand, increasing the liquidity deficit will tend to increase the interbank market rate, as the average liquidity shock of a borrower relative to the aggregate volume of trade decreases. On the other, this effect is tampered by the change in the surplus from trade. For a normal distribution we obtain $\bar{i_m}'(x) > 0$ for all x as shown in the graph below (same parameters as before, magenta curve shows $\bar{i}_m(\bar{m}-m)$ where m in on the x-axis relative to the

[14] That is,

$$1-F(\bar{m}-m) = \frac{1}{2Q(\bar{m}-m)}\int_{-\bar{v}}^{\bar{m}-m} -v\frac{dF(v)}{F(\bar{m}-m)}$$

while

$$F(\bar{m}-m) = \frac{1}{2Q(\bar{m}-m)}\int_{\bar{m}-m}^{\bar{v}} v\frac{dF(v)}{F(\bar{m}-m)} = \frac{1}{2Q(\bar{m}-m)}\left\{\frac{E[v]}{F(\bar{m}-m)} - \int_{-\bar{v}}^{\bar{m}-m} v\frac{dF(v)}{F(\bar{m}-m)}\right\}$$

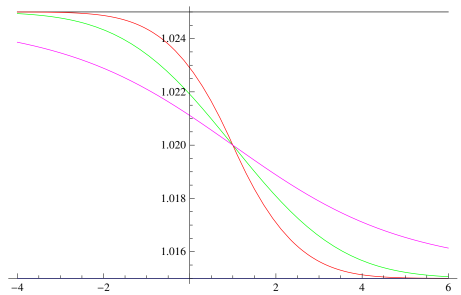


Figure 12: Interbank rate

usual Poole's rate - green - and the auction rate - red).

Volatility We now compute the rate volatility, using the weighted variance,

$$\sigma_w^2(i_m) = \int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{q(v_\ell, v_b)}{\tilde{Q}(\bar{m}, m)} [i_m(v_\ell, v_b) - \bar{i_m}]^2 \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)},$$

and in the Appendix, we show that whenever $c = F(\bar{m} - m) - \frac{(\bar{m}-m)}{2\tilde{Q}(\bar{m}-m)} \geq 0$ (which is always the case when $m > \bar{m}$), then

$$\sigma_w^2(i_m) \leq (i_\ell - i_d)^2 c(1 + c).$$

In particular, with neutral provision of liquidity, the volatility is less than $\frac{3}{4}(i_\ell - i_d)^2$. Therefore, if the corridor width is 100 basis points, then the volatility is less than 0.75 basis point.

To find a lower bound for the OTC rate volatility, we restrict our attention to the case where there is a liquidity surplus, or $m > \bar{m}$. In this case, we show in the Appendix that

$$\sigma_w^2(i_m) \geq (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{(v_\ell - v_b)}{2\tilde{Q}} \left(\frac{v_\ell - \bar{m} - m}{v_\ell - v_b} \right)^2 \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)} - 2c + c^2 \right]$$

Notice that the first term in bracket is positive but less than unity.[15] From there it follows that $\sigma_w^2(i_m) \geq (i_\ell - i_d)^2 c(c - 2)$. However, this may be a loose bound if $c < 2$. Figure 13 plots the standard deviation of OTC rates whenever the shocks are normally distributed (left panel) or uniformly distributed (right panel).

[15] i.e. $\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{(v_\ell - v_b)}{2\tilde{Q}} \left(\frac{v_\ell - \bar{m} - m}{v_\ell - v_b} \right)^2 \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)} \leq 1.$

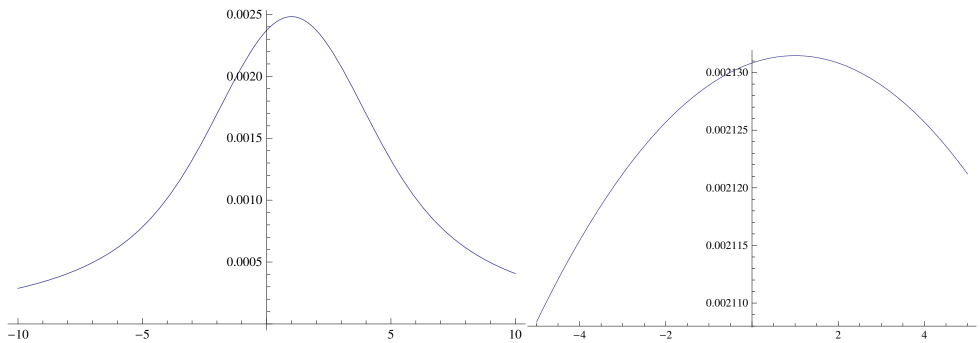


Figure 13: OTC rates volatility (standard deviation) (left: normal shocks; right: uniform)

5 Counterparty risk

We now introduce the probability that banks default. To introduce counterparty risk, we assume that each bank knows that it can disappear with probability $\delta \in (0,1)$, whether they are buyers or sellers. We assume that the central bank still charge i_ℓ when a bank accesses its lending facility, irrespective of δ.

Again, as banks are risk neutral, we assume that, *if they trade*, banks equate their reserves holdings so that (1) will hold, even with default. Lenders will adjust the rate they charge in order to compensate for the expected loss from a default and we use i_m^δ to denote the rate they charge when there is counterparty risk. In the Appendix, we show that the OTC rate as a function of (m_ℓ, m_b) are defined as follows

$$i_m^\delta(m_\ell, m_b) = i_m(m_\ell, m_b) + \frac{\delta}{(2-\delta)}\left[1 + i_m(m_\ell, m_b)\right] \tag{11}$$

where we have defined $i_m(m_\ell, m_b)$ in (2). Hence, the probability of default introduces a risk premium that is proportional to the risk-free rate.[16] Notice that when δ is high, we will get $i_m^\delta(m_\ell, m_b) > i_\ell$ in which case the borrower will prefer to borrow from the lending facility. In other words, banks will not trade – i.e. $q^\delta(m_\ell, m_b) = 0$ – whenever

[16]The reason why the risk premium is not increasing to infinity as $\delta \to 1$ is that lenders are also disappearing with almost certainty in that case, so they don't really care to be reimbursed or not.

$i_m^\delta \geq i_\ell$ or, equivalently, whenever[17]

$$1 + i_m(m_\ell, m_b) \geq \left(1 - \frac{\delta}{2}\right)(1 + i_\ell)\,. \tag{12}$$

Using (2) and (11), $q^\delta(m_\ell, m_b)$ is defined as

$$q^\delta(m_\ell, m_b) = \begin{cases} 0 & \text{if } m_b < m_\ell < \bar{m} \\ 0 & \text{if } m_b < \bar{m} < m_\ell \text{ and } 1 + i_m \geq \left(1 - \frac{\delta}{2}\right)(1 + i_\ell) \\ 0 & \text{if } \bar{m} < m_b < m_\ell \\ \frac{m_\ell - m_b}{2} & \text{otherwise} \end{cases}$$

Notice that trade collapses when both banks have enough reserves to satisfy their reserves requirement: in this case the borrower would have to pay the risk premium while only getting a benefit of i_d on the amount borrowed. Therefore the potential borrower would not want to borrow in this case. Using (2) and arranging, we can rewrite the *no trade condition* (12) when $m_b < \bar{m} < m_\ell$ as

$$(\Delta - 1)(m_\ell - \bar{m}) \leq \bar{m} - m_b, \tag{13}$$

where

$$\Delta = \frac{2(i_\ell - i_d)}{\delta(1 + i_\ell)}. \tag{14}$$

Therefore, trading will collapse when the corridor shrinks to zero, as we have assumed $m_\ell > \bar{m} > m_b$. Now, when $\bar{m} < m_b < m_\ell$, we can rewrite (12) as $\delta(1 + i_\ell) > 2(i_\ell - i_d)$. Since $i_m \geq i_d$, notice that if this last inequality holds then banks will never trade, under no circumstances, as (13) is also trivially satisfied. Therefore, banks never trade if the corridor is small relative to δ. This is our first result regarding counterparty risk.

Lemma 1. *There is no trade on the interbank market if* $\delta \geq \frac{2(i_\ell - i_d)}{1 + i_\ell}$.

Therefore, if the corridor becomes very small while the default rate increases (a situation that occurred during the recent crisis), then the interbank market may well

[17] We have used (1) to compute the rates (11). Technically, there could still be some trade $q^\delta < \frac{m_\ell - m_b}{2}$ at the rate $i^\delta = i_\ell$, when (12) is satisfied. However, there is no surplus from trade in this case, as the borrower is indifferent between borrowing $q^\delta < q$ from the lender at $i^\delta = i_\ell$ and accessing the lending facility. Therefore, the borrower's surplus is nil. Since the borrower and the lender equate their surplus, the lender's payoff is also nil. Therefore, we chose to ignore those trades by setting $q^\delta = 0$ in those cases.

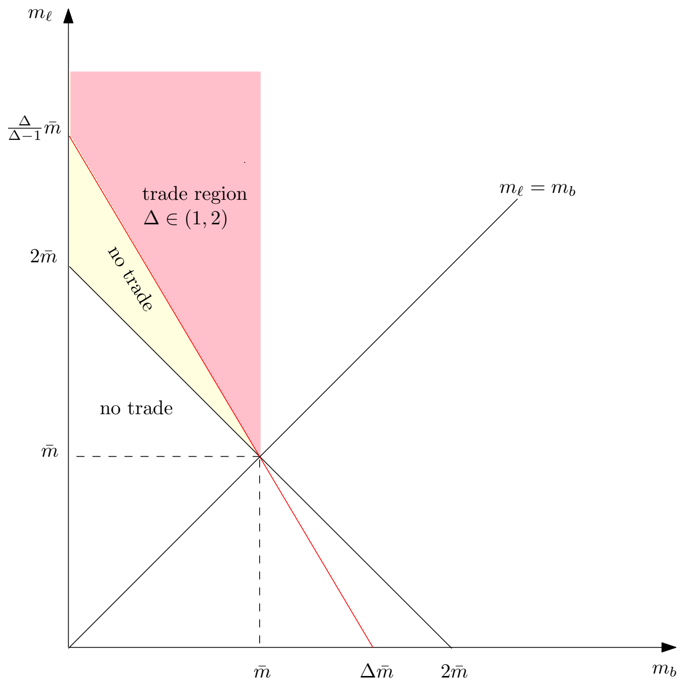


Figure 14: Trade region with high counterparty risk $\delta \geq \frac{i_\ell - i_d}{1+i_\ell}$

cease to function altogether.

To make the analysis a little more interesting, we will assume from now that $\Delta > 1$, so that trade would occur if the "conditions are right," (i.e. if (13) does not hold). When there is a risk of default and when $\bar{m} > m_b$, the *trading* condition is

$$m_\ell - \bar{m} > \frac{\bar{m} - m_b}{\Delta - 1}. \tag{15}$$

In particular, if $\Delta \leq 2$ then the right hand side of (15) is always greater than $\bar{m} - m_b$. Therefore, if there is sufficient counterparty risk (i.e. $\Delta \leq 2$) and $\bar{m} > m_b$ and when there is trade, then we are necessarily in the case where $m_\ell + m_b > 2\bar{m}$. In words, with high counterparty risk, lenders only lend when they are sure to have enough reserves, even after they extend a loan. Figure 14 shows the regions of pairs (m_ℓ, m_b) where there is trade in the case of high counterparty risk. The "no trade" regions are the ones where banks do not trade because of counterparty risk. As the figure illustrates, with a high risk of default, banks will not trade, even though there is sufficient reserves within the pair to cover the reserves requirements of both banks. As Δ decreases to 1 the red curve becomes a straight vertical line at $m_b = \bar{m}$ and trade collapses.

With little counterparty risk, i.e. $\delta < \frac{i_\ell - i_d}{1+i_\ell}$, Figure 15 shows that banks do not trade

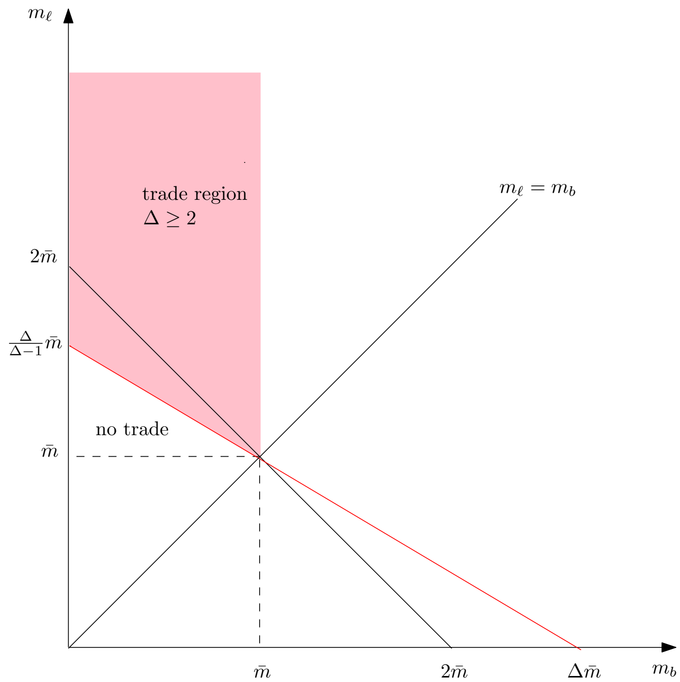


Figure 15: Trade region with low counterparty risk $\delta < \frac{i_\ell - i_d}{1+i_\ell}$

only if the lender bank cannot sufficiently cover the reserves needs of the borrower. In this case, when $\bar{m} > m_b$ and when there is trade, then we are necessarily in the case where $m^*(\Delta) < m_\ell + m_b \leq 2\bar{m}$, for some $m^*(\Delta)$. Figure 15 shows the regions of pairs (m_ℓ, m_b) where there is trade. As Δ increases to infinity (or δ decreases to zero), the red curve becomes a straight horizontal line at $m_\ell = \bar{m}$ and trade converges to the no-risk case.

We can use (1) and (2) to obtain the payoff of the borrower and the lender banks. Denoting the payoff of a borrower bank holding m_b and meeting a lender bank holding m_ℓ by $P^b(m_\ell, m_b; \delta)$, we have (using the fact that there is no trade when $m_b + m_\ell < 2\bar{m}$),

$$\frac{P^b(m_\ell, m_b; \delta)}{1-\delta} = \begin{cases} (1+i_d)m_b & \text{if } m_\ell > m_b > \bar{m} \\ P^b(m_\ell, m_b; 0) - \frac{\delta}{2-\delta}(1+i_m)q & \text{if } \begin{cases} m_b < \bar{m} < m_\ell & \text{and} \\ m_\ell - \bar{m} > \frac{\bar{m}-m_b}{\Delta-1} \end{cases} \\ (1+i_\ell)(m_b - \bar{m}) + (1+i_d)\bar{m} & \text{if } \begin{cases} m_b < \bar{m} < m_\ell & \text{and} \\ m_\ell - \bar{m} \leq \frac{\bar{m}-m_b}{\Delta-1} \end{cases} \\ (1+i_\ell)(m_b - \bar{m}) + (1+i_d)\bar{m} & \text{if } m_b < m_\ell < \bar{m} \end{cases} \tag{16}$$

and the payoff of the lender bank is

$$\frac{P^{\ell}(m_\ell, m_b; \delta)}{1-\delta} = \begin{cases} (1+i_d)m_\ell & \text{if } m_\ell > m_b > \bar{m} \\ P^{\ell}(m_\ell, m_b; 0) + \frac{\delta}{2-\delta}(1+i_m)q & \text{if } \begin{cases} m_b < \bar{m} < m_\ell & \text{and} \\ m_\ell - \bar{m} > \frac{\bar{m}-m_b}{\Delta-1} \end{cases} \\ (1+i_d)m_\ell & \text{if } \begin{cases} m_b < \bar{m} < m_\ell & \text{and} \\ m_\ell - \bar{m} \leq \frac{\bar{m}-m_b}{\Delta-1} \end{cases} \\ (1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} & \text{if } m_b < m_\ell < \bar{m} \end{cases} \tag{17}$$

where i_m is given by (2) in the specified region and q is defined in (1). Notice that, *when there trade*, counterparty risk increases (resp. decreases) the payoff of the lender (resp. borrower) by the expected loss in case default occurs.

Assuming that banks choose to become lenders whenever $m + v \geq \hat{m}$ for some $\hat{m}$ (and borrowers otherwise), we can now compute the value of becoming a borrower (resp. a lender). For a borrower, if $\hat{m} > \tilde{m} > \bar{m}$ (i.e. the threshold is above $\bar{m}$ and the borrower has more reserves than necessary to achieve the requirement) then $V^b(\tilde{m}; \delta) = (1-\delta)(1+i_d)\bar{m}$. If $\tilde{m} < \bar{m}$ then

$$\begin{aligned} \frac{V^b(\tilde{m};\delta)}{1-\delta} &= V^b(\tilde{m};0) - \theta(n) \int_{\bar{m}+\frac{\bar{m}-\tilde{m}}{(\Delta-1)}}^{m+\bar{v}} \frac{\delta}{2-\delta}(1+i_m)q^{\delta}(m_\ell, \tilde{m}) dF_\ell(m_\ell) \\ & \quad -\theta(n) \int_{\hat{m}}^{\bar{m}+\frac{\bar{m}-\tilde{m}}{(\Delta-1)}} \left\{ P^b(m_\ell, \tilde{m}; 0) - [(1+i_\ell)(\tilde{m}-\bar{m}) + (1+i_d)\bar{m}] \right\} dF_\ell(m_\ell) \end{aligned}$$

where $F_\ell(m)$ is the distribution of lenders' money holdings. There are two effects of counterparty risk for the value of becoming a borrower: First the borrower has to pay a risk premium whenever he can borrow, and second he will not be able to borrow from some lenders (while he would have borrowed in the absence of counterparty risk). Therefore, the value of holding some reserves is the value without counterparty risk, minus the risk premium when the borrower trades, and minus the loss of payoff when the borrower rejects a trade that is too expensive. Notice that $V^b(\bar{m}; \delta)$ when $\hat{m} = \bar{m}$ is simply equal to

$$V^b(\bar{m};\delta)|_{\hat{m}=\bar{m}} = (1+i_d)\bar{m}$$

as we use the fact that $q(m_\ell, \bar{m}; \delta) = 0$ in this case, and from (3) we have $P^b(m_\ell, \bar{m}; 0) =$

$(1+i_d)\bar{m}$ in the region of interest.

We can now turn to lenders. The value of becoming a lender when $\hat{m} < m_\ell < \bar{m}$ is $V^\ell(m_\ell;\delta) = (1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m}$. Otherwise, the value is given by

$$\begin{aligned}\frac{V^\ell(\tilde{m};\delta)}{1-\delta} &= V^\ell(\tilde{m};0) + \frac{n}{1-n}\theta(n)\int_{\bar{m}-(\Delta-1)(\tilde{m}-\bar{m})}^{\hat{m}} \frac{\delta}{2-\delta}(1+i_m)q^\delta(\tilde{m},m_b)dF_b(m_b) \\ &\quad -\frac{n}{1-n}\theta(n)\int_{m-\bar{v}}^{\bar{m}-(\Delta-1)(\tilde{m}-\bar{m})} \left[P^\ell(\tilde{m},m_b;0) - (1+i_d)\tilde{m}\right] dF_b(m_b).\end{aligned}$$

where $F_b(m)$ is the distribution of borrowers' money holdings. Counterparty risk is increasing the value of reserves for a lender by the risk premium, but it also makes it smaller as lenders cannot trade as often. Again notice that $V^\ell(\bar{m};\delta)$ when $\hat{m} = \bar{m}$ is simply equal to

$$V^\ell(\bar{m};\delta)|_{\hat{m}=\bar{m}} = (1+i_d)\bar{m}$$

as we use the fact $F_b(\tilde{m}) = \frac{F(\tilde{m}-m)}{F(\hat{m}-m)}$ when the threshold is $\hat{m}$. Therefore, we again have the following result regarding the choice to become a borrower or a lender,

Proposition 3. *With counterparty risk, all banks with reserves below $\bar{m}$ choose to become borrowers, while banks with reserves above $\bar{m}$ choose to become lenders. The number of borrowers is $n = F(\bar{m} - m)$.*

Notice that counterparty risk increases the value of cash for the borrower, because (1) the borrower pays a higher rate on every borrowed reserves, and (2) it increases the likelihood to get a loan (i. e. to fall in the region where the lender is willing to extend a loan at a rate lower than i_ℓ). Relative to the case with no risk, the effect of counterparty risk on lenders is unclear: While they would extract more resources from borrowers if they can lend, lending becomes more difficult, as some borrowers will not borrow at a rate above i_ℓ. For the sake of completeness, we compute the marginal value of reserves with counterparty risk in the Appendix. We find that for all $\delta > 0$,

$$\frac{\partial V^b(\tilde{m};\delta)}{\partial\tilde{m}} > (1-\delta)\frac{\partial V^b(\tilde{m};0)}{\partial\tilde{m}}$$

while it is not possible to say if $\frac{\partial V^\ell(\tilde{m};\delta)}{\partial\tilde{m}}$ is larger or smaller than $(1-\delta)\frac{\partial V^\ell(\tilde{m};0)}{\partial\tilde{m}}$.

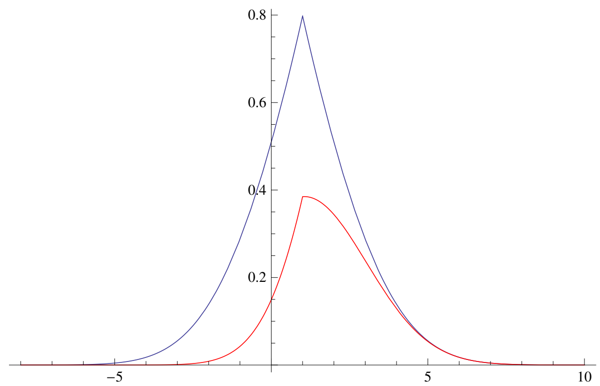


Figure 16: Effect of 1% default probability on the trading volume (red curve)

5.1 Market volume with counterparty risk

Using the model, we can now describe how the aggregate liquidity deficit $\bar{m} - m > 0$ impacts the volume of trade on the OTC interbank market. The aggregate trade volume is given by the total size of trades $\tilde{Q}$ times the number of matches. Given our matching function we obtain that the aggregate trade volume is $Q(\bar{m} - m; \delta) = \min\{n, 1-n\}\tilde{Q}(\bar{m} - m; \delta)$ where $\tilde{Q}$ is the average trade size *when a trade occurs*. Since there is only trading when (15) is satisfied, $\tilde{Q}$ is

$$\tilde{Q}(\bar{m} - m; \delta) = \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}+\frac{\bar{m}-m_b}{\Delta - 1}}^{m+\bar{v}} q(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b)$$

and since $\bar{m} \geq m_b$ and $\Delta'(\delta) < 0$ we obtain naturally that counterparty risk decreases the market volume for all liquidity conditions,

$$\frac{\partial \tilde{Q}(\bar{m} - m; \delta)}{\partial \delta} < 0.$$

Also, notice that the decline in market volume is even more pronounced as liquidity is scarce, i.e. $\bar{m} - m_b = \bar{m} - m - v_b > 0$ is large.

The following figure shows the aggregate trading volume with no risk ($\delta = 0$ in blue) and with $\delta = 1\%$ (red) as shown a little counterparty risk has a large impact on the trading volume when the liquidity conditions are neutral, while the impact is diminished when there is ample liquidity (recall that m is on the $x-$axis).

5.2 Weighted average OTC rates with counterparty risk

$$i_m^\delta(m_\ell, m_b) = i_m(m_\ell, m_b) + \frac{\delta}{(2-\delta)}\left[1 + i_m(m_\ell, m_b)\right]$$

This is $i_m^{\bar{\delta}}$ which satisfies

$$i_m^{\bar{\delta}} = \min\{F(\bar{m}-m), 1-F(\bar{m}-m)\} \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q^\delta(m_b, m_\ell; \delta)}{Q(\bar{m}-m; \delta)} i_m^\delta(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b).$$

Since there is only trading when (15) is satisfied, we can simplify the weighted mean as

$$i_m^{\bar{\delta}} \;=\; \frac{\delta}{2-\delta} + \frac{2}{2-\delta} \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}+\frac{\bar{m}-m_b}{\Delta-1}}^{m+\bar{v}} \frac{m_\ell - m_b}{2\tilde{Q}(\bar{m}-m; \delta)} i_m(m_\ell, m_b) dF_\ell(m_\ell) dF_b(m_b)$$

This expression highlights two the counteracting effects of counterparty risk on the weighted average OTC rates: The direct effect of counterparty risk is to increase all the rates as we have seen. The indirect (negative) effect works through the weights: All trades where m_ℓ is relatively small are not taking place. As a result, the aggregate volume is decreasing, but also more weight is given to those trade with relative large m_ℓ. However, these are the trades with relatively lower rates. As a result, the integral is smaller with counterparty risk than without. The overall effect of counterparty risk on the weighted rate therefore depends (once again) on the distribution of shocks. With relatively large shocks, the average weighted OTC rate will be larger with counterparty risk.

5.3 Volatility of OTC rates with counterparty risk

We now compute the rate volatility, using the weighted variance,

$$\sigma_w^2(i_m) = \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q^\delta(m_\ell, m_b)}{\tilde{Q}(\bar{m}, m; \delta)} [i_m^\delta(m_\ell, m_b) - i_m^{\bar{\delta}}]^2 dF(m_\ell) dF(m_b),$$

which can be simplified as

$$\begin{aligned}
\sigma_w^2(i_m) &= \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q^\delta(m_\ell, m_b)}{\tilde{Q}(\bar{m}, m; \delta)} [i_m^\delta(m_\ell, m_b) - i_m^{\bar{\delta}}]^2 dF(m_\ell) dF(m_b), \\
&= \left(\frac{2}{2-\delta}\right)^2 \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}+\frac{\bar{m}-m_b}{\Delta-1}}^{m+\bar{v}} \frac{m_\ell - m_b}{\tilde{Q}(\bar{m}, m; \delta)} \left[\frac{\delta}{2-\delta} + i_m(m_\ell, m_b) - i_m^{\bar{\delta}}\right]^2 dF(m_\ell) dF(m_b)
\end{aligned}$$

Can we say $\sigma_w^2(i_m)$ increases with δ?

6 Conclusion

We presented a model of the interbank market with directed search. We showed that this framework is able to replicate basic statistics of the interbank market, both in normal and exceptional circumstances. It has the ability to match the data on volume, rates and volatility, even in the presence of counterparty risk. We showed that counterparty risk decreases aggregate trading volume and increases rates, but also that a little risk can go a long way in paralyzing the interbank market. Our approach also allows us to contrast our result from the workhorse model of Poole (1968) where volume, volatility, and counterparty risk are absent.

Of course we made some strong assumptions....

A main contribution of our paper is to show that the best model of the interbank market is something like this: banks choose whether they want to borrow or lend, and then they pick a borrower/lender at random. Purely random search or perfect pairing would not match the data. In this sense, there seems to be some leeway to improve the functioning of the money market. Then, an important question is why do banks choose not to use brokers, or in the case they do, why are brokers not more efficient in matching banks?

Appendix

OTC rates

In this section we show that the OTC rates are given by (2).

There are four types of trades that we have to consider: (1) meetings where $m_b + m_\ell \geq 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell > \bar{m}$, (2) meetings where $m_b + m_\ell \geq 2\bar{m}$, $m_b > \bar{m}$, and $m_\ell > \bar{m}$, (3) meetings where $m_b + m_\ell < 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell > \bar{m}$, and (4) meetings where $m_b + m_\ell < 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell < \bar{m}$. In meetings (1)-(2) the pair of banks has enough reserves for each bank to satisfy their reserve requirements $\bar{m}$. In meetings (3)-(4) however, there is not enough reserves in the pair for both banks to satisfy their reserve requirements.

We now solve the rate in meeting (1). Since the rates are given by equalizing surplus from trade, rates in meetings (1) will have to satisfy

$$\begin{aligned}(m_b + q - \bar{m})(1+i_d) + \bar{m}(1+i_d) - q(1+i_m) - [-(\bar{m} - m_b)(1+i_\ell) + \bar{m}(1+i_d)] &= \\ (m_\ell - q - \bar{m})(1+i_d) + \bar{m}(1+i_d) + q(1+i_m) - [(m_\ell - \bar{m})(1+i_d) + \bar{m}(1+i_d)]\end{aligned}$$

Since there are enough reserves for both banks in the pair to satisfy their reserve requirements, both banks exit the interbank market with excess reserves. The borrower exits with $m_b + q - \bar{m}$ excess reserves and the lender exits with $m_\ell - q - \bar{m}$. Since excess reserves are remunerated at the rate i_d we get the first term in both surplus. The other terms read in the same way. Notice the difference between the outside option of the borrower and the lender: while the borrower would have to borrow $\bar{m} - m_b$, the lender could deposit $m_\ell - \bar{m}$. Simplifying using (1) we obtain

$$1 + i_m = \frac{m_\ell - \bar{m}}{m_\ell - m_b}(1+i_d) + \frac{\bar{m} - m_b}{m_\ell - m_b}(1+i_\ell).$$

We now solve for the rate in meeting (2). Since the rates are given by equalizing surplus from trade, rates in meetings (2) will have to satisfy

$$\begin{aligned}(m_b + q - \bar{m})(1+i_d) + \bar{m}(1+i_d) - q(1+i_m) - [(m_b - \bar{m})(1+i_d) + \bar{m}(1+i_d)] &= \\ (m_\ell - q - \bar{m})(1+i_d) + \bar{m}(1+i_d) + q(1+i_m) - [(m_\ell - \bar{m})(1+i_d) + \bar{m}(1+i_d)]\end{aligned}$$

The only difference with case (1) is in the outside option of the borrower: in case (2), even if the borrower does not borrow, he still has excess reserves. Simplifying, we obtain

$$1 + i_m = 1 + i_d.$$

We now solve for the rate in meeting (3). In this case, both the borrower and the lender have to borrow at the lending facility after they exit the interbank market. Since the rates are given by equalizing surplus from trade, rates in meetings (3) will have to satisfy

$$\begin{aligned}-(\bar{m} - (m_b + q))(1+i_\ell) + \bar{m}(1+i_d) - q(1+i_m) - [-(\bar{m} - m_b)(1+i_\ell) + \bar{m}(1+i_d)] &= \\ -(\bar{m} - (m_\ell - q))(1+i_\ell) + \bar{m}(1+i_d) + q(1+i_m) - [(m_\ell - \bar{m})(1+i_d) + \bar{m}(1+i_d)]\end{aligned}$$

Simplifying, we obtain

$$1+i_m=\frac{m_\ell-\bar{m}}{m_\ell-m_b}(1+i_d)+\frac{\bar{m}-m_b}{m_\ell-m_b}(1+i_\ell).$$

Finally, we now solve for the rate in meeting (4). In this case, both the borrower and the lender have to borrow at the lending facility after they exit the interbank market, whether or not they traded on the interbank market. Since the rates are given by equalizing surplus from trade, rates in meetings (4) will have to satisfy

$$\begin{aligned}-(\bar{m}-(m_b+q))(1+i_\ell)+\bar{m}(1+i_d)-q(1+i_m)-[-(\bar{m}-m_b)(1+i_\ell)+\bar{m}(1+i_d)] &= \\ -(\bar{m}-(m_\ell-q))(1+i_\ell)+\bar{m}(1+i_d)+q(1+i_m)-[-(\bar{m}-m_\ell)(1+i_\ell)+\bar{m}(1+i_d)]\end{aligned}$$

Simplifying, we obtain

$$1+i_m=1+i_\ell.$$

Marginal value of reserves perfect pairing

Using (8) we obtain

$$W(\tilde{m})=I+\int_{-\bar{v}}^{\bar{v}}(1+i_m(v+\tilde{m}-m))(v+\tilde{m}-m)dF(v)$$

where $I=(1+i_d)\bar{m}+(m-\bar{m})[\mathbb{I}_{\{m>\bar{m}\}}(1+i_d)-\mathbb{I}_{\{m<\bar{m}\}}(1+i_\ell)]$ is the payoff when there is no liquidity shock. Using (5) we obtain again four cases for $i_m(v+\tilde{m}-m)$:

(1) If $m-\bar{m}>\tilde{m}-m>0$ then

$$1+i_m(v+\tilde{m}-m)=\begin{cases}1+i_d & \text{if } v+\tilde{m}-m<m-\bar{m}\\ 1+\frac{i_\ell+i_d}{2}+\frac{\bar{m}-m}{2(v+\tilde{m}-m)}(i_\ell-i_d) & \text{if } v+\tilde{m}-m>m-\bar{m}\end{cases}\tag{18}$$

(2) If $\tilde{m}-m>m-\bar{m}>0$ then the interest rate is

$$1+i_m(v+\tilde{m}-m)=1+\frac{i_\ell+i_d}{2}+\frac{\bar{m}-m}{2(v+\tilde{m}-m)}(i_\ell-i_d).$$

(3) If $m - \bar{m} < \tilde{m} - m < 0$ then the interest rate is given by

$$1 + i_m(v + \tilde{m} - m) = \begin{cases} 1 + \frac{i_\ell + i_d}{2} + \frac{\bar{m} - m}{2(v + \tilde{m} - m)}(i_\ell - i_d) & \text{if } v + \tilde{m} - m > \bar{m} - m \\ 1 + i_\ell & \text{if } v + \tilde{m} - m < \bar{m} - m \end{cases} \tag{19}$$

(4) If $\tilde{m} - m < m - \bar{m} < 0$ then the interest rate is again

$$1 + i_m(v + \tilde{m} - m) = 1 + \frac{i_\ell + i_d}{2} + \frac{\bar{m} - m}{2(v + \tilde{m} - m)}(i_\ell - i_d).$$

As we want to know the marginal value of reserves around m, notice that we only have to consider cases (1) and (3), as the other two would yield to a contradiction (as we will set $\tilde{m} = m$). In case (1) where $m - \bar{m} > \tilde{m} - m$ we have

$$\begin{aligned} W(\tilde{m}) &= I + \int_{-(2m-\bar{m}-\tilde{m})}^{2m-\bar{m}-\tilde{m}} (1 + i_d)(v + \tilde{m} - m)dF(v) \\ &+ \int_{-\bar{v}}^{-(2m-\bar{m}-\tilde{m})} \left[\left(1 + \frac{i_\ell + i_d}{2}\right)(v + \tilde{m} - m) + \frac{\bar{m} - m}{2}(i_\ell - i_d) \right] dF(v) \\ &+ \int_{2m-\bar{m}-\tilde{m}}^{\bar{v}} \left[\left(1 + \frac{i_\ell + i_d}{2}\right)(v + \tilde{m} - m) + \frac{\bar{m} - m}{2}(i_\ell - i_d) \right] dF(v) \end{aligned}$$

with marginal utility

$$W'(\tilde{m}) = (1 + i_d)\left[2F(2m - \bar{m} - \tilde{m}) - 1\right] + \left(1 + \frac{i_\ell + i_d}{2}\right) 2\left[1 - F(2m - \bar{m} - \tilde{m})\right]$$

In case (3) where $m - \bar{m} < \tilde{m} - m < 0$, we have

$$\begin{aligned} W(\tilde{m}) &= I + \int_{-(\bar{m}-\tilde{m})}^{\bar{m}-\tilde{m}} (1 + i_\ell)(v + \tilde{m} - m)dF(v) \\ &+ \int_{-\bar{v}}^{-(\bar{m}-\tilde{m})} \left[\left(1 + \frac{i_\ell + i_d}{2}\right)(v + \tilde{m} - m) + \frac{\bar{m} - m}{2}(i_\ell - i_d) \right] dF(v) \\ &+ \int_{\bar{m}-\tilde{m}}^{\bar{v}} \left[\left(1 + \frac{i_\ell + i_d}{2}\right)(v + \tilde{m} - m) + \frac{\bar{m} - m}{2}(i_\ell - i_d) \right] dF(v) \end{aligned}$$

with marginal utility

$$W'(\tilde{m}) = (1 + i_\ell)\left[2F(\bar{m} - \tilde{m}) - 1\right] + \left(1 + \frac{i_\ell + i_d}{2}\right) 2\left[1 - F(\bar{m} - \tilde{m})\right]$$

In equilibrium, $\tilde{m} = m$, so that if $m > \bar{m}$ then the marginal value of reserves is

$$W'(m) = (1 + i_d)\left[2F(m - \bar{m}) - 1\right] + \left(1 + \frac{i_\ell + i_d}{2}\right) 2\left[1 - F(m - \bar{m})\right]$$

while if $m < \bar{m}$ then the marginal value of reserves is

$$W'(m) = (1 + i_\ell)\left[2F(\bar{m} - m) - 1\right] + \left(1 + \frac{i_\ell + i_d}{2}\right) 2\left[1 - F(\bar{m} - m)\right]$$

Using the fact that the distribution of shocks is symmetric, we obtain $F(\bar{m} - m) = 1 - F(m - \bar{m})$ so that we can simplify the last two expressions to simply

$$W'(m) = (1 + i_d)F(m - \bar{m}) + (1 + i_\ell)\left[1 - F(m - \bar{m})\right].$$

Rate volatility with perfect pairing

$$\begin{aligned}
\sigma_w^2(i_m) &= \int_{|\bar{m}-m|}^{\bar{v}} \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} v \left[\frac{1}{v^2} + \frac{\left[1 - F(|\bar{m}-m|)\right]^2}{Q(\bar{m}-m)^2} - \frac{2}{v}\frac{1 - F(|\bar{m}-m|)}{Q(\bar{m}-m)}\right] dF(v) \\
&= \int_{|\bar{m}-m|}^{\bar{v}} \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} \left[\frac{1}{v} + \frac{\left[1 - F(|\bar{m}-m|)\right]^2}{Q(\bar{m}-m)^2} v - 2\frac{1 - F(|\bar{m}-m|)}{Q(\bar{m}-m)}\right] dF(v) \\
&= \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} \left[\int_{|\bar{m}-m|}^{\bar{v}} \frac{1}{v} dF(v) + \frac{\left[1 - F(|\bar{m}-m|)\right]^2}{Q(\bar{m}-m)^2} \int_{|\bar{m}-m|}^{\bar{v}} v dF(v) - 2\frac{[1 - F(|\bar{m}-m}{Q(\bar{m}-m)}\right. \\
&= \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} \left[\int_{|\bar{m}-m|}^{\bar{v}} \frac{1}{v} dF(v) - \frac{\left[1 - F(|\bar{m}-m|)\right]^2}{Q(\bar{m}-m)}\right] \\
&\leq \frac{(\bar{m}-m)^2}{Q(\bar{m}-m)} \frac{(i_\ell - i_d)^2}{4} \left[\frac{1 - F(|\bar{m}-m|)}{|\bar{m}-m|} - \frac{\left[1 - F(|\bar{m}-m|)\right]^2}{Q(\bar{m}-m)}\right] \\
&= \frac{(i_\ell - i_d)^2}{4} \frac{|\bar{m}-m|\left[1 - F(|\bar{m}-m|)\right]}{Q(\bar{m}-m)} \left[1 - \frac{|\bar{m}-m|\left[1 - F(|\bar{m}-m|)\right]}{Q(\bar{m}-m)}\right]
\end{aligned}$$

Since

$$Q(\bar{m} - m) \geq |\bar{m} - m|\left(1 - F(|\bar{m} - m|)\right)$$

we have

$$\sigma_w^2(i_m) \leq \frac{(i_\ell - i_d)^2}{16}$$

Proof of Proposition 1.

We guess and verify that $\hat{m} = \bar{m}$ (i.e. all banks with enough reserves to satisfy the reserves requirement will become lenders and inversely, while those holding $\bar{m}$ will be indifferent). When $\hat{m} = \bar{m}$ we have

$$\begin{aligned} V^b(\tilde{m}) &= \theta(n)\int_{\bar{m}}^{m+\bar{v}} \mathbb{I}_{\{\tilde{m}+m_\ell>2\bar{m}\}}\left[\bar{m}(1+i_d)+\frac{\tilde{m}-\bar{m}}{2}(i_\ell-i_d)\right]dF_\ell(m_\ell) \\ &\quad +\theta(n)\int_{\bar{m}}^{m+\bar{v}} \mathbb{I}_{\{\tilde{m}+m_\ell<2\bar{m}\}}\left[\tilde{m}(1+i_\ell)+\frac{m_\ell-\bar{m}}{2}(i_\ell-i_d)-\bar{m}(i_\ell-i_d)\right]dF_\ell(m_\ell) \\ &\quad +(1-\theta(n))\left[(\tilde{m}-\bar{m})(1+i_\ell)+\bar{m}(1+i_d)\right] \end{aligned}$$

where $F_\ell(m_\ell)$ is the distribution of reserves holding of lenders, and where we have used the payoff of the borrower in each possible state. We can now do a change of variable to obtain

$$\begin{aligned} V^b(\tilde{m}) &= \theta(n)\int_{\bar{m}-m}^{\bar{m}-m+\bar{m}-\tilde{m}}\left[\tilde{m}(1+i_\ell)+\frac{m_\ell(v)-\bar{m}}{2}(i_\ell-i_d)-\bar{m}(i_\ell-i_d)\right]\frac{dF(v)}{1-F(\bar{m}-m)} \\ &\quad +\theta(n)\int_{\bar{m}-m+\bar{m}-\tilde{m}}^{\bar{v}}\left[\bar{m}(1+i_d)+\frac{\tilde{m}-\bar{m}}{2}(i_\ell-i_d)\right]\frac{dF(v)}{1-F(\bar{m}-m)} \\ &\quad +(1-\theta(n))\left[(\tilde{m}-\bar{m})(1+i_\ell)+\bar{m}(1+i_d)\right] \end{aligned} \tag{20}$$

Notice that

$$V^b(\bar{m}) = \theta(n)\int_{\bar{m}-m}^{\bar{v}}\bar{m}(1+i_d)\frac{dF(v)}{1-F(\bar{m}-m)}+(1-\theta(n))\bar{m}(1+i_d) \tag{21}$$

and we also have

$$\frac{\partial V^b(\tilde{m})}{\partial\tilde{m}} = 1+i_\ell-\theta(n)\frac{(i_\ell-i_d)}{2}\frac{1-F(2\bar{m}-m-\tilde{m})}{1-F(\bar{m}-m)}. \tag{22}$$

Turning to the lenders, and using similar steps, we obtain

$$\begin{aligned} V^\ell(\tilde{m}) &= \frac{n}{1-n}\theta(n)\int_{\bar{m}-m+\bar{m}-\tilde{m}}^{\bar{m}-m}\left[\tilde{m}(1+i_d)+\frac{\bar{m}-m_b(v)}{2}(i_\ell-i_d)-\bar{m}(i_\ell-i_d)\right]\frac{dF(v)}{F(\bar{m}-m)} \\ &\quad +\frac{n}{1-n}\theta(n)\int_{-\bar{v}}^{\bar{m}-m+\bar{m}-\tilde{m}}\left[\tilde{m}(1+i_d)+\frac{\tilde{m}-\bar{m}}{2}(i_\ell-i_d)\right]\frac{dF(v)}{F(\bar{m}-m)} \\ &\quad +(1-\frac{n}{1-n}\theta(n))\tilde{m}(1+i_d) \end{aligned} \tag{23}$$

with

$$V^{\ell}(\bar{m}) \;=\; \frac{n}{1-n}\theta(n)\int_{-\bar{v}}^{\bar{m}-m}\bar{m}(1+i_d)\frac{dF(v)}{F(\bar{m}-m)}+(1-\frac{n}{1-n}\theta(n))\bar{m}(1+i_d)(24)$$

and

$$\frac{\partial V^{\ell}(\tilde{m})}{\partial \tilde{m}} = 1+i_d+\frac{n}{1-n}\theta(n)\frac{(i_\ell-i_d)}{2}\frac{F(2\bar{m}-m-\tilde{m})}{F(\bar{m}-m)}. \tag{25}$$

Now, using (21) and (24), as well as the fact that $n=\int_{-\bar{v}}^{\bar{m}-m}dF(v)=F(\bar{m}-m)$, we can see that $V^b(\bar{m})=V^{\ell}(\bar{m})$, which verifies our guess (to really verify it we should also compute $V^b(\tilde{m})$ for $\tilde{m}>\bar{m}$ - but I computed V^b when the threshold is not $\bar{m}$ and we find a contradiction with $V^b(\hat{m})=V^{\ell}(\hat{m})$).

Proof of Proposition 2

Changing variable to integrate over the liquidity shocks v_ℓ and v_b instead of reserves, we obtain,

$$\tilde{Q}(\bar{m}-m)=\int_{-\bar{v}}^{\bar{m}-m}\int_{\bar{m}-m}^{\bar{v}}\frac{v_\ell-v_b}{2}\frac{dF(v_\ell)}{1-F(\bar{m}-m)}\frac{dF(v_b)}{F(\bar{m}-m)}$$

and integrating we have

$$\tilde{Q}(\bar{m}-m)=\frac{E[v]-\int_{-\bar{v}}^{\bar{m}-m}vdF(v)}{2F(\bar{m}-m)[1-F(\bar{m}-m)]}$$

and assuming that the mean liquidity shock is zero, i.e. $E[v]=0$, we obtain

$$\tilde{Q}(\bar{m}-m)=\frac{-\int_{-\bar{v}}^{\bar{m}-m}vdF(v)}{2F(\bar{m}-m)[1-F(\bar{m}-m)]}.$$

Now we are interested in the derivative of $Q(x)=\min\{F(x);1-F(x)\}\tilde{Q}(x)$. So $Q(x)$ is differentiable almost everywhere, as it has a kink at $x=0$. Assuming that the distribution function $F(v)$ is symmetric, we have $F(x)<1-F(x)$ whenever $x<0$, while $F(x)>1-F(x)$ whenever $x>0$. Therefore, for all $x<0$ we obtain

$$Q(x)\mid_{x<0}\;=\frac{-\int_{-\bar{v}}^{x}vdF(v)}{2[1-F(x)]},$$

with derivative

$$2Q'(x)\,|_{x<0} \quad = \quad \frac{[1-F(x)]\,[-xf(x)] + f(x)\left[-\int_{-\bar{v}}^{x} v dF(v)\right]}{[1-F(x)]^2}.$$

Hence, $Q'(x) > 0$ whenever $x < 0$. Also, $\lim_{x\to -\bar{v}} Q(x) = 0$.

Now, for all $x > 0$ we obtain

$$Q(x)\,|_{x>0} = \frac{-\int_{-\bar{v}}^{x} v dF(v)}{2F(x)},$$

with derivative

$$2Q'(x)\,|_{x>0} \quad = \quad \frac{F(x)\,[-xf(x)] - f(x)\left[-\int_{-\bar{v}}^{x} v dF(v)\right]}{F(x)^2}$$

and since $x > 0$ we obtain that $Q'(x) < 0$ whenever $x > 0$. Also, $\lim_{x\to \bar{v}} Q(x) = 0$, as the mean shock is zero.

Average OTC rate

$$\bar{i_m} = \int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} \frac{q(m_b, m_\ell)}{\tilde{Q}(\bar{m}-m)} i_m(m_b, m_\ell) dF_\ell(m_\ell) dF_b(m_b)$$

and using (1) and (10) combined with (2),

$$\begin{aligned}
1+\bar{i_m} &= \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[\int_{m-\bar{v}}^{\bar{m}} \int_{\bar{m}}^{m+\bar{v}} (m_\ell - m_b + (m_\ell - \bar{m})i_d + (\bar{m}-m_b)i_\ell)\, dF_\ell(m_\ell) dF_b(m_b)\right] \\
1+\bar{i_m} &= 1 + \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} [(m+v_\ell-\bar{m})i_d + (\bar{m}-m-v_b)i_\ell]\, \frac{dF(v_\ell)}{1-F(\bar{m}-m)} \frac{dF(v_b)}{F(\bar{m}-m)}\right. \\
&= 1 + \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[\int_{\bar{m}-m}^{\bar{v}} (m+v_\ell-\bar{m})i_d \frac{dF(v_\ell)}{1-F} + \int_{-\bar{v}}^{\bar{m}-m} (\bar{m}-m-v_b)i_\ell \frac{dF(v_b)}{F}\right] \\
&= 1 + \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[(\bar{m}-m)(i_\ell - i_d) + i_d \int_{\bar{m}-m}^{\bar{v}} v\frac{dF(v)}{1-F} - i_\ell \int_{-\bar{v}}^{\bar{m}-m} v\frac{dF(v)}{F}\right]
\end{aligned}$$

where we use $F = F(\bar{m}-m)$ as a shorthand. Therefore

$$\bar{i_m} \quad = \quad \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[(\bar{m}-m)(i_\ell - i_d) + \frac{i_d}{1-F}E[v] - \int_{-\bar{v}}^{\bar{m}-m} v dF(v)\left(\frac{i_d}{1-F} + \frac{i_\ell}{F}\right)\right]$$

Hence, using (10) we have

$$\bar{i_m} = (1-F)i_\ell + Fi_d + \frac{1}{2\tilde{Q}(\bar{m}-m)}\left[(\bar{m}-m)(i_\ell - i_d) + \frac{i_d}{1-F}E[v]\right]$$

and if $E[v] = 0$ we obtain

$$\begin{aligned}\bar{i_m} &= (1-F(\bar{m}-m))i_\ell + F(\bar{m}-m)i_d + \frac{(\bar{m}-m)}{\tilde{Q}(\bar{m}-m)}\frac{(i_\ell - i_d)}{2} \\ &= F(\bar{m}-m)i_\ell + [1-F(\bar{m}-m)]\,i_d + \left[1-2F(\bar{m}-m) + \frac{(\bar{m}-m)}{2\tilde{Q}(\bar{m}-m)}\right](i_\ell - i_d)\end{aligned}$$

Hence, the average OTC rate is equal to the rate in Poole (1968) whenever $\bar{m} = m$. The relative position of the average OTC rate with respect to the Poole rate then depends on the expression in brackets. Using $x = \bar{m} - m$ we have

$$\bar{i_m}'(x) = f(x)(i_d - i_\ell) + \frac{(i_\ell - i_d)}{2}\frac{\tilde{Q}(x) - x\tilde{Q}'(x)}{\tilde{Q}(x)^2}$$

and

$$\tilde{Q}'(x) = \frac{f(x)}{2}\frac{F(x)[1-F(x)]\,[-x] - [1-2F(x)]\left[-\int_{-\bar{v}}^{x} v dF(v)\right]}{F(x)^2[1-F(x)]^2}$$

Therefore $x > 0$ implies $\tilde{Q}'(x) < 0$ and $x < 0$ implies $\tilde{Q}'(x) > 0$, so that $x\tilde{Q}'(x) < 0$.

Volatility

For computational purpose, we can rearrange the expression for the volatility as follows:

$$
\begin{aligned}
&\frac{q(v_\ell, v_b)}{\tilde{Q}(\bar{m}, m)}[i_m(v_\ell, v_b) - \bar{i}_m]^2 \\
&= (i_\ell - i_d)^2 \frac{v_\ell - v_b}{2\tilde{Q}} \left[\frac{\bar{m} - m - v_\ell}{v_\ell - v_b} + F(\bar{m} - m) - \frac{(\bar{m} - m)}{2\tilde{Q}(\bar{m} - m)} \right]^2 \\
&= (i_\ell - i_d)^2 \frac{v_\ell - v_b}{2\tilde{Q}} \left[\frac{\bar{m} - m - v_\ell}{v_\ell - v_b} + c \right]^2 \\
&= (i_\ell - i_d)^2 \frac{v_\ell - v_b}{2\tilde{Q}} \left\{ \left[\frac{\bar{m} - m - v_\ell}{v_\ell - v_b} \right]^2 + c^2 + 2 \frac{\bar{m} - m - v_\ell}{v_\ell - v_b} c \right\} \\
&= (i_\ell - i_d)^2 \left\{ \frac{v_\ell - v_b}{2\tilde{Q}} \left[\frac{\bar{m} - m - v_\ell}{v_\ell - v_b} \right]^2 + \frac{v_\ell - v_b}{2\tilde{Q}} c^2 + \frac{1}{\tilde{Q}} (\bar{m} - m - v_\ell) c \right\}
\end{aligned}
$$

where

$$
c = F(\bar{m} - m) - \frac{(\bar{m} - m)}{2\tilde{Q}(\bar{m} - m)}
$$

Notice that, as $\tilde{Q} \geq 0$, we obtain $c \geq 0$ whenever $m \geq \bar{m}$, while $c \leq F(\bar{m} - m)$ otherwise. In the sequel, we will assume that $c \geq 0$, as we are mostly interested in the case where $m \geq \bar{m}$. So

$$
\sigma_w^2(\bar{i}_m) = \int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{q(v_\ell, v_b)}{\tilde{Q}(\bar{m}, m)}[i_m(v_\ell, v_b) - \bar{i}_m]^2 \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)}
$$

is equal to

$$
\begin{aligned}
\sigma_w^2(\bar{i}_m) &= (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \left\{ \frac{1}{2\tilde{Q}} \frac{(\bar{m} - m - v_\ell)^2}{v_\ell - v_b} + \frac{1}{\tilde{Q}} (\bar{m} - m - v_\ell) c \right\} \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m} \right. \\
&= (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{1}{2\tilde{Q}} \frac{(\bar{m} - m - v_\ell)^2}{v_\ell - v_b} \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)} + \int_{\bar{m}-m}^{\bar{v}} \frac{(\bar{m} - m - v_\ell)}{\tilde{Q}} \right. \\
&\leq (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{1}{2\tilde{Q}} \frac{(\bar{m} - m - v_\ell)^2}{v_\ell - v_b} \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} \frac{dF(v_b)}{F(\bar{m} - m)} + c^2 \right] \\
&\leq (i_\ell - i_d)^2 \left[\int_{\bar{m}-m}^{\bar{v}} \frac{1}{2\tilde{Q}} \frac{(\bar{m} - m - v_\ell)^2}{v_\ell - (\bar{m} - m)} \frac{dF(v_\ell)}{1 - F(\bar{m} - m)} + c^2 \right]
\end{aligned}
$$

where the last inequality follows from the fact that $v_b \leq \bar{m} - m$ over the space of integration. Therefore

$$
\begin{aligned}
\sigma_w^2(\bar{i}_m) &\leq (i_\ell - i_d)^2 \left[\int_{\bar{m}-m}^{\bar{v}} \frac{1}{2\tilde{Q}} \left[v_\ell - (\bar{m}-m) \right] \frac{dF(v_\ell)}{1-F(\bar{m}-m)} + c^2 \right] \\
&= (i_\ell - i_d)^2 \left[\frac{1}{2\tilde{Q}} \int_{\bar{m}-m}^{\bar{v}} v_\ell \frac{dF(v_\ell)}{1-F(\bar{m}-m)} - \frac{(\bar{m}-m)}{2\tilde{Q}} + c^2 \right] \\
&= (i_\ell - i_d)^2 \left[\frac{1}{2\tilde{Q}} \left[\frac{E[v] - \int_{-\bar{v}}^{\bar{m}-m} v_\ell dF(v_\ell)}{1-F(\bar{m}-m)} \right] - \frac{(\bar{m}-m)}{2\tilde{Q}} + c^2 \right] \\
&= (i_\ell - i_d)^2 \left[\frac{1}{2\tilde{Q}} \left[2\tilde{Q} F(\bar{m}-m) \right] - \frac{(\bar{m}-m)}{2\tilde{Q}} + c^2 \right] \\
&= (i_\ell - i_d)^2 \left[F(\bar{m}-m) - \frac{(\bar{m}-m)}{2\tilde{Q}(\bar{m}-m)} + c^2 \right] \\
&= (i_\ell - i_d)^2 (c + c^2)
\end{aligned}
$$

We now describe a lower bound for the OTC rate volatility:

$$
\begin{aligned}
\sigma_w^2(\bar{i}_m) &= (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \left\{ \frac{v_\ell - v_b}{2\tilde{Q}} \left(\frac{v_\ell - (\bar{m}-m)}{v_\ell - v_b} \right)^2 + \frac{1}{\tilde{Q}} (\bar{m} - m - v_\ell) c \right\} \frac{dF(v_\ell)}{1-F(\bar{m}-m)} \right. \\
&\geq (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \left\{ \frac{v_\ell - v_b}{2\tilde{Q}} \left(\frac{v_\ell - (\bar{m}-m)}{v_\ell - v_b} \right)^2 + \frac{1}{\tilde{Q}} (v_b - v_\ell) c \right\} \frac{dF(v_\ell)}{1-F(\bar{m}-m)} \right. \\
&= (i_\ell - i_d)^2 \left[\int_{-\bar{v}}^{\bar{m}-m} \int_{\bar{m}-m}^{\bar{v}} \frac{(v_\ell - v_b)}{2\tilde{Q}} \left(\frac{v_\ell - \bar{m} - m}{v_\ell - v_b} \right)^2 \frac{dF(v_\ell)}{1-F(\bar{m}-m)} \frac{dF(v_b)}{F(\bar{m}-m)} - 2c + c^2 \right]
\end{aligned}
$$

counterparty risk

To find the risk premium, we again have to consider four types of trades: (1) meetings where $m_b + m_\ell \geq 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell > \bar{m}$, (2) meetings where $m_b + m_\ell \geq 2\bar{m}$, $m_b > \bar{m}$, and $m_\ell > \bar{m}$, (3) meetings where $m_b + m_\ell < 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell > \bar{m}$, and (4) meetings where $m_b + m_\ell < 2\bar{m}$, $m_b < \bar{m}$, and $m_\ell < \bar{m}$. We assume that if the borrower defaults, then he gets nothing.

We now solve the rate in meeting (1). Since the rates are given by equalizing surplus from trade, rates in meetings (1) will have to satisfy

$$
(1-\delta) \left\{ (m_b + q - \bar{m})(1+i_d) + \bar{m}(1+i_d) - q(1+i_m^\delta) - \left[-(\bar{m} - m_b)(1+i_\ell) + \bar{m}(1+i_d) \right] \right\} =
$$
$$
(1-\delta) \left\{ (m_\ell - q - \bar{m})(1+i_d) + \bar{m}(1+i_d) + (1-\delta) q (1+i_m^\delta) - \left[(m_\ell - \bar{m})(1+i_d) + \bar{m}(1+i_d) \right] \right\}
$$

notice that the lender only gets paid when it does not disappear *and* the borrower does not disappear either. We assume that the borrower always has to pay, as this is akin to the fact that all liabilities are still due, even in bankruptcy case. We can simplify the expression above to find

$$\begin{aligned}
1+i_m^\delta &= \frac{m_\ell-\bar{m}}{m_\ell-m_b}(1+i_d)+\frac{\bar{m}-m_b}{m_\ell-m_b}(1+i_\ell)+\delta\frac{1+i_m^\delta}{2} \\
1+i_m^\delta &= \frac{2}{2-\delta}(1+i_m)
\end{aligned}$$

We now solve for the rate in meeting (2). Since the rates are given by equalizing surplus from trade, rates in meetings (2) will have to satisfy

$$\begin{aligned}
&(1-\delta)\left\{(m_b+q-\bar{m})(1+i_d)+\bar{m}(1+i_d)-q(1+i_m^\delta)-[(m_b-\bar{m})(1+i_d)+\bar{m}(1+i_d)]\right\} = \\
&(1-\delta)\left\{(m_\ell-q-\bar{m})(1+i_d)+\bar{m}(1+i_d)+(1-\delta)q(1+i_m^\delta)-[(m_\ell-\bar{m})(1+i_d)+\bar{m}(1+i_d)]\right\}
\end{aligned}$$

Simplifying, we obtain

$$\begin{aligned}
1+i_m^\delta &= \frac{2}{2-\delta}(1+i_d) \\
1+i_m^\delta &= \frac{2}{2-\delta}(1+i_m)
\end{aligned}$$

We now solve for the rate in meeting (3). In this case, both the borrower and the lender have to borrow at the lending facility after they exit the interbank market. Rates in meetings (3) will have to satisfy

$$\begin{aligned}
&(1-\delta)\left\{-(\bar{m}-(m_b+q))(1+i_\ell)+\bar{m}(1+i_d)-q(1+i_m^\delta)-[-(\bar{m}-m_b)(1+i_\ell)+\bar{m}(1+i_d)]\right\} = \\
&(1-\delta)\left\{-(\bar{m}-(m_\ell-q))(1+i_\ell)+\bar{m}(1+i_d)+(1-\delta)q(1+i_m^\delta)-[(m_\ell-\bar{m})(1+i_d)+\bar{m}(1+i_d)]\right\}
\end{aligned}$$

Simplifying, we obtain

$$\begin{aligned}
1+i_m^\delta &= \frac{2}{2-\delta}\left[\frac{(m_\ell-\bar{m})}{(m_\ell-m_b)}(1+i_d)+\frac{(\bar{m}-m_b)}{(m_\ell-m_b)}(1+i_\ell)\right] \\
1+i_m^\delta &= \frac{2}{2-\delta}(1+i_m)
\end{aligned}$$

Finally, we now solve for the rate in meeting (4). In this case, both the borrower and the lender have to borrow at the lending facility after they exit the interbank market, whether or not they traded on the interbank market. Rates in meetings (4) will have to satisfy

$$(1-\delta)\left\{-(\bar{m}-(m_b+q))(1+i_\ell)+\bar{m}(1+i_d)-q(1+i_m^\delta)-[-(\bar{m}-m_b)(1+i_\ell)+\bar{m}(1+i_d)]\right\} =$$
$$(1-\delta)\left\{-(\bar{m}-(m_\ell-q))(1+i_\ell)+\bar{m}(1+i_d)+(1-\delta)q(1+i_m^\delta)-[-(\bar{m}-m_\ell)(1+i_\ell)+\bar{m}(1+i_d)]\right\}$$

Simplifying, we obtain

$$\begin{aligned} 1+i_m^\delta &= \frac{2}{2-\delta}(1+i_\ell) \\ &= \frac{2}{2-\delta}(1+i_m) \end{aligned}$$

Payoff with counterparty risk We now compute the payoffs, under each cases. In case (1) for the borrower:

$$
\begin{aligned}
(1+i_d)(m_b + \frac{m_\ell - m_b}{2} - \bar{m}) + (1+i_d)\bar{m} - \frac{m_\ell - m_b}{2}(1+i_m^\delta) &= \\
(1+i_d)\left(m_b + \frac{m_\ell - m_b}{2}\right) - \frac{m_\ell - m_b}{2}(1+i_m^\delta) &= \\
(1+i_d)m_b + \frac{m_\ell - m_b}{2}\left[1 + i_d - \left(\frac{2}{2-\delta}\right)(1+i_m)\right] &= \\
(1+i_d)m_b + \frac{m_\ell - m_b}{2}\left[1 + i_d - \left(\frac{2}{2-\delta}\right)\left(\frac{m_\ell - \bar{m}}{m_\ell - m_b}(1+i_d) + \frac{\bar{m} - m_b}{m_\ell - m_b}(1+i_\ell)\right)\right] &= \\
(1+i_d)m_b + \frac{1}{2}\left[(1+i_d)(m_\ell - m_b) - \left(\frac{2}{2-\delta}\right)((m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)(1+i_\ell))\right] &= \\
(1+i_d)m_b + \frac{1}{2}\left[(1+i_d)(m_\ell - \bar{m} + \bar{m} - m_b) - \left(\frac{2}{2-\delta}\right)((m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)(1+i_\ell))\right] &= \\
(1+i_d)m_b + \frac{1}{2}\left[(m_\ell - \bar{m})(1+i_d)\left(1 - \frac{2}{2-\delta}\right) + (\bar{m} - m_b)\left[(1+i_d) - \left(\frac{2}{2-\delta}\right)(1+i_\ell)\right]\right] &= \\
(1+i_d)m_b + \frac{1}{2}\left[-\frac{\delta}{2-\delta}(m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)\frac{2(i_d - i_\ell) - \delta(1+i_d)}{(2-\delta)}\right] &= \\
(1+i_d)m_b - \frac{\delta}{2-\delta}\frac{(m_\ell - \bar{m})}{2}(1+i_d) - \frac{(\bar{m} - m_b)}{2}\frac{2(i_\ell - i_d) + \delta(1+i_d)}{(2-\delta)} &= \\
(1+i_d)m_b - \frac{(\bar{m} - m_b)}{(2-\delta)}(i_\ell - i_d) - \frac{\delta}{2-\delta}\frac{(m_\ell - m_b)}{2}(1+i_d) &= \\
(1+i_d)m_b - \frac{(\bar{m} - m_b)}{2}(i_\ell - i_d) - \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(i_\ell - i_d) + \frac{(m_\ell - m_b)}{2}(1+i_d)\right] &= \\
P^b(m_\ell, m_b) - \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(1 + i_\ell - (1+i_d)) + \frac{(m_\ell - \bar{m} + \bar{m} - m_b)}{2}(1+i_d)\right] &= \\
P^b(m_\ell, m_b) - \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(1+i_\ell) + \frac{(m_\ell - \bar{m})}{2}(1+i_d)\right] &=
\end{aligned}
$$

In case (1) for the lender:

$$
\begin{aligned}
(1+i_d)(m_\ell - \frac{m_\ell - m_b}{2} - \bar{m}) + (1+i_d)\bar{m} + \frac{m_\ell - m_b}{2}(1+i_m^\delta) &= \\
(1+i_d)\left(m_\ell - \frac{m_\ell - m_b}{2}\right) + \frac{m_\ell - m_b}{2}(1+i_m^\delta) &= \\
(1+i_d)m_\ell - \frac{m_\ell - m_b}{2}\left[1+i_d - \left(\frac{2}{2-\delta}\right)(1+i_m)\right] &= \\
(1+i_d)m_\ell - \frac{m_\ell - m_b}{2}\left[1+i_d - \left(\frac{2}{2-\delta}\right)\left(\frac{m_\ell - \bar{m}}{m_\ell - m_b}(1+i_d) + \frac{\bar{m} - m_b}{m_\ell - m_b}(1+i_\ell)\right)\right] &= \\
(1+i_d)m_\ell - \frac{1}{2}\left[(1+i_d)\left(m_\ell - m_b\right) - \left(\frac{2}{2-\delta}\right)((m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)(1+i_\ell))\right] &= \\
(1+i_d)m_\ell - \frac{1}{2}\left[(1+i_d)\left(m_\ell - \bar{m} + \bar{m} - m_b\right) - \left(\frac{2}{2-\delta}\right)((m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)(1+i_\ell))\right] &= \\
(1+i_d)m_\ell - \frac{1}{2}\left[(m_\ell - \bar{m})(1+i_d)\left(1 - \frac{2}{2-\delta}\right) + (\bar{m} - m_b)\left[(1+i_d) - \left(\frac{2}{2-\delta}\right)(1+i_\ell)\right]\right] &= \\
(1+i_d)m_\ell - \frac{1}{2}\left[-\frac{\delta}{2-\delta}(m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)\frac{2(i_d - i_\ell) - \delta(1+i_d)}{(2-\delta)}\right] &= \\
(1+i_d)m_\ell + \frac{\delta}{2-\delta}\frac{(m_\ell - \bar{m})}{2}(1+i_d) + \frac{(\bar{m} - m_b)}{2}\frac{2(i_\ell - i_d) + \delta(1+i_d)}{(2-\delta)} &= \\
(1+i_d)m_\ell + \frac{(\bar{m} - m_b)}{(2-\delta)}(i_\ell - i_d) + \frac{\delta}{2-\delta}\frac{(m_\ell - m_b)}{2}(1+i_d) &= \\
(1+i_d)m_\ell + \frac{(\bar{m} - m_b)}{2}(i_\ell - i_d) + \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(i_\ell - i_d) + \frac{(m_\ell - m_b)}{2}(1+i_d)\right] &= \\
P^\ell(m_\ell, m_b) + \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(i_\ell - i_d) + \frac{(m_\ell - m_b)}{2}(1+i_d)\right] &= \\
P^\ell(m_\ell, m_b) + \frac{\delta}{2-\delta}\left[\frac{(\bar{m} - m_b)}{2}(1+i_\ell) + \frac{(m_\ell - \bar{m})}{2}(1+i_d)\right] &
\end{aligned}
$$

In case (3) for the borrower:

$$(1+i_\ell)(m_b+\frac{m_\ell-m_b}{2}-\bar{m})+(1+i_d)\bar{m}-\frac{m_\ell-m_b}{2}(1$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{m_\ell-m_b}{2}\left[(1+i_\ell)-(1+\right.$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{m_\ell-m_b}{2}\left[(1+i_\ell)-\left(\frac{2}{2-\delta}\right)(1+\right.$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{m_\ell-m_b}{2}\left[(1+i_\ell)-\left(\frac{2}{2-\delta}\right)\left(\frac{m_\ell-\bar{m}}{m_\ell-m_b}(1+i_d)+\frac{\bar{m}-m_b}{m_\ell-m_b}(1+\right.\right.$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+(1+i_\ell)\frac{m_\ell-\bar{m}+\bar{m}-m_b}{2}-\left(\frac{1}{2-\delta}\right)((m_\ell-\bar{m})(1+i_d)+(\bar{m}-m_b)(1$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{(m_\ell-\bar{m})}{2}\left[(1+i_\ell)-\frac{2}{2-\delta}(1+i_d)\right]-(1+i_\ell)\frac{(\bar{m}-m_b)}{2}$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{(m_\ell-\bar{m})}{(2-\delta)}(i_\ell-i_d)-\frac{(m_\ell-m_b)}{2}\frac{\delta}{(2-\delta)}(1$$

$$(1+i_\ell)(m_b-\bar{m})+(1+i_d)\bar{m}+\frac{(m_\ell-\bar{m})}{2}(i_\ell-i_d)+\frac{\delta}{2-\delta}\left[\frac{(m_\ell-\bar{m})}{2}(i_\ell-i_d)-\frac{(m_\ell-m_b)}{2}(1-\right.$$

$$P^b(m_\ell,m_b)+\frac{\delta}{2-\delta}\left[\frac{(m_\ell-\bar{m})}{2}(i_\ell-i_d)-\frac{(m_\ell-m_b)}{2}(1-\right.$$

$$P^b(m_\ell,m_b)+\frac{\delta}{2-\delta}\left[\frac{(m_\ell-\bar{m})}{2}(1+i_\ell-(1+i_d))-\frac{(m_\ell-\bar{m}+\bar{m}-m_b)}{2}(1-\right.$$

$$P^b(m_\ell,m_b)-\frac{\delta}{2-\delta}\left[\frac{(m_\ell-\bar{m})}{2}(1+i_d)+\frac{(\bar{m}-m_b)}{2}(1-\right.$$

In case (3) for the lender:

$$(1+i_\ell)(m_\ell - \frac{m_\ell - m_b}{2} - \bar{m}) + (1+i_d)\bar{m} + \frac{m_\ell - m_b}{2}(1$$

$$(1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} - \frac{m_\ell - m_b}{2}\big[(1+i_\ell) - (1+$$

$$(1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} - \frac{m_\ell - m_b}{2}\left[(1+i_\ell) - \left(\frac{2}{2-\delta}\right)(1+\right.$$

$$(1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} - \frac{m_\ell - m_b}{2}\left[(1+i_\ell) - \left(\frac{2}{2-\delta}\right)\left(\frac{m_\ell - \bar{m}}{m_\ell - m_b}(1+i_d) + \frac{\bar{m} - m_b}{m_\ell - m_b}(1+\right.\right.$$

$$(1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} - (1+i_\ell)\frac{m_\ell - \bar{m} + \bar{m} - m_b}{2} + \left(\frac{1}{2-\delta}\right)((m_\ell - \bar{m})(1+i_d) + (\bar{m} - m_b)(1$$

$$(1+i_\ell)(m_\ell - \bar{m}) + (1+i_d)\bar{m} - \frac{(m_\ell - \bar{m})}{2}\left[(1+i_\ell) - \frac{2}{2-\delta}(1+i_d)\right] + (1+i_\ell)\frac{(\bar{m} - m_b)}{2}$$

$$(1+i_\ell)\frac{(m_\ell - \bar{m})}{2} + (1+i_d)\bar{m} + \frac{(m_\ell - \bar{m})}{2}\frac{2}{2-\delta}(1+i_d) + (1+i_\ell)\frac{(\bar{m} - m_b)}{2}$$

$$(1+i_d)m_\ell + (1+i_\ell)\frac{(m_\ell - \bar{m})}{2} - (1+i_d)(m_\ell - \bar{m}) + \frac{(m_\ell - \bar{m})}{2}\frac{2}{2-\delta}(1+i_d) + (1+i_\ell)\frac{(\bar{m} - m_b)}{2}$$

$$(1+i_d)m_\ell + \frac{(m_\ell - \bar{m})}{2}\left[(i_\ell - i_d) + \frac{\delta}{2-\delta}(1+i_d)\right] + \frac{\delta}{2-\delta}(1+i_\ell)\frac{(\bar{m} -}{2}$$

$$(1+i_d)m_\ell + \frac{(m_\ell - \bar{m})}{2}(i_\ell - i_d) + \frac{(m_\ell - \bar{m})}{2}\frac{\delta}{2-\delta}(1+i_d) + \frac{\delta}{2-\delta}(1+i_\ell)\frac{(\bar{m} -}{2}$$

$$(1+i_d)m_\ell + \frac{(m_\ell - \bar{m})}{2}(i_\ell - i_d) + \frac{\delta}{2-\delta}\left[\frac{(m_\ell - \bar{m})}{2}(1+i_d) + \frac{(\bar{m} - m_b)}{2}(1\right.$$

$$P^\ell(m_\ell, m_b) + \frac{\delta}{2-\delta}\left[\frac{(m_\ell - \bar{m})}{2}(1+i_d) + \frac{(\bar{m} - m_b)}{2}(1\right.$$

Value functions

Since $\hat{m} = \bar{m}$ we can use (3) and (4) to simplify the value functions as follows

$$\begin{aligned}
\frac{V^b(\tilde{m};\delta)}{1-\delta} &= V^b(\tilde{m};0) - \theta(n)\int_{\bar{m}+\frac{\bar{m}-\tilde{m}}{(\Delta-1)}}^{m+\bar{v}} \frac{\delta}{2-\delta}(1+i_m)q^\delta(m_\ell,\tilde{m})dF_\ell(m_\ell) \\
&\quad -\theta(n)\int_{\bar{m}}^{\bar{m}+\frac{\bar{m}-\tilde{m}}{(\Delta-1)}} \left\{P^b(m_\ell,\tilde{m};0) - [(1+i_\ell)(\tilde{m}-\bar{m}) + (1+i_d)\bar{m}]\right\} dF_\ell(m_\ell)
\end{aligned}$$

Notice that $q^\delta(\bar{m} - \frac{\bar{m}-\tilde{m}}{(\Delta-1)}, \tilde{m}) = 0$ and also $P^b(\bar{m} - \frac{\bar{m}-\tilde{m}}{(\Delta-1)}, \tilde{m}; 0) = (1+i_\ell)(\tilde{m} - \bar{m}) + (1+i_d)\bar{m}$. Hence the derivative of $V^b(\tilde{m};\delta)$ is

$$\begin{aligned}\frac{1}{1-\delta}\frac{\partial V^b(\tilde{m};\delta)}{\partial \tilde{m}} &= \frac{\partial V^b(\tilde{m})}{\partial \tilde{m}} - \theta(n)\int_{\bar{m}}^{\bar{m}+\frac{\bar{m}-\tilde{m}}{\Delta-1}}\left[\frac{\partial P^b(m_\ell,\tilde{m};0)}{\partial \tilde{m}} - (1+i_\ell)\right]dF_\ell(m_\ell) \\ &+\theta(n)\int_{\bar{m}+\frac{\bar{m}-\tilde{m}}{(\Delta-1)}}^{m+\bar{v}}\frac{\delta}{2-\delta}\frac{(1+i_\ell)}{2}dF_\ell(m_\ell)\end{aligned}$$

Notice that $\frac{\partial P^b(m_\ell,\tilde{m};0)}{\partial \tilde{m}} - (1+i_\ell) \leq 0$, so that all the extra terms (multiplied by $\theta(n)$) are positive. Hence

$$\frac{\partial V^b(\tilde{m};\delta)}{\partial \tilde{m}} > (1-\delta)\frac{\partial V^b(\tilde{m})}{\partial \tilde{m}}$$

We can now turn to lenders. Since $\hat{m} = \bar{m}$, we can rewrite the value of becoming a lender as

$$\begin{aligned}\frac{V^\ell(\tilde{m};\delta)}{1-\delta} &= V^\ell(\tilde{m};0) + \frac{n}{1-n}\theta(n)\int_{\bar{m}-(\Delta-1)(\tilde{m}-\bar{m})}^{\bar{m}}\frac{\delta}{2-\delta}(1+i_m)q^\delta(\tilde{m},m_b)dF_b(m_b) \\ &-\frac{n}{1-n}\theta(n)\int_{m-\bar{v}}^{\bar{m}-(\Delta-1)(\tilde{m}-\bar{m})}\left[P^\ell(\tilde{m},m_b;0) - (1+i_d)\tilde{m}\right]dF_b(m_b).\end{aligned}$$

and as $q^\delta(\tilde{m}, \bar{m}-(\Delta-1)(\tilde{m}-\bar{m})) = 0$, and $P^\ell(\tilde{m}, \bar{m}-m-(\Delta-1)(\tilde{m}-\bar{m}); 0) = (1+i_d)\tilde{m}$, the derivative is

$$\begin{aligned}\frac{1}{1-\delta}\frac{\partial V^\ell(\tilde{m};\delta)}{\partial \tilde{m}} &= \frac{\partial V^\ell(\tilde{m};0)}{\partial \tilde{m}} \quad +\frac{n}{1-n}\theta(n)\int_{\bar{m}-m-(\Delta-1)(\tilde{m}-\bar{m})}^{\bar{m}-m}\frac{\delta}{2-\delta}\frac{(1+i_d)}{2}F_b(m_b) \\ &\quad -\frac{n}{1-n}\theta(n)\int_{m-\bar{v}}^{\bar{m}-(\Delta-1)(\tilde{m}-\bar{m})}\left[\frac{\partial P^\ell(\tilde{m},m_b;0)}{\partial \tilde{m}} - (1+i_d)\right]dF_b(m_b),\end{aligned}$$

since $\frac{\partial P^\ell(\tilde{m},m_b;0)}{\partial \tilde{m}} \geq (1+i_d)$ in the region of interest, we cannot say if an additional unit of reserves is more valuable with or without counterparty risk.

Auction rate with counterparty risk

To find each bank's willingness to pay, we need to define the value of exiting this centralized stage with reserves m. This is

$$\frac{W(m)}{1-\delta} = \int_{-\bar{v}}^{\bar{m}-m} V^b(m+v)dF(v) + \int_{\bar{m}-m}^{\bar{v}} V^\ell(m+v)dF(v).$$

Therefore,

$$W'(m) = \int_{-\bar{v}}^{\bar{m}-m} \frac{\partial V^b(m+v;\delta)}{\partial m} dF(v) + \int_{\bar{m}-m}^{\bar{v}} \frac{\partial V^\ell(m+v;\delta)}{\partial m} dF(v)$$

References

Afonso, Gara and Ricardo Lagos, 2012. "Trade Dynamics in the Market for Federal Funds," Staff Report 549, Federal Reserve Bank of New York.

Angelini, P, A Nobili and C Picillo, 2011. "The Interbank Market after August 2007: What Has Changed, and Why?" *Journal of Money, Credit and Banking*, 43, 923 – 958.

Ashcraft, Adam B., and Darrell Duffie. 2007. "Systemic Illiquidity in the Federal Funds Market." *American Economic Review* 97(2) (May): 221–25.

Bech, Morten and Todd Keister (2013) "The liquidity coverage ratio and monetary policy implementation", BIS working paper xxx.

Bech, Morten and Klee, Elizabeth, 2011. "The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market," *Journal of Monetary Economics*, vol. 58, no. 5, p. 415-536, July.

Bech, Morten and Cyril Monnet, 2013. The impact of unconventional monetary policy on the overnight interbank market." BIS mimeo.

Berentsen, Aleksander and Cyril Monnet, Cyril, 2008. "Monetary policy in a channel system," Journal of Monetary Economics, Elsevier, vol. 55(6), pages 1067-1080

Bindseil, Ulrich. 2004. *Monetary Policy Implementation in the Euro Area.*

Bonner, Clemens and Sylvester Eijffinger, 2012. "The Impact of Liquidity Regulation on Interbank Money Markets," CEPR Discussion Papers 9124, C.E.P.R. Discussion Papers.

Borio, Claudio and Piti Disyatat, 2010. "Unconventional Monetary Policies: An Appraisal," Manchester School, University of Manchester, vol. 78(s1), pp. 53-89,

Christensen, Jens, Jose Lopez and Glenn Rudebusch, 2009. "Do central bank liquidity facilities affect interbank lending rates?," Working Paper Series 2009-13, Federal Reserve Bank of San Francisco.

Clinton, Kevin. 1997. "Implementation of Monetary Policy in a Regime with Zero Reserve requirements," Bank of Canada Working Paper 97-8.

Cœuré, Benoît, 2013a. "Ensuring the smooth functioning of money markets" Speech at the 17th Global Securities Financing Summit, Luxembourg, 16 January 2013. Available at www.ecb.int.

Cœuré, Benoît, 2013b. “Where to exit to? Monetary policy implementation after the crisis” Speech at the 15th Geneva Conference on the World Economy: “Exit strategies: time to think about them”, 3 May, Available at www.ecb.int.

Debelle, Guy, 2008. “Market Operations in the Past Year”. Speech at the 2008 FTA Congress, Melbourne, October 31.

Ennis, Huberto, and Todd Keister, 2008. “Understanding Monetary Policy Implementation,” Federal Reserve Bank of Richmond *Economic Quarterly*, Summer.

Kreicher Lawrence, Robert McCauley and Patrick McGuire (2013). “The 2011 FDIC assessment on banks managed liabilities: interest rate and balance-sheet responses”, BIS Working Papers No 413, May.

Heider, Florian, Maria Hoerova and Cornellia Holthausen, 2009. “Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk”, ECB Working Paper Series No. 1126.

Ishi, Kotaro, Fujita, Kenji and Stone, Mark, 2011. “Should Unconventional Balance Sheet Policies be Added to the Central Bank Toolkit? A Review of the Experience So Far,” IMF Working Papers No 11/145

Jeffery, Christopher, 2012. “Interbank market ‘almost dead’ in Europe, says Banque de France’s Mongars”, Central Banking 29 March. Available www.centralbanking.com/central-banking/news/2164700/interbank-market-almost-dead-europe-banque-france-s-mongars.

Maraffino, Guiseppe and Laurent Fransolet, 2012. “Euro Money Markets Weekly - ECB: to exit or not?”, *Barclays Interest Rate Research* 2 April.

McAndrews, Jamie, Asani Sakar and Zhu Wang, 2008. “The effect of the term auction facility on the London Inter-Bank Offer rate” Federal Reserve Bank of New York – Staff Reports, no. 335.

Michaud, F and Christian Upper, 2008. “What drives interbank rates? Evidence from the Libor panel”, *BIS Quarterly Review*, March, 47-58.

Poole, William. 1968. “Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy,” *Journal of Finance*, vol. 23, no. 5, December, p. 769-791.

Reid, Christopher, 2007. “The Canadian Overnight Market: Recent Evolution and Structural Changes”, Bank of Canada Review, Spring, pp. 15-29.

Schwarz, Krista (2010). “Mind the Gap: Disentangling Credit and Liquidity in Risk Spreads” Available at SSRN: http://ssrn.com/abstract=1486240

Stigum, Marcia, and Credenzi, Anthony. 2007. *Stigum's Money Market*, New York: McGraw Hill.

Taylor, John and John Williams, 2008. "Further results on a black swan in the money market", Discussion Paper 07-046, Stanford Institute for Economic Policy Research.

Taylor, John and JohnWilliams, 2009. "A black Swan in the money market", *American Economic Journal: Macroeconomics*, 1, 58-83.

Whitesell, William, 2006. "Interest rate corridors and reserves," *Journal of Monetary Economics*, Elsevier, vol. 53(6), 1177-1195.

Woodford, Michael. 2001. "Monetary Policy in the Information Economy," Symposium on Economic Policy for the Information Economy, Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, September.

Zhang, Nellie (2012). "Estimating the Demand for Settlement Balances in the Canadian Large Value Transfer System". Bank of Canada Working Paper 2015-15.