The Ever-Changing Challenges to Price Stability*

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Abstract

U.S. inflation risk is non-symmetric and varies considerably over time. Monetary and fiscal policies along with non-policy factors, such as unit labor costs, long-run interest rates, the unemployment gap, and commodity prices, are key drivers of the inflation risk. Macroeconomic predictors affect the long-run mean of inflation chiefly by influencing the shape and the skewness of the predictive distribution of long-run inflation. Inflation stabilization requires periodic revisions to the monetary and fiscal framework to counterbalance persistent shifts in the inflation risk. Failing to offset the inflation risk led to the large upside inflation risk of the 1960s and the 1970s. Our findings suggest that the Phillips curve is nonlinear and its slope is affected by policy and non-policy factors that have bearings on short-term volatility and risk of inflation.

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1 Introduction

After being largely neglected for almost three decades, inflation has become again a major concern for households, firms, and policymakers in many countries. It is not only the high inflation rate to raise concerns but it is also the general perception that the economy may be entering into a new regime where spikes in inflation become recurrent. Figure 1 lends support to this perception by showing the thickening of the right tail of the subjective distribution of inflation outcomes from the Survey of Professional Forecasters (SPF) and the Michigan Surveys of Consumers (MSC) in the most recent period.[1] In the past decade, the odds of low inflation or even deflation outcomes were considerably high, posing new challenges to policymakers (Clarida 2020; Schnabel 2021) and spurring a copious academic literature on the implications of recurrently binding lower-bound constraints on nominal interest rate for monetary policy (Gust et al. 2017; Bernanke et al. 2019). Despite the centrality of the topic, the economic literature studying the dynamics of inflation risks and the macroeconomic conditions conducive to these dynamics is surprisingly thin.

In this paper, we estimate the time-varying mean, variance, and asymmetry of the long- and short-run predictive distribution of U.S. core Personal Consumption Expenditure (PCE) inflation. Having the full conditional distribution at each point in time also allows us to evaluate the balance of risks to the inflation outlooks. We find that shifts in the balance of risk to inflation have been large and frequent in the U.S. postwar period. Our model allows us to distinguish between short- and long-run moments of inflation. The long-run mean and variance of inflation increase in the 1960s and in the 1970s and fall in the following decades. Similarly, long-run skewness remained substantially positive from mid-1960s through the end of 1980s, implying a long-run balance of risk clearly tilted towards the upside. Nevertheless, we find that long-run risk has been varying a lot in the past three decades. In the second part of the sample, until 2020, the balance of risks has become increasingly negative, driven by a swift decline in inflation skewness, which turned negative during the mid-1990s.

Our results show that swings in the long-run risks to inflation are largely predicted by changes in monetary and fiscal policy along with some non-policy factors, such as the structure of the labor market and the level of the nominal interest rates. Short-run risks are influenced by the unemployment gap, the monetary policy stance, and the international prices of commodities. Similar factors influence the long-run and short-term volatility of inflation. While the stance of monetary policy and the unemployment gap explain the lion share of the fluctuations in the first and second moment of short-run inflation, the skewness is influenced primarily by monetary policy and international commodity prices.

[1] Notice that the two surveys are rather different in their structure. Specifically, the MSC questionnaire focuses more on the expected probability on increases in the price levels. The sample median and mean for the MSC are about 3.5% and 4.5%, respectively. Therefore, we define left tail as the probability of inflation expectations below 1.5% for the SPF and below 3.5% for MSC, central corresponds to expectations in the $[1.5\%, 2.5\%)$ interval for SPF and $[3.5\%, 4.5\%)$ for MSC, whereas the right tail is defined as expectations above 2.5% and 4.5% for SPF and MSC, respectively.

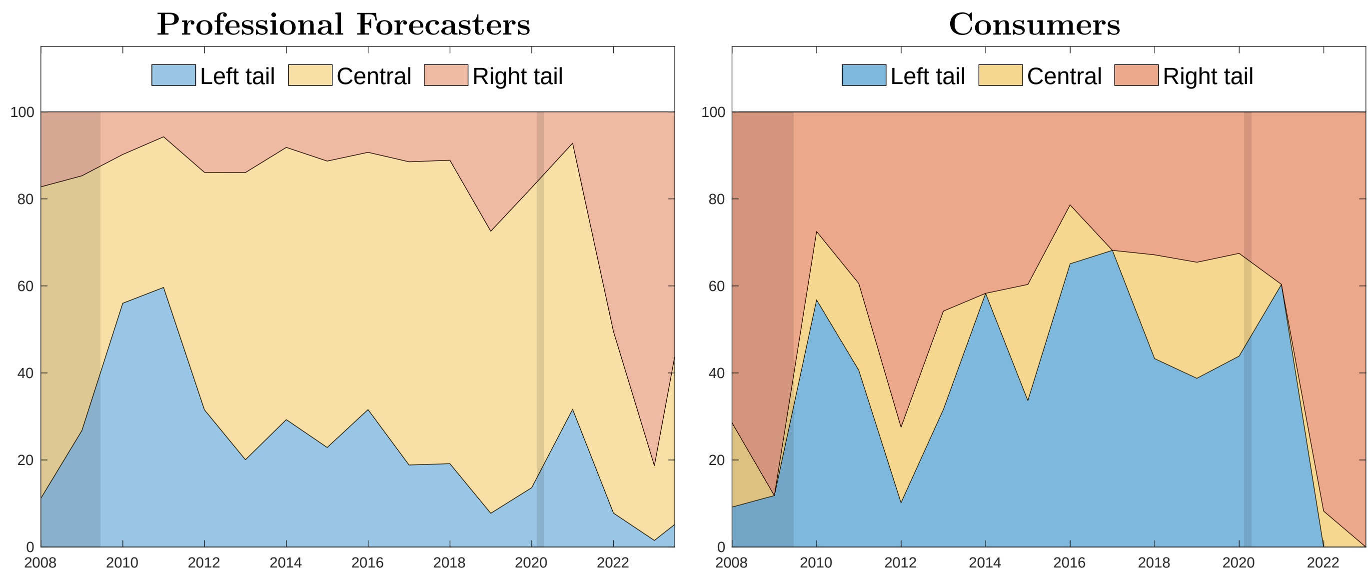


Figure 1: Inflation expectations

Note: The panels report Survey of Professional Forecasters (SPF, left) and University of Michigan's Survey of Consumers Finances' interval forecasts from 2008. We define left tail as the probability of inflation expectations below 1.5% for the SPF and below 3.5% for MSC, central corresponds to expectations in the $[1.5\%, 2.5\%)$ interval for SPF and $[3.5\%, 4.5\%)$ for MSC, whereas the right tail is defined as expectations above 2.5% and 4.5% for SPF and MSC, respectively. Gray shaded areas represent NBER recessions.

Non-policy factors, such as the level of the long-run interest rates and unit labor costs, have played an important role in propping up long-run inflation volatility and in tilting the long-run inflation risk to the upside in the 1960s and in the 1970s. The high levels of the interest rates suggest that the central bank has to raise its interest rate considerably to stabilize inflation, imposing large costs on the financial sector. Heightened unit labor costs captures persistent cost pressures for firms stemming from the labor market.

The monetary and fiscal framework was ill-set and exacerbated the upside risk to inflation through the end of 1970s. In the 1980s these policy factors contributed to lowering the variance and the positive skewness of long-run inflation and, in the 1990s, caused the long-run inflation risk to become tilted toward the negative side. The contribution of monetary and fiscal policies started to reverse by the end of that decade and in the post-Great Recession recovery, when they again tilted long-run inflation risk to the upside. In that period, the skewness remained on net negative because of the low long-run real interest rate. This last finding is consistent with the notion that in a low interest rate environment recurrent zero-lower-bound episodes lead to negatively skewed inflation. Long-run monetary and fiscal factors partially offset this downward pressures on the long-run skewness of inflation consistently with the recent structural study by Bianchi, Faccini, and Melosi (2023).

The central tendency or location of the predictive distribution of long-run inflation is largely unaffected by our predictors. This finding implies that changes in the long-run mean of inflation are chiefly explained by movements in the risk driven by predictors. Consequently, those factors that predict changes in the shape and skewness of the distribution of long-run inflation are also the drivers of the long-run mean of inflation.

Furthermore, we show that it is optimal for policymakers to review their policy frameworks from time to time so as to offset the frequent swings in the balance of risk to inflation. Examples of such frameworks include monetary policy make-up strategies – such as the average inflation targeting or asymmetric strategies – that aim to counterbalance the deflationary risk due to proximity of interest rates to their lower bound (e.g., Mertens and Williams 2019, Duarte et al. 2020, and Bianchi et al. 2021). While in the 1960s and 1970s the monetary and fiscal framework exacerbates the inflation risk on the upside, in the 1990s it pushes the balance of risk to inflation to the negative territory. In the 2020s, monetary and fiscal factors offset the risk of deflation resulting chiefly from the low interest rate environment. In regards to the short-term risk to inflation, monetary policy has contributed less to it in the most recent period as the central bank appears to have become more effective in adjusting the stance of monetary policy so as to counterbalance the effects of the unemployment gap and international commodities prices on the short-term risk to inflation.

We draw three main implications from our results. First, there is no such a thing as a one-size-fit-all policy framework; that is, a monetary policy strategy that remains optimal forever. Second, fiscal policy plays an important role in shaping the long-run distribution of inflation and its contribution to long-run inflation risk is almost invariably found to be correlated with that of monetary policy. The latter result holds true both when monetary and fiscal policies contributed to tilting the inflation risk to the upside in the 1970s and in the 2010s and when they contributed to pushing the balance of risk to the negative territory in the 1960s and in the 1990s. Taken together, these findings suggest the existence of monetary and fiscal regimes that evolve over time in a fairly coordinated manner as some structural studies have found (Bianchi and Ilut 2017 and Bianchi and Melosi 2017).

Third, we find evidence suggesting that the Phillips curve is nonlinear and its slope may depend on monetary policy. The elasticity of the short-run inflation expectations to the unemployment gap is time-varying in our model and crucially depends on short-run inflation volatility and on the balance of risk. This elasticity is connected to the slope of the Phillips curve. Everything else being equal, a more positively skewed distribution of inflation leads to a larger pass-through from the labor market to short-term inflation expectations; that is, a steeper Phillips curve. Similar effects are predicted if short-run volatility falls. In addition, a lower inflation volatility causes the slope to become less sensitive to inflation risk.

Interestingly, the model predicts that the unemployment gap is one of the leading predictors of short-run inflation uncertainty. A tight (loose) labor market is almost invariably associated with higher (lower) uncertainty regarding inflation. Since higher (lower) uncertainty corresponds to a steeper (flatter) Phillips curve, the model suggests a nonlinear link between expectations about short-term inflation and the unemployment gap. Specifically, inflation expectations increase at a faster pace as the unemployment gap becomes more negative (i.e., the labor market get tighter and tighter). Conversely, inflation expectations falls at a slower pace as the unemployment gap becomes larger and larger.

The other leading predictor of inflation uncertainty is monetary policy, which is found to mostly contribute to increasing inflation uncertainty in the 1960s and 1970s and to decreasing it in the subsequent decades. This finding in combination with the positive relation between the slope of the Phillips curve and inflation uncertainty suggests that monetary policy has been an important factor behind the increase in the slope of the Phillips curve in the 1970s and its fall in the most recent period.

Monetary policy also affects the time-varying slope of the Phillips curve by changing the balance of risk of inflation. In the 1960s and in the 1970s, monetary policy contributed to increasing the positive skewness of inflation whereas, in the subsequent decades, it has mostly contributed to tilting the inflation risk to the negative side. Given the high uncertainty of inflation in the 1960s and 1970s, the monetary-policy-led upside risk of inflation causes the Phillips curve to become steeper.

The relevance of modeling inflation risk dates back to the seminal work of Engle (1982) on time-varying inflation volatility. This has been proven to be a necessary feature of any model for inflation forecasting to produce accurate point (Stock and Watson, 2007) and density forecasts (see, e.g., Rossi, 2021). Only recently the attention has moved to the modelling of the whole density of inflation outcomes (Manzan and Zerom, 2013, 2015; Lopez-Salido and Loria, 2020; Korobilis et al., 2021). We follow Delle Monache et al. (2021) and introduce a parametric model for the whole density of US core PCE. The model allows for asymmetric innovations, drawn from a Skew-t distribution (see Arellano-Valle et al., 2005), and relies on the score-driven framework of Harvey (2013) and Creal et al. (2013) to set up laws of motion for the parameters, as in Delle Monache and Petrella (2017).[2] Following Stock and Watson (2007), we allow time-varying moments to feature trend components, mainly driven by structural policies, in line with Cogley and Sbordone (2008), and cyclical variations, aimed at capturing transitory, short-lived factors that can temporarily affect price dynamics ("cost-push" and demand forces, as in Gordon, 1970). We also allow a number of short- and long-run predictors to drive the dynamics of the time-varying parameters in order to assess the economic factors driving changes in the mean, the variance, and the downside and upside risk of inflation.

We consider a broad set of predictors with the objective of testing some of the leading theories of inflation. The long-run real interest rate may constrained monetary policy actions. For instance, the low nominal interest rate environment of the last decade has increased the frequency and duration of the zero lower bound (ZLB) spells, imparting a deflationary bias to inflation (e.g., negative skewness) according to theoretical macro models (Adam and Billi, 2007; Bianchi et al.,

[2]Score-driven dynamics provide, under some general conditions, optimal updates in the informational theoretic sense (Blasques et al., 2014). It is important to notice that the specification considered by Blasques et al. (2014) is a rather simple one, with only time-varying scale.

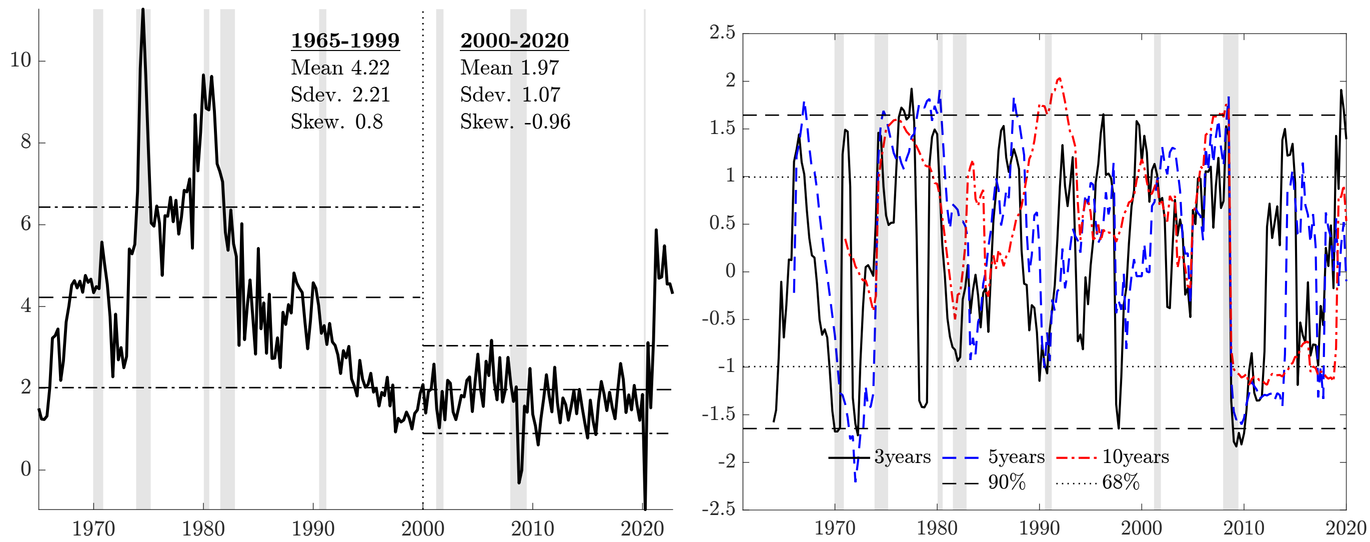


Figure 2: Inflation distributions

Note: The left panel show US core PCE from 1965Q1 to 2020Q1. The right panel reports rolling Bai and Ng (2005) test statistics, using windows of 3, 5 and 10 years, and the the 68 and 95% critical values. Gray shaded areas represent NBER recessions.

2021). When long-run nominal interest rates are very high, the central bank ought to increase the policy rate considerably to rein in inflation. But large monetary tightening puts stress on the financial system, which central banks typically want to avoid. The central bank's reluctance to move interest rates high enough to control inflation may be a factor tilting inflation risk to the upside. We consider Unit Labor Costs (ULC), capturing the role played by labor market conditions in affecting the cost of hiring and retaining workers. This predictor is consistent with the theory behind the Phillips Curve. The growth rate of money is shown by classical theories to contribute to inflation (see, e.g., Estrella and Mishkin, 1997). We also include a measure of fiscal stance in the US. The link between fiscal aggregates and inflation dynamics has been studied by proponents of the fiscal theory of the price level (Sargent and Wallace, 1981; Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 2001; Cochrane, 1998, 2001; Schmitt-Grohe and Uribe, 2000; Bassetto, 2002; Reis, 2016; Bassetto and Sargent, 2020, among many others).

2 Price stability: some facts

Over the last 60 years, US core PCE inflation has experienced substantial variability. The great inflation of the 1970s sees to be in stark contrast with the slow pace at which prices have grown in the aftermath of the Great Financial Crisis. Figure 2 shows two interesting facts about inflation data. In the left panel we split the sample around 2000 and report the mean, standard deviation and skewness of the subsamples; for the second sample, the post-2000, we compute moments with data until the end of 2022 to prevent the most recent data to dominate the statistics. The sample average has more than halved, moving from about 4.5% to just below 2%; since the FED formally set a 2% inflation target in 2012, this value has further reduced to 1.6%.[3] The values show a clear break in the sample average, but also a dramatic shift in the the higher-order moments. Inflation volatility has reduced two-fold, from about 2.2 to around 1.0, whereas sample skewness has changed sign, moving from 0.8 to -0.95.[4] We further investigate the change in inflation skewness in the right panel of Figure 2. We compute Bai and Ng (2005) statistics for the sample skewness, along with 68% (dotted) and 90% (dashed) critical values, computed with rolling windows of 3, 5 and 10 years. Results support evidence of time variations in sample skewness, with the time series of the test statistics showing substantial variation over the sample, with several periods of significant values.

The right panel of Figure 2 shows a sharp fall in inflation skewness towards the end of the sample, pointing at excess downside risks in the inflation outlook. Since the turn of the century, core inflation has consistently undershot the Fed's 2% target. These results directly speak to the evidence reported in Figure 1. The plots clearly show a "*deflationary bias*" in inflation expectations, both for SPF and MSC, which lost their anchor at the target level of 2%, as observed by Yellen (2015) and Reis (2021), and experiencing a surge in disagreement (see, e.g., Allayioti et al., 2023).[5] This dynamics has started to quickly revert over the last few quarters, signalling increasing risks to the upside.

3 Skewed inflation risk and monetary policy

The deflationary bias observed since the Great Financial Crisis has called for policy makers to renew their commitment to maintain price stability. Over the last two years, two of the major Central Banks, the Federal Reserve and the European Central Bank, have reviewed their approach to keeping inflation close to a target level of about 2% over the medium horizon. The major update coming out of such *strategy reviews* was a move from a symmetric inflation target, that is the Central Banks reacted equally to above- or below-target inflation realizations (the so called "bygones-be-bygones"), in favor of an *asymmetric* target, according to which, for example, the policy maker will allow inflation to run higher than target after periods of below target levels (see, e.g., FED, 2021; Reichlin et al., 2021).

Within this new policy environment, the optimal monetary policy response to inflation fluctuation needs to take into account the evidence of asymmetric risks to the inflation outlook. We consider an environment where the agents and the monetary authority track the predictive distribution of inflation, and therefore inflation risk, forming expectations considering the possible

[3]It is well established that before 2012 the FOMC informally targeted inflation around 2% (see, e.g., Bullard, 2018).

[4]To help the interpretation, we calibrate two Skew-Normal distributions (see, e.g., Mudholkar and Hutson, 2000) to match the sample moments in the two subperiods. A change in skewness from 0.8 to -0.95 is equivalent to an increase in the probability of observing realizations below the mode from 0.15 to 0.75.

[5]For example, SPF forecasts averaged around 1.75% over the 2000-2019 period.

presence of non-Gaussianity and nonlinearities.[6] We start with a simple case, in line with the traditional Barro and Gordon (1983) framework, where the Central Bank action can affect current inflation, via inflationary surprises, $\varepsilon_{t+1} = \pi_{t+1} - \mu_{t+1|t}$, defined as a deviation of current inflation from the modal inflation forecast, $\mu_{t|t-1}$. When the loss function is quadratic, it is optimal for the Central Bank to anchor inflation expectations to the preannounced inflation target,

$$E_t \pi_{t+1} = \pi^*. \tag{1}$$

Let us now assume that inflation expectations are formed by means of a generic linear learning rule, $f_\mu \left(\mu_{t|t-1}, \varepsilon_t\right) = b_\mu \mu_{t|t-1} + a_\mu \varepsilon_t$.[7] When innovations are Gaussian, $E_t \pi_{t+1} = \mu_{t+1|t}$ and the optimal level of inflation surprises can be expressed as

$$\mu_{t+1|t} = \pi^* = b_\mu \mu_{t|t-1} + a_\mu \varepsilon_t,$$

such that

$$\varepsilon_t = \frac{1}{a_\mu} \left(\pi^* - b_\mu \mu_{t|t-1}\right),$$

and the optimal level of inflation surprises depends on i) the perceived persistence in inflation expectations, and thus on the the deviation of future expected inflation from the optimal level, absent any intervention (i.e. $\pi^* - b_\mu \mu_{t|t-1}$) and ii) the speed at which inflation surprises are incorporated into revisions of future expected inflation (regulated by the coefficient, a_μ).

Let us now consider the situation where inflation risk is not symmetric and can be characterized by a general class of asymmetric distribution with fixed scale, σ, and shape, ρ, parameter (Fechner, 1897).[8] In this environment, the modal forecast (i.e., the most likely scenario) deviates from the expected forecast, such that $E_t \pi_{t+1} = \mu_{t+1|t} + g(\sigma, \rho, \upsilon)$, where $g(\cdot)$ denotes the impact of higher order terms on expected inflation which tilts the mean forecast in the direction of the skewness. In this environment the optimal inflation surprise that achieves the objective in Equation (1) sets

$$\varepsilon_t = \frac{1}{a_\mu} \left[\pi^* - b_\mu \mu_{t|t-1} + g(\sigma, \rho, \upsilon)\right], \tag{2}$$

meaning that higher moments affect the optimal policy actions whereby the Central Bank optimal inflation surprise needs to offset the perceived skewness in inflation risk, represented by the function $g(\cdot)$. In a situation with a positive inflation risk (i.e. $\rho > 0$), the optimal response requires that

[6]In the next Section we will introduce a parsimonious model that can be considered as an approximation to a more general Bayesian learning in an environment characterized by non-Gaussianity and nonlinearities (see Buccheri et al., 2021).

[7]This simple autoregressive specification is consistent with the traditional Bayesian updating, which uses the Kalman filter to update expectations in a linear-Guassian environment, in this case updates are proportional to the prediction errors.

[8]Asymmetric densities have been widely used to communicate future inflation outlooks since the mid-1990s (Wallis, 1999). See Wallis (2014) for an overview of the history of asymmetric (two-piece) distributions.

Table 1: Müller and Watson (2018) long-run covariability

	Location		Dispersion		Asymmetry	
	Sample	Quantile	Sample	Quantile	Sample	Quantile
	Low frequency					
ΔULC	**0.445**	**0.448**	**0.538**	0.213	0.273	0.028
	[0.027,0.719]	[0.028,0.721]	[0.184,0.813]	[−0.158,0.539]	[−0.103,0.593]	[−0.379,0.443]
FSD	−0.133	−0.157	−0.200	−0.030	**0.454**	**0.503**
	[−0.511,0.210]	[−0.524,0.209]	[−0.529,0.158]	[−0.413,0.337]	[0.082,0.714]	[0.115,0.782]
ΔM3N	0.150	0.161	0.213	−0.022	−0.031	**−0.412**
	[−0.212,0.503]	[−0.209,0.524]	[−0.150,0.533]	[−0.413,0.365]	[−0.429,0.334]	[−0.669,−0.001]
LRR	**0.825**	**0.813**	**0.651**	**0.447**	**0.461**	0.273
	[0.539,0.931]	[0.513,0.924]	[0.301,0.866]	[0.013,0.724]	[0.036,0.772]	[−0.102,0.596]

Note: The table reports sample correlations (*All frequency*) and the long-run covariabilities of (Low frequency, Müller and Watson, 2018) between predictors and inflation moments. We consider mean and the median as measures of location, sample standard deviation and the interquantile range for the dispersion and (rescaled) sample and quantile skewness for the asymmetry. 68% confidence intervals are reported in brackets. Values in **bold** indicate significant results. **ΔULC**: unit labor cost; **FSD**: federal surplus/deficit; **M3N**: M3 aggregate growth.

the policy maker sets the modal forecast below the inflation target to offset excess upside risk. In a more general setting with time-varying risks, the optimal policy is achieved when

$$E_t\pi_{t+1} = \mu_{t+1|t} + g\left(\sigma_{t+1|t}, \rho_{t+1|t}, v\right) = \pi^*, \tag{3}$$

making it optimal for the Central Bank to systematically set policies that allow the modal forecast to overshoot (undershoot) the target when there is positive (negative) excess inflation risk.

4 Determinants of trend skewness in inflation

We run a preliminary analysis about the historical drivers of inflation dynamics, considering some selected explanatory variables, related to the most prominent theories of inflation. We use quarterly figures starting from 1965Q1 until 2020Q1. Inflation is measured as the annualized price inflation rate of the Personal Consumption Expenditures (PCE), excluding food and energy prices in the US.[9] We consider changes in Unit Labor Costs, often embedded in most structural models to study inflation dynamics as suggested by the theory of the Phillips curve. Fiscal policy is measured as the ratio of primary surplus to the stock of the Federal debt. According to the fiscal theory of the price level, if this ratio is negative, inflation has to rise to prevent debt from growing exponentially. In that, values above the real average interest rate paid on the public debt characterize periods when the fiscal authority is raising primary surplus at a pace that is commensurate to stabilize

[9]We prefer PCE over the Consumer Price Index (CPI) due to the relevance of the former in the policy decision of the Federal Reserve.

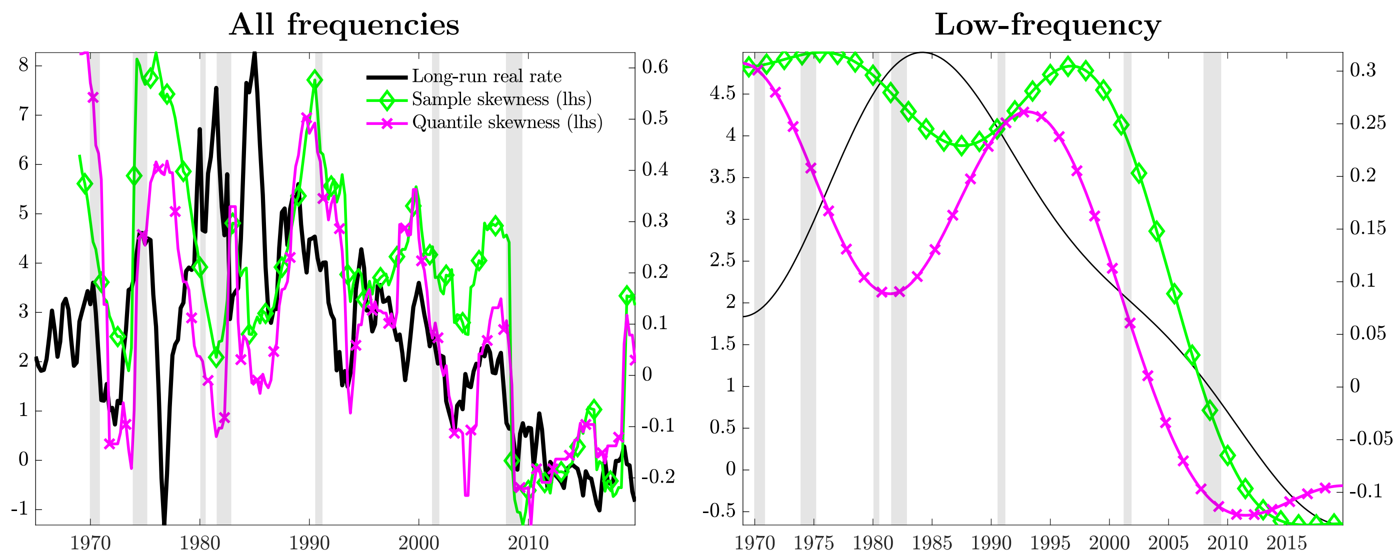


Figure 3: Skewness and the long-run real rate

Note: The left plot compare the long-run real rate and two non-parametric measures of time-varying skewness: rolling sample skewness (green) and rolling quantile skewness at the 5% level (magenta). The right panel reports the low frequency of the measures. Gray shaded areas represent NBER recessions.

the debt, so that the real value of government debt falls over time. We include the long-run real interest rate as a direct measure of the cost of refinancing for firms and households, and a gauge of the broad financial conditions; historical data are reconstructed using data from Liu and Wu (2021). Monetary policy is considered via changes in the M3 monetary aggregate. The analysis rests on assessing pairwise covariablity between the predictors and measures of location, dispersion and asymmetry of inflation's distribution, computed over rolling windows of 10 years. We consider long-run covariability by means of the approach of Müller and Watson (2018). This methodology captures long-run covariability by estimating the sample moments of cosine-weighted averages of the data that smooth out short-run fluctuations. In this application we captured periodicities of approximately 80 quarters, (20 years, $q = 6$). Table 1 shows the low-frequency covariabilities. Interestingly, the variables related with different magnitudes to the different measures of inflation moments. To visualize such comovements, Figure 3 reports the time-series plots of the long-run real rate (black) and the non-parametric measures of inflation skewness, considering all frequencies in the left panel, and only low-frequencies in the right panel. Whereas in the early sample the comovement is not striking, starting from the second half of the 1980s the series follow similar patterns, with a clear downward trend in inflation skewness, that seem to be anticipated by falling real rates.

5 Model specification

Inflation dynamics is commonly modelled via the New Keynesian Phillips Curve (NKPC),

$$\pi_t = \mathbb{E}_t \, \pi_{t+1} + \psi X_t, \tag{4}$$

which prescribes a linear relation between current inflation and a set of predictors, including measures of slack. Whereas the NKPC implies a structural relation between inflation and the predictors, already Gordon (1981) found Equation (4) to successfully fit the data (see also Atkeson et al., 2001; Stock and Watson, 2008; Faust and Wright, 2013). We propose to go beyond the linear relation implied by Equation (4) and set up a Phillips Curve-type model for the full distribution of inflation, which includes a time-varying trend and stochastic volatility, in the spirit of Stock and Watson (2007). Let $\pi_t = 400 \log(p_t/p_{t-1})$ be annualized, quarter-on-quarter (core) inflation,

$$\pi_t = \mu_{t|t-1} + \varepsilon_t, \quad \varepsilon_t \sim Skt_\nu(0, \sigma^2_{t|t-1}, \varrho_{t|t-1}), \tag{5}$$

where the innovation ε_t is distributed as a Skew-t distribution (Arellano-Valle et al., 2005; Gómez et al., 2007) with constant degrees of freedom ν and time-varying location μ_t, scale σ_t, and shape ϱ_t parameters, estimated conditional to time $t-1$.[10] The conditional log-likelihood function of the observation at time t is:

$$\ell_t = \log p(\pi_t|\theta, \Pi_{t-1}) = \log \mathcal{C}(\eta) - \frac{1}{2}\log \sigma^2_t - \frac{1+\eta}{2\eta} \begin{cases} \log\left[1 + \frac{\eta\varepsilon_t^2}{(1+\varrho_t)^2\sigma_t^2}\right], & \varepsilon_t \geq 0 \\ \log\left[1 + \frac{\eta\varepsilon_t^2}{(1-\varrho_t)^2\sigma_t^2}\right], & \varepsilon_t < 0 \end{cases}, \tag{6}$$

with $\eta = \frac{1}{\nu}$ being the inverse of the degrees of freedom, $\mathcal{C}(\eta) = \frac{\Gamma\left(\frac{1+\eta}{2\eta}\right)}{\sqrt{\frac{\pi}{\eta}}\Gamma\left(\frac{1}{2\eta}\right)}$, $\Gamma(\cdot)$ is the Gamma function, and $sgn(\cdot)$ is the sign function. The vector θ collects the static parameters, and Π_{t-1} is the information set including past inflation and past parameter values, $\mathtt{f_t} = (\mu_\mathtt{t}, \sigma^2_\mathtt{t}, \varrho_\mathtt{t})'$. The distribution of inflation realizations is positively (negatively) skewed for $\varrho > 0$ ($\varrho < 0$). This specification allows as special cases the symmetric Student-t distribution when $\varrho_t = 0$, Mudholkar and Hutson (2000) epsilon-Skew-Gaussian for $\nu \to \infty$ and the Gaussian density when both conditions hold. Thus, we allow for, but do not impose, asymmetric innovation terms.

To ensure $\sigma_t > 0$ and $\varrho_t \in [-1, 1]$, we model $\delta_t = \log(\sigma_t)$ and $\gamma_t = \operatorname{arctanh}(\varrho_t)$, such that $f_t = (\mu_t, \delta_t, \gamma_t)'$ and

$$f_{t+1} = \bar{f}_{t+1} + \beta X_t. \tag{7}$$

$$\bar{f}_{t+1} = \bar{f}_t + \kappa s_t, \tag{8}$$

[10] Henceforth, we will simplify the notation "$t|t-1$" into "t".

such that the parameters are linear functions of observed variables, X_t, plus a *residual trend component*, $\bar{f}_t$, following a score-driven random walk specification; this nests the limiting case where the long-run component is entirely in line with observable variables. The scaled score, s_t, is defined as $s_t = \mathcal{S}_t \nabla_t$, where:

$$\nabla_t = \frac{\partial \ell_t}{\partial f_t} \frac{\partial f_t}{\partial \mathtt{f_t}}, \qquad \mathcal{S}_t = diag(\mathcal{I}_t)^{-1},$$

with ∇_t being a vector of scores, namely the gradient of the likelihood function ℓ_t with respect to the dynamic parameters, while the scaling matrix $\mathcal{S}_t$ is proportional to the Moore-Penrose pseudo-inverse of the diagonal of the Information matrix, $\mathcal{I}_t = \mathbb{E}\left[\nabla \nabla'\right]$. In Appendix A we provide detailed derivations for the score and a discussion of how current news about inflation (i.e., $\pi_t - \mu_t$) are translated into updates of the conditional distribution of future inflation.

Basic Features of the Model. Equations (5) to (8) describes a model that build on the UCSV model of Stock and Watson (2007) but considers a broader definition of risk to inflation, by allowing for time-varying skewness in the predictive distribution of inflation.[11] Whereas Equation (4) assumes a linear relation for the expected value of inflation, an attractive feature of the Skew-t distribution is that both mean and variance are non-linear functions of the shape of the entire distribution. Specifically, one can show that:

$$\mathbb{E}(\pi_t | \Pi_{t-1}) = \mu_t + g(\eta) \sigma_t \varrho_t, \qquad g(\eta) = \frac{4\mathcal{C}(\eta)}{1-\eta}, \tag{9}$$

$$Var(\pi_t | \Pi_{t-1}) = \sigma_t^2 \left(\frac{1}{1-2\eta} + h(\eta) \varrho_t^2 \right), \qquad h(\eta) = \frac{3}{1-2\eta} - g(\eta)^2. \tag{10}$$

Both equations include a part equal to the mean and variance of a standard Student-t distribution, μ_t and $\frac{\sigma_t^2}{1-2\eta}$ respectively, plus non-linear functions of the shape parameter. A positive asymmetry coefficient triggers a positive correction of the location parameter, so that the mean of the distribution lies right of the mode, implying that the chance of observing values greater than the modal one is greater than that of observing smaller values; the converse is true for values of $\varrho_t < 0$. It is important to notice that developments of the asymmetry parameters are magnified by larger values of σ_t for the expected value, which is always positively affected by shifts of the shape parameter (i.e. $\frac{\partial \mathbb{E}(y_t | Y_{t-1})}{\partial \varrho_t} > 0, \forall \varrho_t, \eta$). Similarly, an increase of asymmetry is associated with an increase in the variance when the distribution is positively skewed (i.e. for $\varrho_t > 0$), and a decrease in the variance when the distribution is negatively skewed (i.e. for $\varrho_t < 0$).[12] Therefore, procyclical variations of the skewness are reflected into a time-varying correlation between mean and volatility.

[11] Notice that Stock and Watson (2007) is a *parameter-driven* model, contrary to our choice, that falls in the category of the *observation-driven*; see Cox (1981).

[12] In fact, $\frac{\partial Var(y_t | Y_{t-1})}{\partial \varrho_t} = 2h(\eta)\sigma_t^2 \varrho_t$, and since $h(\eta) > 0$ for $\nu > 3$, the shift in the variance will be of the same sign as the level of the shape parameter (thus of the same sign to the level of the conditional skewness).

Estimation. The parameters of the model and the associated conditional distribution of inflation are estimated using Bayesian methods. Loadings on the score component are Gamma distributed, with mean 0.1 and variance 0.005. This choice ensures that the filter is invertible (Blasques et al., 2022), that is, it (a) reduces the possibility of overshooting in the direction of the (local) optimum, and (b) assumes conservative views on parameters time variation.[13] We assume Normal priors for the loadings associated to the predictors, with means centered around zero, and standard deviations being drawn from half-Cauchy distributions, in the spirit of the *Hosre-shoe* shrinkage prior (Carvalho et al., 2010). Lastly, we assume an inverse gamma prior for η. We set up an adaptive Random-Walk Metropolis-Hastings algorithm (ARWMH, Haario et al., 1999). Credible sets for both static and time-varying parameters are obtained from the empirical distribution functions arising from the resampling. See Delle Monache et al. (2021) for a detailed discussion.

6 Short- and long-run drivers of inflation risk

We start from the evidence in Table 1 and consider two broad sets of predictors: variable that predict long-run movements in inflation, such as the low-frequency components -computed in the previous Section- of changes in Unit Labor Costs (ΔUCL), Federal Surplus/Deficit (FSD), change in money supply (ΔM3N) and the Long-term Rate (LRR), and variables that relate to cyclical variations in the moments of inflation such as the unemployment gap (UGP), the FFR/2y-rate spread (MPS),[14] the short-run component (e.g., the cyclical part around the low-frequency component) of changes in Unit Labor Costs (ΔUCL), International Commodity Price (ICI) and Real Exchange Rate (RER). Figure 4 reports the time-varying moments of the conditional distribution of inflation, with full moment being in blue, while long-run component are reported in red. The time-varying mean matches the well-documented narrative for trend inflation (see, e.g., Stock and Watson, 2016), which started to increase in the mid-1960s and then sluggishly fell starting from the early 1980s through the mid-1990s when it stabilized around 2 %. Following the Great Recession, trend inflation crept down and remained below 2% for an entire decade. In the pandemic, trend inflation soared considerably, reaching values unseen since the early 1980s.

Inflation volatility was heightened in the 1970s and peaked twice between the mid 1970s and mid 1980s. In the 1980s volatility started its decline until it reached extremely low value in the 1990s and in the first half of the 2000s, consistent with the *Great Moderation* narrative of McConnell and Perez-Quiros (2000) and Stock and Watson (2002). In the following decade, volatility inflation

[13] We further enforce the following condition to hold via a rejection step:

$$\frac{1}{T}\sum_{t=1}^{T}\log\left|I + K\frac{\partial s_t}{\partial f_t'}(\theta)\right| < 0.$$

[14] The FFR/2y-rate spread is an indicator of the monetary policy stance. In our sample it is also strongly correlated with the short-run component of the real rate.

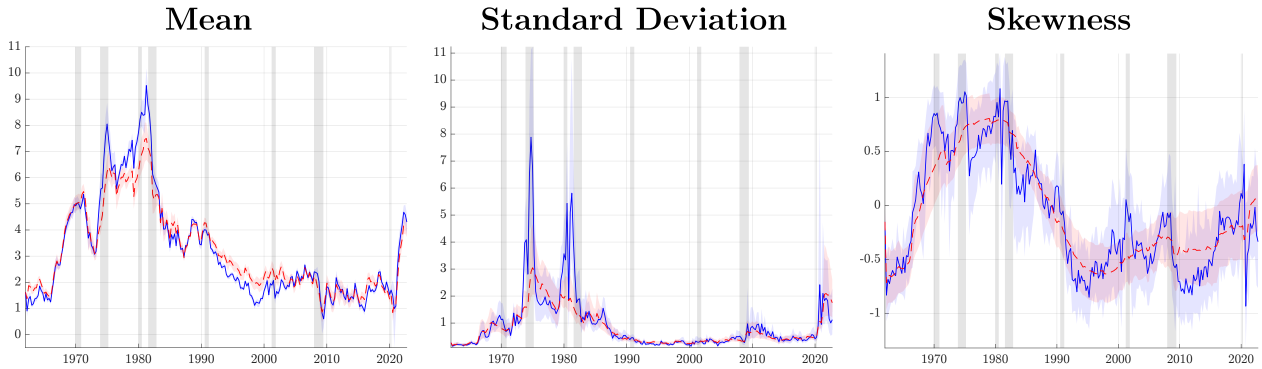


Figure 4: Time-varying moments with predictors

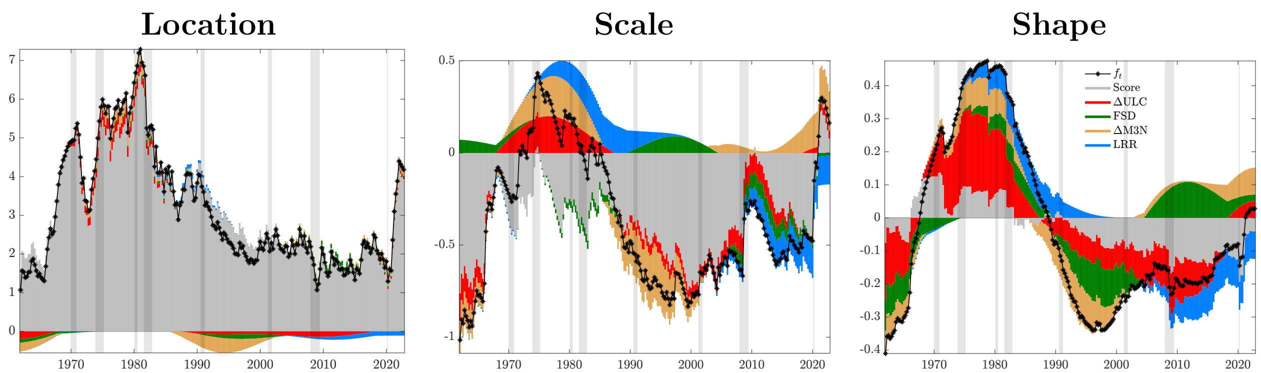
Note: Moments of the conditional distribution of inflation, in blue, and associated long-run components, in red. Shadings around the curve report the 68-90% credible sets. NBER recessions are shaded in gray.

increased moderately and experienced a discrete increase in the pandemic period at the end of our sample.

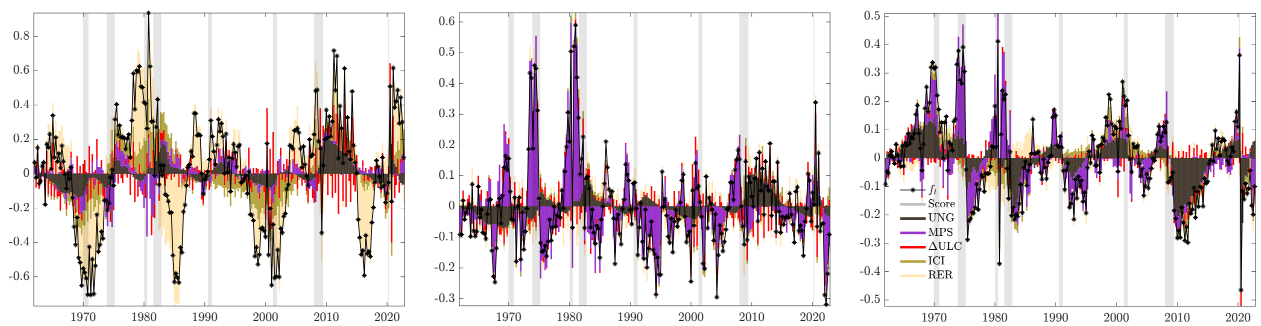
Let us turn our attention to the skewness on the right panel of Figure 4. Skewness follows a fascinating pattern with three distinct phases: the early 1960s, the 1970s-1980s, and the most recent period. The distribution of inflation transitioned from being tilted to the left to being tilted to the right in the mid 1960s. The risk of inflation remained skewed to the upside through the end of the 1980s. From the beginning of 1990s through the end of the sample, our estimated distribution of inflation was tilted to the left, suggesting that inflation have chiefly surprised to the downside in the most recent period.

To better understand the impact of predictors in shaping the moments of inflation outlook, Figure 5 reports a decomposition of the models' parameters, μ_t, γ_t and δ_t, into what is attributed to each predictor and, for the long-run components a part reflecting the endogenous update driven by past information (i.e. the score component). Swings in the long-run risks to inflation are largely predicted by changes in monetary and fiscal policy along with some non-policy factors, such as the structure of the labor market and the level of the nominal interest rates. Short-run risks are influenced by the unemployment gap, the monetary policy stance, and the international prices of commodities. Similar factors influence the long-run and short-term volatility of inflation. While the stance of monetary policy and the unemployment gap explain the lion share of the fluctuations in the first and second moment of short-run inflation, the dynamics of short-run inflation skewness is influenced primarily by monetary policy and international commodity prices.

Non-policy factors, such as the level of the long-run interest rates and unit labor costs, have played an important role in propping up long-run inflation volatility and in tilting the long-run inflation risk to the upside in the 1960s and in the 1970s. The high levels of the interest rates suggest that the central bank has to raise its interest rate considerably to stabilize inflation, imposing large costs on the financial sector. Heightened unit labor costs captures persistent cost pressures for firms stemming from the labor market.



(a) Low-frequency components



(b) High-frequency components

Figure 5: Time-varying parameter decomposition

Note: The plots report the decomposition of the unrestricted parameters into a score-driven component (blue) and predictor-specific contributions. Top panels report the decompositions of short-run parameters. Bottom panels plots long-run parameters. Gray shaded areas represent NBER recessions. **Δ ULC**, changes in unit labor costs; **FSD**, fiscal surplus/deficit; **ΔM3N**, money growth; **LRR**, long-run real rate; **UNG**, unemployment gap; **MPS**, monetary policy stance; **ICI**, international commodity index; **RER**, real exchange rate.

The monetary and fiscal framework exacerbated the upside risk to inflation due to non-policy factors through the end of 1970s. In the 1980s these policy factors contributed to lowering the variance and the positive skewness of long-run inflation and, in the 1990s, caused the long-run inflation risk to become tilted toward the negative side. The contribution of monetary and fiscal policies started to reverse by the end of that decade and in the post-Great Recession recovery, when they again tilted long-run inflation risk to the upside. In that period, the skewness remained on net negative because of the low long-run real interest rate. This last finding is consistent with the notion that in a low interest rate environment recurrent zero-lower-bound episodes lead to negatively skewed inflation (Adam and Billi 2007 and Nakov 2008). Long-run monetary and fiscal factors partially offset this downward pressures on the long-run skewness of inflation consistently with the recent structural study by Bianchi, Faccini, and Melosi (2023).

Among the short-run predictors, the unemployment gap and the monetary policy stance drive most of the cyclical variation. Phillips curve relations emerge in the dynamics of the location as periods when the unemployment rate is above the NAIRU depress the level of inflation, greatly reducing the cyclical variability of the predictive densities.

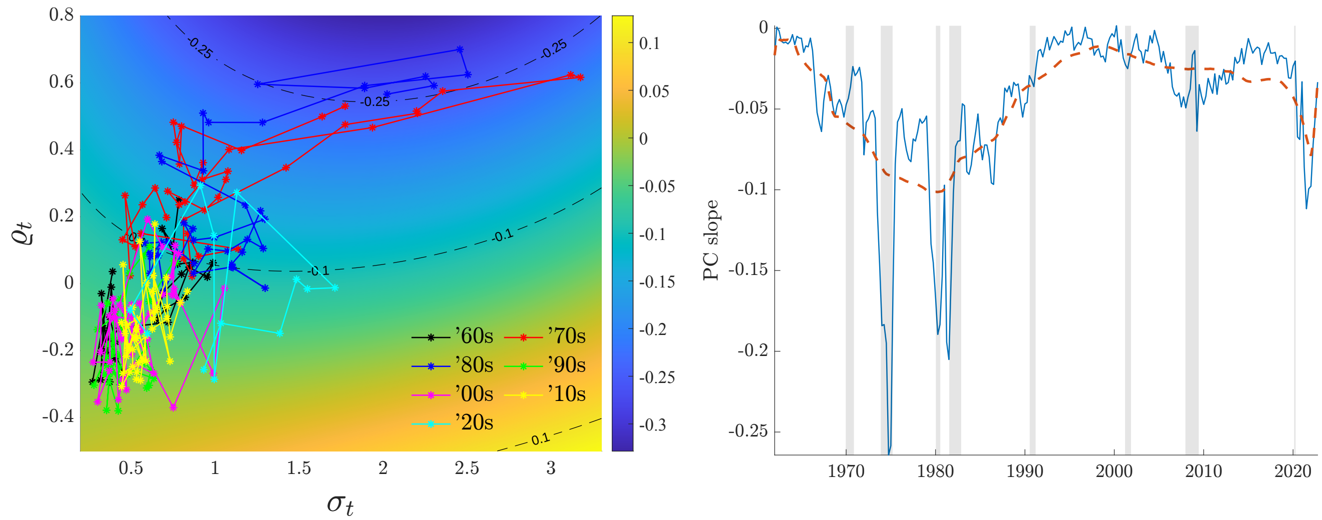


Figure 6: Short-run elasticities with respect to Unemployment Gap

Note: The left panel reports the Phillips Curve slope as a function of the levels of asymmetry (y-axis) and scale (x-axis); slopes values are reported as level curves. The right panel shows the time series plot of the PC slope against the unemployment gap. Gray shaded areas represent NBER recessions.

Dynamic response to predictors and the slope of the Phillips curve. Within this specification of the model, $\gamma_{t+1} = \ln(\sigma_{t+1})$ and $\delta_{t+1} = \text{arctanh}(\varrho_{t+1})$ are linear functions of the exogenous predictors. As a consequence, these predictors will shape the distribution of expected inflation, depending on the current value of the distribution's parameters. Given Equation (9), we can compute the elasticities of expected value and variance to changes in the generic predictor x_t:

$$\frac{\partial E_t\left(\pi_{t+1}\right)}{\partial x_t} = \beta_{\mu x} + g\left(\eta\right)\left[\varrho_{t+1}\frac{\partial \sigma_{t+1}}{\partial \gamma_{t+1}}\beta_{\gamma x} + \sigma_{t+1}\frac{\partial \varrho_{t+1}}{\partial \delta_{t+1}}\beta_{\delta x}\right]. \tag{11}$$

This coefficient varies over time and can potentially change sign, being it a function of ϱ_{t+1}. Thus, for example, in periods of positively skewed inflation (e.g., the 1970s), average inflation could have responded to developments in some predictors rather then others, whilst being less affected by the same predictors in periods characterized by symmetric or negatively skewed distributions. Interestingly, if we interpret the correlation between current slack and expected inflation as the "slope of the Phillips Curve", this can potentially be time-varying, being a function of volatility and skewness of the conditional distribution of inflation.[15]

The right panel of Figure 6 reports the dynamic response of the conditional mean to the unemployment gap. The figure can be interpreted as the time-varying slope of a New Keynesian Phillips curve. In the left panel, we report the estimated volatility (horizontal axis) and skewness

[15]In as similar fashion, from Equation (10), we can derive

$$\frac{\partial Var_t\left(\pi_{t+1}\right)}{\partial x_t} = \left(\frac{1}{1-2\eta} + h\left(\eta\right)\varrho_{t+1}^2\right)\frac{\partial \sigma_{t+1}^2}{\partial \gamma_{t+1}}\beta_{\sigma x} + \sigma_{t+1}^2 h\left(\eta\right)\frac{\partial \varrho_{t+1}^2}{\partial \delta_{t+1}}\beta_{\delta x}.$$

(vertical axis) and the estimated Phillips curve's slope – the black dashed lines in the graph.

Looking at how the slope of the Phillips curve (the black dashed line) is affected by the skewness only if the inflation volatility is sufficiently large. In the 1970s and 1980s (the red and blue lines), the slope of the Phillips curve was large because the volatility of inflation was elevated and, concomitantly, inflation skewness was at its highest levels. In the subsequent decades, the sharp fall in inflation volatility implies that changes in the skewness have considerably less impact on the slope of the Phillips curve. Note how the black dashed lines bend upward as inflation volatility on the horizontal axis is lowered.[16]

To sum up, there are two main takeaways from this analysis of the Phillips curve's slope. First, the slope of the Phillips curve depends on time-varying inflation volatility. Thus, by raising inflation volatility, the pandemic period has indeed steepened the curve.

Second, we find suggestive evidence that the Phillips curve is not an exploitable relation for policymakers. To the extent that sound monetary policy lowers the volatility of inflation, the slope of the Phillips curve reflects the effectiveness of a country's monetary framework.

7 Balance of risk and policy counterfactuals

In Section 3, we restricted out example to a simple quadratic loss function (Equation (1)) for its appealing for its properties. Here, following Kilian and Manganelli (2007, 2008), we depart from this assumption and assume that the Central Bank defines its loss in terms of upside and downside risk ($UR(\pi_{t+1})$ and $DR(\pi_{t+1})$, respectively) with respect to an inflation target, π^*:

$$L(\pi_{t+1}) = \{aE_t[DR(\pi_{t+1})] + (1-a)E_t[UR(\pi_{t+1})]\}$$

with $a \in [0,1]$. From the first order conditions it follows that the optimal policy consists of balancing upside and downside risks:

$$-a\frac{\partial E_t[DR(\pi_{t+1})]}{\partial \pi_t} = (1-a)\frac{\partial E_t[UR(\pi_{t+1})]}{\partial \pi_t}, \tag{12}$$

so that inflation surprises perfectly offsets expected losses of overshooting or undershooting the target; these costs can be generically defined by the upside and downside risk.

Assuming the policy maker weight equally upside and downside deviations ($a = \frac{1}{2}$),

$$E_t[DR(\pi_{t+1})] = \int_{-\infty}^{\pi^*} (\pi^* - \pi_{t+1})^2 dF_\pi \quad \text{and} \quad E_t[UR(\pi_{t+1})] = \int_{\pi^*}^{\infty} (\pi_{t+1} - \pi^*)^2 dF_\pi, \tag{13}$$

[16] Evidence of the flattening of the New Keynesian Phillips curve is shown by Galí and Gambetti (2019), Stock and Watson (2020), and Del Negro et al. (2020) among others. Our model suggests that the extremely low volatility of inflation caused the flattening of the Phillips curve in the pre-Pandemic period.

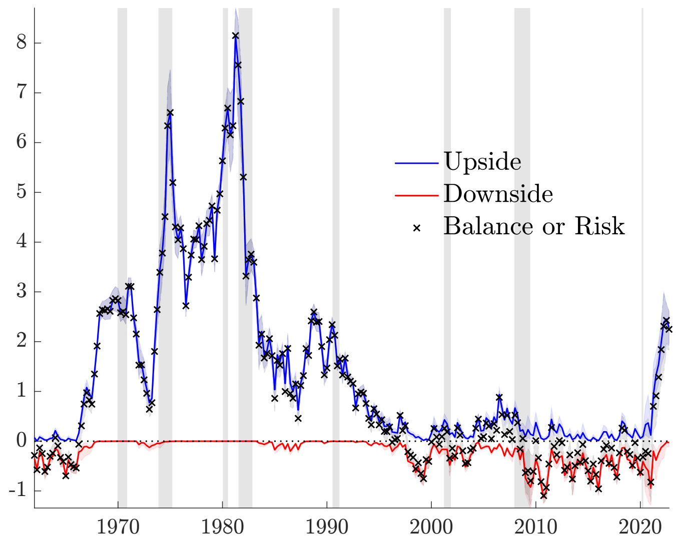


Figure 7: Balance of risk

Note: The panel reports the estimated upside- (blue) and downside-risk (red) components (defined in Equation (13)), and the corresponding balance of risks (black). Gray shaded areas represent NBER recessions.

where F_π denotes the predictive distribution of inflation, summarizing inflation risk. The optimal policy requires the Central Bank to set

$$\int_{-\infty}^{\pi^*} (\pi^* - \pi_{t+1})\, dF_\pi = \int_{\pi^*}^{\infty} (\pi_{t+1} - \pi^*)\, dF_\pi \tag{14}$$

which implies the Central Bank's expectations of positive and negative deviations from the target to be, ex-ante, equal in magnitude. When the predictive distribution of inflation is symmetric, this is achieved by setting expected inflation to the expected modal value. Conversely, when inflation risk is skewed, the optimality condition (Equation (14)) implies that larger deviations expected on the side of the skewness are – partially – offset by tilting the modal value in the opposite direction of the skew of the distribution. For instance, a positively skewed risk in inflation requires the Central Bank to target a mode below the target, in order to compensate for the higher chance of positive prediction, on average. The availability of full distributions for inflation outcomes allows us to evaluate the balance of risk over time. We set the inflation target, π^*, to 2% and compute the measures described in Equation (14). We report the balance of risk in Figure 7.

The Figure clearly shows that the balance of risks has been massively tilted towards the upside across all the 1970s, but has steadily reversed since the Volker disinflation. Downside risks to inflation took over in the post-Great Financial Crisis. Over the last few quarters, we estimate upside risks of magnitudes similar to those experienced in the early part of the sample. Decomposing the balance of risk over the long- and short-run components highlights a clear predominance of long-run risks. Nontheless, short-run risk were higher during the 1970s, but have significantly flattened ever-since, suggesting that the depressed inflationary environment that has characterized the last 10 years is the result of "structural" imbalances rather than transitory shocks.

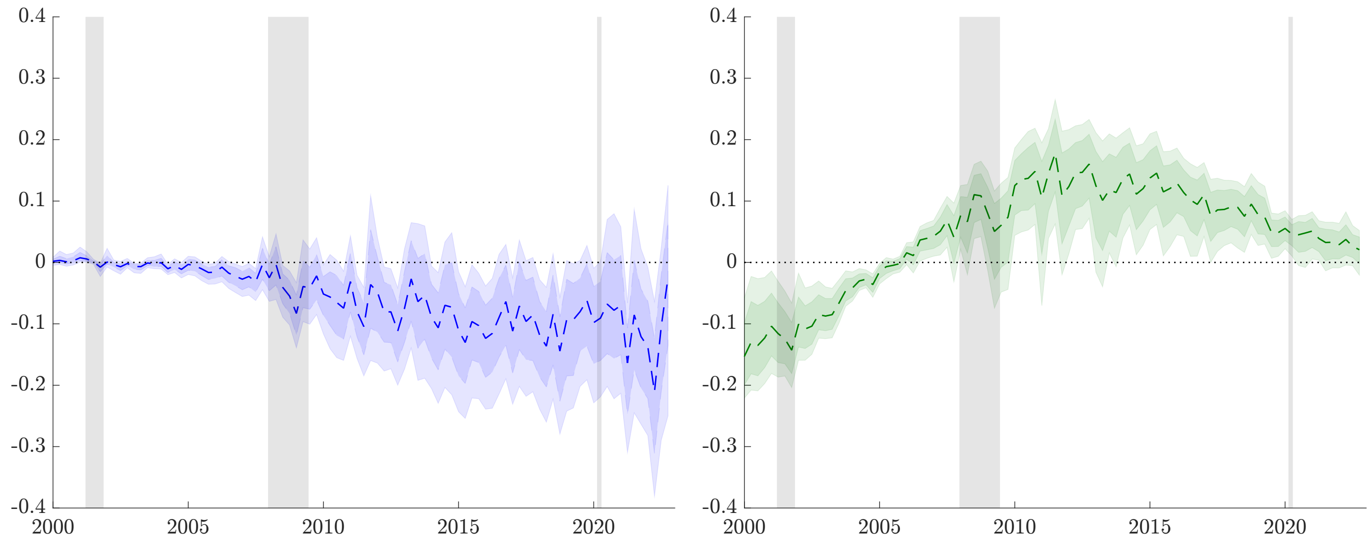


Figure 8: Balance of risk, counterfactuals

Note: The panels report the median difference between the counterfactual and the actual (see Figure 7) balances of risks. The left panel assumes a scenario in which the long-run real rate does not move into negative territory, but hits a hard floor at zero. On the right, the scenario assumes a muted fiscal response over the last 20 years. Bands represent 90% credible bands. Gray shaded areas represent NBER recessions.

Counterfactuals. In addition to evaluate the balance of risk, the inclusion of explanatory variables and the flexibility of the model allows us to draw couterfactual balances of risk. Specifically, this tool is suited to aid policy makers with the assessment of the effect of future policies on the balance of risk, based on historical correlations. In Figure 8 we report the difference between counterfactual balances of risks and the actual estimate in Figure 7. In the left panel, we assume that the long-run real rate does not move into negative territory, but hits a hard floor at zero. The result highlight how depressed real rates generate downside risks to inflation. That is, the counterfactual balance of risk appears less tilted towards the downside, highlighting the importance of falling real rates, which constraint the action of monetary policy from below. In the right panel we evaluate a counterfactual balance of risks assuming fiscal policy would remain muted since the turn of the century. The result show that the fiscal expansion in the aftermath of Great Financial Crisis under the Obama administration has had an inflationary effect, that is it generated risks towards the upside. Notably, Figure 8 shows how the fiscal stance might have played an important role in mitigating the downward bias imposed by long spells of nominal rates close to their effective lower bound.

8 Concluding remarks

We have estimated a model of inflation where innovations distribute as a Skew-t distribution. This model allows us to track changes in the mean, variance, and asymmetry of the predictive distribution of inflation over time. Furthermore, we can expand the model to study what macroeconomic factors help predict changes in these moments at different horizons.

We find that non-policy factors, such as unit labor costs, long-run real interest rates, the unemployment gap, and commodity prices, are key drivers of inflation risks. Failing to offset inflation in the 1960s and in 1970s led to a large and persistent increase upside risk of inflation.

A Score-driven framework

Assume that the variable y_t is generated by the observation density $\mathcal{D}(\theta, f_t)$, with θ collecting the static parameters of the distribution. The score-driven setting postulates the dynamics of the time-varying parameters, f_t, being:

$$f_{t+1} = \varpi + \sum_{i=0}^{p-1} \alpha_i s_{t-i} + \sum_{j=0}^{q-1} \beta_j f_{t-j}, \tag{15}$$

which we refer to as $GAS(p, q)$ dynamics. The scaled score s_t is a non-linear function of past observations and past parameters' values. For $\ell_t = \log \mathcal{D}(\theta, f_t)$, we define:

$$s_t = \mathcal{S}_t \nabla_t, \quad \nabla_t = \frac{\partial \ell_t}{\partial f_t}, \quad \mathcal{S}_t = \mathcal{I}_t^{-1} = -\,\mathbb{E}\left(\frac{\partial^2 \ell_t}{\partial f_t \partial f_t'}\right)^{-1},$$

where ∇_t corresponds to the gradient vector of the log-likelihood function, ℓ_t, and the scaling matrix $\mathcal{S}_{t-1}$ is proportional to the square-root generalized inverse of the Information matrix $\mathcal{I}_{t-1}$. Within this framework, the parameters are updated in the direction of the steepest ascent, in order to maximize the local fit of the model.

A.1 Model's specifics

Given the log-likelihood in Equation (6), the elements of the gradient ∇_t, with respect to location, squared scale and asymmetry, are:

$$\frac{\partial \ell_t}{\partial \mu_t} = \frac{1}{\sigma_t^2} w_t \varepsilon_t, \qquad \frac{\partial \ell_t}{\partial \sigma_t^2} = \frac{1}{2\sigma_t^4}(w_t \varepsilon_t^2 - \sigma_t^2), \qquad \frac{\partial \ell_t}{\partial \varrho_t} = -\frac{1}{\sigma_t^2} \frac{sgn(\varepsilon_t)}{(1 - sgn(\varepsilon_t)\varrho_t)} w_t \varepsilon_t^2, \tag{16}$$

where $w_t = \frac{(1+\eta)}{(1-sgn(\varepsilon_t)\varrho_t)^2 + \eta\zeta_t^2}$ and ζ_t denotes the scaled prediction error, $\zeta_t = \frac{\varepsilon_t}{\sigma_t}$. The associated information matrix reads as follows:

$$\mathcal{I}_t = \begin{bmatrix} \frac{(1+\eta)}{(1+3\eta)(1-\varrho_t^2)\sigma_t^2} & 0 & -\frac{4\mathcal{C}(1+\eta)}{\sigma_t(1-\varrho_t^2)(1+3\eta)} \\ 0 & \frac{1}{2(1+3\eta)\sigma_t^4} & 0 \\ -\frac{4\mathcal{C}(1+\eta)}{\sigma_t(1-\varrho_t^2)(1+3\eta)} & 0 & \frac{3(1+\eta)}{(1-\varrho_t^2)(1+3\eta)} \end{bmatrix}. \tag{17}$$

In order to ensure the scale σ_t to be positive and the shape ϱ_t to lie within the unit circle, we apply time-invariant, invertible and twice differentiable *"link functions"* to these parameters. In practice, we model $\gamma_t = \log(\sigma_t)$ and $\delta_t = \text{arctanh}(\varrho_t)$, such that the vector of time-varying parameters becomes $f_t = (\mu_t, \gamma_t, \delta_t)'$. Moreover, we follow Lucas and Zhang (2016) in scaling the score only using the diagonal elements of the information matrix. Therefore, the associated scaled score vector is

$$s_t = (J_t' diag(\mathcal{I}_t) J_t)^{-1} J_t' \nabla_t = \begin{bmatrix} s_{\mu t} \\ s_{\gamma t} \\ s_{\delta t} \end{bmatrix} = w_t \zeta_t (1+3\eta) \begin{bmatrix} \frac{(3\sigma_t(1-\varrho_t^2)-4sgn(\varepsilon_t)\mathcal{C}(1+sgn(\varepsilon_t)\varrho_t)\zeta_t)}{(3-16\mathcal{C}^2)(1+\eta)} \\ \\ \frac{\sigma_t^2(w\zeta_t^2-1)}{w_t\zeta_t} \\ \\ \frac{(4\mathcal{C}(1-\varrho_t^2)-sgn(\varepsilon_t)\sigma_t(1+sgn(\varepsilon_t)\varrho_t)\zeta_t)}{(3-16\mathcal{C}^2)(1+\eta)} \end{bmatrix}, \tag{18}$$

where $J_t = \frac{\partial(\mu_t, \sigma_t^2, \varrho_t)}{\partial(\mu_t, \gamma_t, \delta_t)'}$ is the Jacobian matrix associated to the link functions. Detailed derivations can be found in Delle Monache et al. (2021). Weights, w_t, penalize extreme standardized innovations depending on the thickness of the tails, as well as volatility and asymmetry estimated conditional to time $t-1$. This asymmetric treatment of the signal of the prediction error is more pronounced as the skewness of the distribution grows larger (i.e., $|\varrho_t| \to 1$).

Robustness to outliers Here we consider the limiting behaviour of the *Skt* scaled scores. We show that for (standardized) forecast errors approaching positive (negative) infinity, the scaled scores either converge to zero, implying a trimming of the outliers, or converge to a positive (negative) constant, akin to Winsorizing extreme observations. Specifically,

$$\lim_{\zeta \to \pm\infty} s_{\mu t} = 0, \tag{19}$$

$$\lim_{\zeta \to \pm\infty} s_{\sigma t} = \frac{1+\eta}{\eta}\sqrt{\frac{(1+3\eta)}{2}}, \tag{20}$$

$$\lim_{\zeta \to \pm\infty} s_{\varrho t} = \mp\frac{1+\eta}{\eta}\sqrt{\frac{(1+3\eta)(1+\varrho_t)}{3(1+\eta)(1-\varrho_t)}}. \tag{21}$$

In line with results for the t distribution, the scaled score for the location trims outliers, preventing any update of the parameter. The limits of the scaled scores for the scale and shape parameter, on the other hand, converge to constant factors. These are functions of the degrees of freedom parameter, as well as the conditional asymmetry parameter at time t for the shape.

Unrestricted parameters' scores Here we provide a proof of the equivalence between the the score vector arising from the restriction imposed on the scale and shape parameters and that arising by imposing constraints on the two-component specification. We will drop the time subscript for the sake of clarity.

Proof. Let consider $\gamma = \log \sigma$ being the log-scale, it follows that $\sigma = \exp \gamma$, and the gradient is

$$\frac{\partial \ell}{\partial \gamma} = \frac{\partial \ell}{\partial \sigma^2}\frac{\partial \sigma^2}{\partial \gamma} = \frac{\partial \ell}{\partial \sigma^2} 2\sigma^2.$$

Let now consider the multiplicative two-component counterpart $\sigma = \bar{\sigma}\tilde{\sigma}$, such that $\gamma = \log \sigma = \log \bar{\sigma} + \log \tilde{\sigma} = \bar{\gamma} + \tilde{\gamma}$. The gradient with respect to the first component reads

$$\frac{\partial \ell}{\partial \bar{\gamma}} = \frac{\partial \ell}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \gamma} \frac{\partial \gamma}{\partial \bar{\gamma}} = \frac{\partial \ell}{\partial \sigma^2} 2\sigma^2.$$

The same applies for the second component. □

Proof. For the shape parameter $\varrho = \tanh \delta$, we model $\delta = \operatorname{arctanh} \varrho$ and the gradient is

$$\frac{\partial \ell}{\partial \delta} = \frac{\partial \ell}{\partial \varrho} \frac{\partial \varrho}{\partial \delta} = \frac{\partial \ell}{\partial \varrho}(1 - \varrho^2).$$

Consider the two-component counterpart $\delta = (\bar{\delta} + \tilde{\delta})$, so that $\varrho = \tanh(\bar{\delta} + \tilde{\delta})$, it is easy to see that that

$$\frac{\partial \ell}{\partial \bar{\delta}} = \frac{\partial \ell}{\partial \varrho} \frac{\partial \varrho}{\partial \delta} \frac{\partial \delta}{\partial \bar{\delta}} = \frac{\partial \ell}{\partial \varrho}(1 - \varrho^2).$$

The same applies for the second component. □

B Additional figures

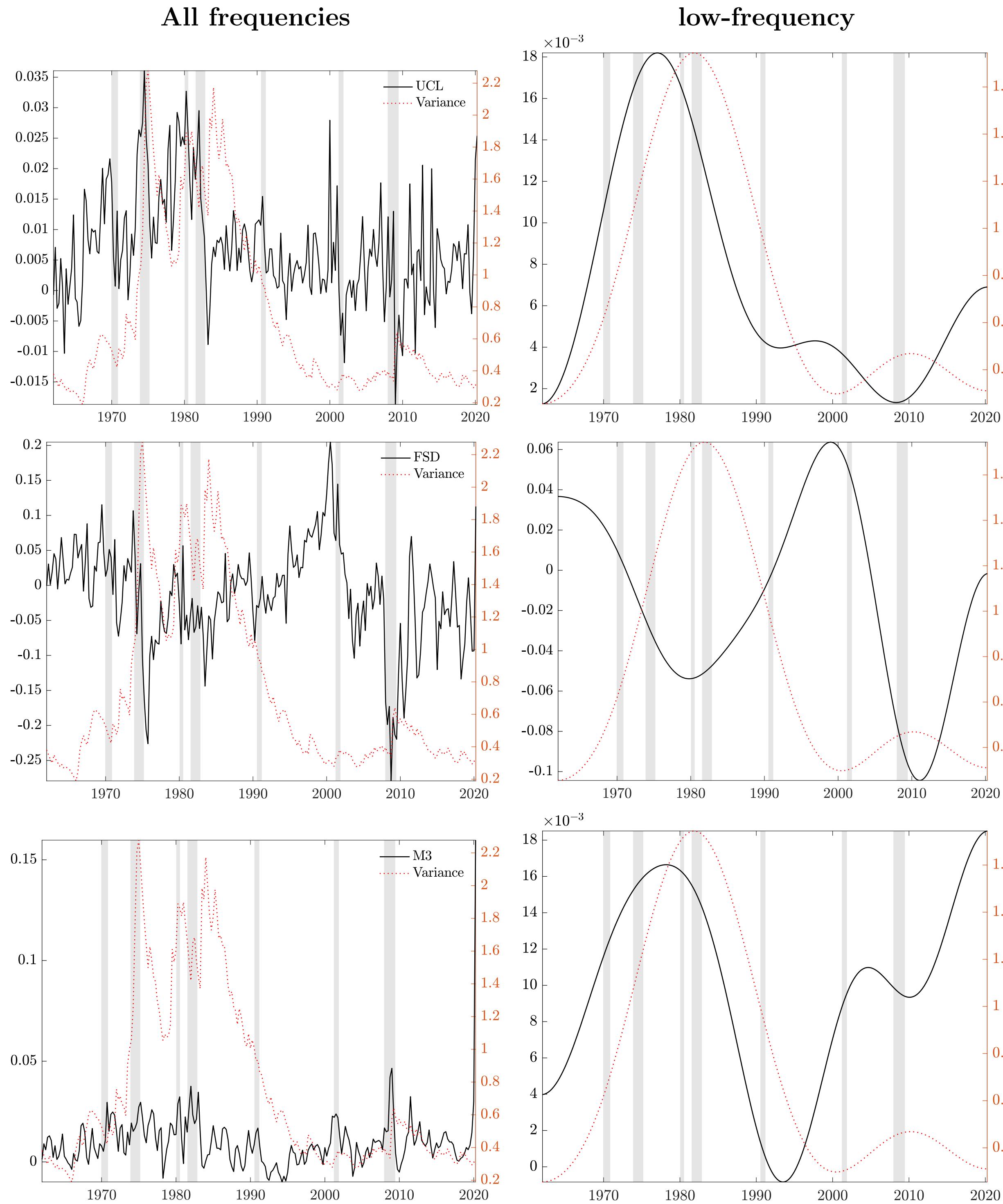


Figure 9: Variance vs. long-run predictors

Note: Variance measures refer to the right axes. The variance estimate used in this exercise is extracted from the baseline specification, and corresponds to the long-run forecast of the moment (i.e., $lim_{h\to\infty} Var(\pi_{t+h|t})$).

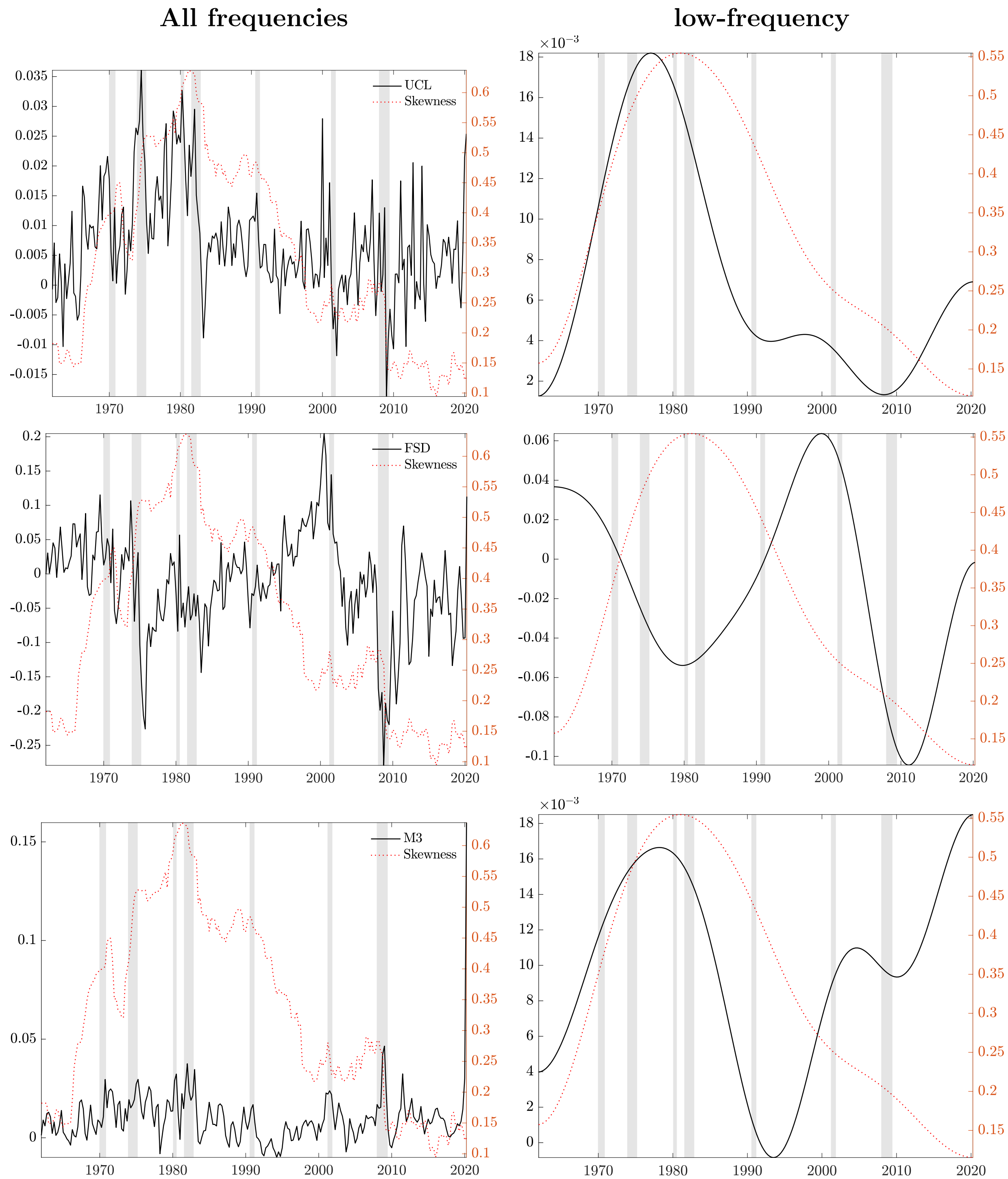


Figure 10: Skewness vs. long-run predictors

Note: Skewness measures refer to the right axes. The skewness estimate used in this exercise is extracted from the baseline specification, and corresponds to the long-run forecast of the moment (i.e., $lim_{h\to\infty} Skew(\pi_{t+h|t})$).

References

Adam, K. and R. M. Billi (2007): "Discretionary monetary policy and the zero lower bound on nominal interest rates," *Journal of monetary Economics*, 54, 728–752.

Allayioti, A., F. Monti, and M. Piffer (2023): "The transmission of monetary policy when agents fear extreme inflation outcomes," Tech. rep., Mimeo.

Arellano-Valle, R. B., H. W. Gómez, and F. A. Quintana (2005): "Statistical inference for a general class of asymmetric distributions," *Journal of Statistical Planning and Inference*, 128, 427–443.

Atkeson, A., L. E. Ohanian, et al. (2001): "Are Phillips curves useful for forecasting inflation?" *Federal Reserve bank of Minneapolis quarterly review*, 25, 2–11.

Bai, J. and S. Ng (2005): "Tests for skewness, kurtosis, and normality for time series data," *Journal of Business & Economic Statistics*, 23, 49–60.

Barro, R. J. and D. B. Gordon (1983): "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91, 589–610.

Bassetto, M. (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70, 2167–2195.

Bassetto, M. and T. J. Sargent (2020): "Shotgun Wedding: Fiscal and Monetary Policy," *Annual Review of Economics*, 12, 659–690.

Bernanke, B. S., M. T. Kiley, and J. M. Roberts (2019): "Monetary Policy Strategies for a Low-Rate Environment," *AEA Papers and Proceedings*, 109, 421–26.

Bianchi, F., R. Faccini, and L. Melosi (2023): "A Fiscal Theory of Persistent Inflation," The Quarterly Journal of Economics, forthcoming.

Bianchi, F. and C. Ilut (2017): "Monetary/Fiscal policy mix and agents' beliefs," *Review of Economic Dynamics*, 26, 113–139.

Bianchi, F. and L. Melosi (2017): "Escaping the Great Recession," *American Economic Review*, 107, 1030–58.

Bianchi, F., L. Melosi, and M. Rottner (2021): "Hitting the elusive inflation target," *Journal of Monetary Economics*, 124, 107–122.

Blasques, F., S. J. Koopman, and A. Lucas (2014): "Stationarity and ergodicity of univariate generalized autoregressive score processes," *Electronic Journal of Statistics*, 8, 1088–1112.

Blasques, F., J. van Brummelen, S. J. Koopman, and A. Lucas (2022): "Maximum likelihood estimation for score-driven models," *Journal of Econometrics*, 227, 325–346.

Buccheri, G., G. Bormetti, F. Corsi, and F. Lillo (2021): "Filtering and Smoothing with Score-Driven Models," .

Bullard, J. B. (2018): "What Is the Best Strategy for Extending the U.S. Economy's Expansion?: a presentation at the CFA Society Chicago—Distinguished Speaker Series Breakfast, Chicago, Ill," Speech 320, Federal Reserve Bank of St. Louis.

Carvalho, C. M., N. G. Polson, and J. G. Scott (2010): "The horseshoe estimator for sparse signals," *Biometrika*, 97, 465–480.

Clarida, R. (2020): "The Federal Reserve's New Monetary Policy Framework: A Robust Evolution." Speech at the Peterson Institute for International Economics, Washington, D.C., August 31, 2020.

Cochrane, J. H. (1998): "A Frictionless Model of U.S. Inflation," in *NBER Macroeconomics Annual 1998*, ed. by B. S. Bernanke and J. J. Rotemberg, Cambridge, MA: MIT Press, 323–384.

——— (2001): "Long Term Debt and Optimal Policy in the Fiscal Theory of Price Level," *Econometrica*, 69, 69–116.

Cogley, T. and A. M. Sbordone (2008): "Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve," *American Economic Review*, 98, 2101–2126.

Cox, D. R. (1981): "Statistical analysis of time series: Some recent developments," *Scandinavian Journal of Statistics*, 93–115.

Creal, D., S. J. Koopman, and A. Lucas (2013): "Generalized autoregressive score models with applications," *Journal of Applied Econometrics*, 28, 777–795.

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020): "What's Up with the Phillips Curve?" *Brookings Papers on Economic Activity.*

Delle Monache, D., A. De Polis, and I. Petrella (2021): "Modeling and forecasting macroeconomic downside risk," *Bank of Italy Temi di Discussione (Working Paper) No*, 1324.

Delle Monache, D. and I. Petrella (2017): "Adaptive models and heavy tails with an application to inflation forecasting," *International Journal of Forecasting*, 33, 482–501.

Duarte, F. M., B. K. Johannsen, L. Melosi, and T. Nakata (2020): "Strengthening the FOMC's Framework in View of the Effective Lower Bound and Some Considerations Related to Time-Inconsistent Strategies," Finance and Economics Discussion Series 2020-067, Board of Governors of the Federal Reserve System (U.S.).

ENGLE, R. F. (1982): "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econometrica: Journal of the Econometric Society*, 987–1007.

ESTRELLA, A. AND F. S. MISHKIN (1997): "Is there a role for monetary aggregates in the conduct of monetary policy?" *Journal of monetary economics*, 40, 279–304.

FAUST, J. AND J. H. WRIGHT (2013): "Forecasting inflation," in *Handbook of economic forecasting*, Elsevier, vol. 2, 2–56.

FECHNER, G. T. (1897): *Kollektivmasslehre*, Engelmann.

FED (2021): "Review of Monetary Policy Strategy, Tools, and Communications," .

GALÍ, J. AND L. GAMBETTI (2019): "Has the US wage Phillips curve flattened? A semi-structural exploration," Tech. rep., National Bureau of Economic Research.

GÓMEZ, H. W., F. J. TORRES, AND H. BOLFARINE (2007): "Large-sample inference for the epsilon-skew-t distribution," *Communications in Statistics—Theory and Methods*, 36, 73–81.

GORDON, R. J. (1970): "The recent acceleration of inflation and its lessons for the future," *Brookings Papers on Economic Activity*, 1970, 8–47.

——— (1981): "Inflation, flexible exchange rates, and the natural rate of unemployment," Tech. rep., National Bureau of Economic Research.

GUST, C., E. HERBST, D. LÓPEZ-SALIDO, AND M. E. SMITH (2017): "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*, 107, 1971–2006.

HAARIO, H., E. SAKSMAN, AND J. TAMMINEN (1999): "Adaptive proposal distribution for random walk Metropolis algorithm," *Computational Statistics*, 14, 375–396.

HARVEY, A. C. (2013): *Dynamic models for volatility and heavy tails: with applications to financial and economic time series*, vol. 52, Cambridge University Press.

KILIAN, L. AND S. MANGANELLI (2007): "Quantifying the risk of deflation," *Journal of Money, Credit and Banking*, 39, 561–590.

——— (2008): "The central banker as a risk manager: Estimating the Federal Reserve's preferences under Greenspan," *Journal of Money, Credit and Banking*, 40, 1103–1129.

KOROBILIS, D., B. LANDAU, A. MUSSO, AND A. PHELLA (2021): "The time-varying evolution of inflation risks," .

LEEPER, E. M. (1991): "Equilibria Under Active and Passive Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

Liu, Y. and J. C. Wu (2021): "Reconstructing the yield curve," *Journal of Financial Economics*, 142, 1395–1425.

Lopez-Salido, D. and F. Loria (2020): "Inflation at risk," .

Lucas, A. and X. Zhang (2016): "Score-driven exponentially weighted moving averages and Value-at-Risk forecasting," *International Journal of Forecasting*, 32, 293–302.

Manzan, S. and D. Zerom (2013): "Are macroeconomic variables useful for forecasting the distribution of US inflation?" *International Journal of Forecasting*, 29, 469–478.

——— (2015): "Asymmetric quantile persistence and predictability: the case of US inflation," *Oxford Bulletin of Economics and Statistics*, 77, 297–318.

McConnell, M. M. and G. Perez-Quiros (2000): "Output fluctuations in the United States: What has changed since the early 1980's?" *American Economic Review*, 90, 1464–1476.

Mertens, T. M. and J. C. Williams (2019): "Tying Down the Anchor: Monetary Policy Rules and the Lower Bound on Interest Rates," Federal Reserve Bank of San Francisco, Working Paper 2019-14, 2019.

Mudholkar, G. S. and A. D. Hutson (2000): "The epsilon–skew–normal distribution for analyzing near-normal data," *Journal of Statistical Planning and Inference*, 83, 291–309.

Müller, U. K. and M. W. Watson (2018): "Long-Run Covariability," *Econometrica*, 86, 775–804.

Nakov, A. (2008): "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 4, 73–127.

Reichlin, L., K. Adam, W. J. McKibbin, M. McMahon, R. Reis, G. Ricco, and W. di Mauro B (2021): "The ECB strategy: The 2021 review and its future," Tech. rep., CEPR Press.

Reis, R. (2016): "QE in the Future: the Central Bank's Balance Sheet in a Fiscal Crisis," Working Paper 22415, National Bureau of Economic Research.

——— (2021): "Losing the inflation anchor," .

Rossi, B. (2021): "Forecasting in the presence of instabilities: How we know whether models predict well and how to improve them," *Journal of Economic Literature*, 59, 1135–90.

Sargent, T. and N. Wallace (1981): "Some Unpleasant Monetarist Arithmetic," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall, 1–17.

Schmitt-Grohe, S. and M. Uribe (2000): "Price level determinacy and monetary policy under a balanced-budget requirement," *Journal of Monetary Economics*, 45, 211–246.

Schnabel, I. (2021): "A New Strategy for A Changing World," Speech at the Peterson Institute For International Economics, Washington, D.C., July 14, 2021.

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

Stock, J. H. and M. W. Watson (2002): "Has the business cycle changed and why?" *NBER Macroeconomics Annual Report*, 17, 159–218.

——— (2007): "Why has US inflation become harder to forecast?" *Journal of Money, Credit and banking*, 39, 3–33.

——— (2008): "Phillips curve inflation forecasts," .

——— (2016): "Core inflation and trend inflation," *Review of Economics and Statistics*, 98, 770–784.

——— (2020): "Slack and cyclically sensitive inflation," *Journal of Money, Credit and Banking*, 52, 393–428.

Wallis, K. F. (1999): "Asymmetric density forecasts of inflation and the Bank of England's fan chart," *National Institute Economic Review*, 167, 106–112.

——— (2014): "The two-piece normal, binormal, or double Gaussian distribution: its origin and rediscoveries," *Statistical Science*, 106–112.

Woodford, M. (1994): "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *Economic Theory*, 4, 345–389.

——— (1995): "Price Level Determinacy without Control of a Monetary Aggregate," *Carnegie-Rochester Series of Public Policy*, 43, 1–46.

——— (2001): "Fiscal Requirements of Price Stability," *Journal of Money, Credit and Banking*, 33, 669–728.

Yellen, J. L. (2015): "Inflation Dynamics and Monetary Policy: A speech at the Philip Gamble Memorial Lecture, University of Massachusetts, Amherst, Amherst, Massachusetts, September 24, 2015," Tech. rep.