Forecasting UK inflation using evidence on the role of energy in productivity and prices

Jennifer L. Castle

Magdalen College, University of Oxford

with David F. Hendry and Andrew B. Martinez

Forecasting @ Risk

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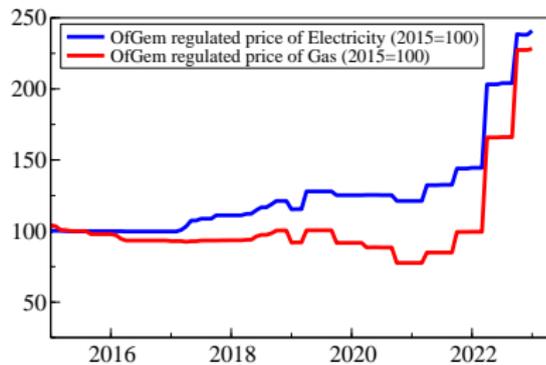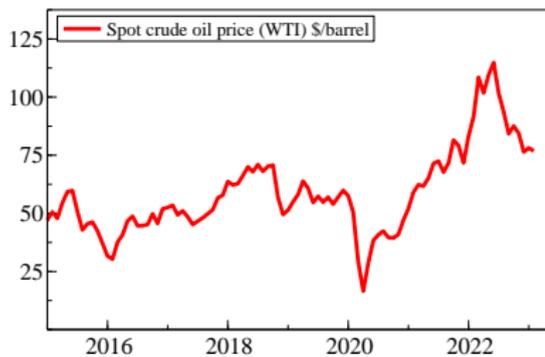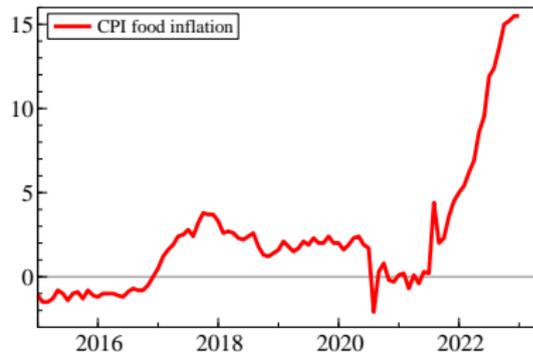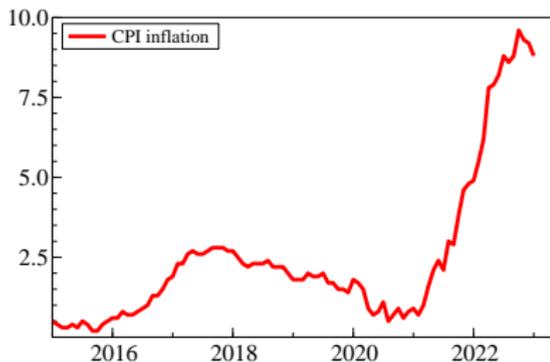
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Drawback:

Annual data: low frequency means cannot rapidly update forecasts as news comes in. Circumvent by calculating projections of inflation based on our empirical models but using recent higher frequency data observations.
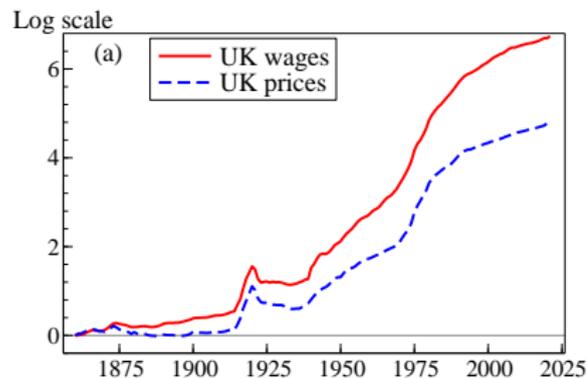
Log scale

(a) UK wages and prices 1860–2021: risen 700 & 100 fold.
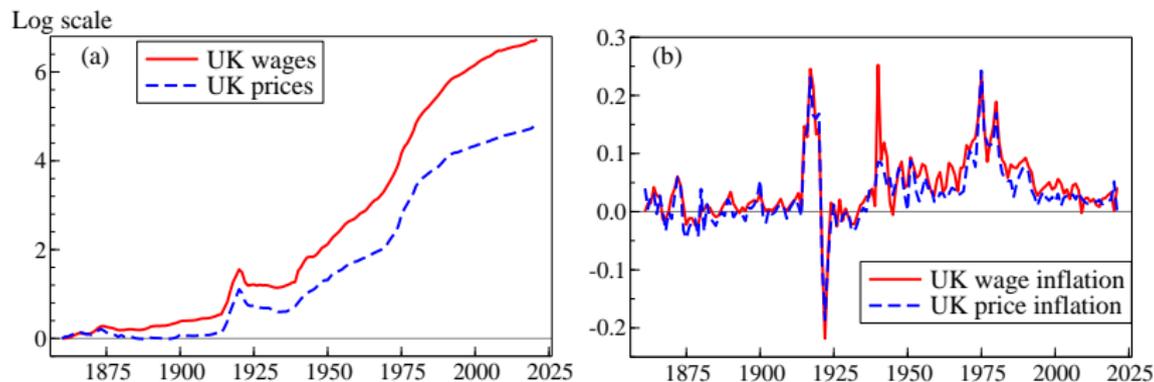
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(a) UK wages, UK prices

(b) UK wage inflation, UK price inflation

(b) Wage and price inflation.
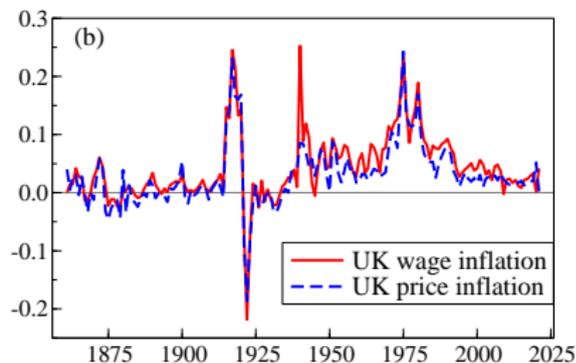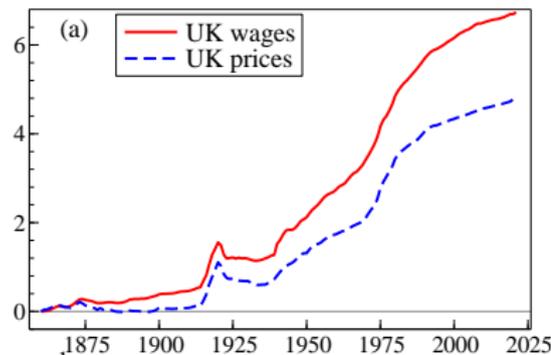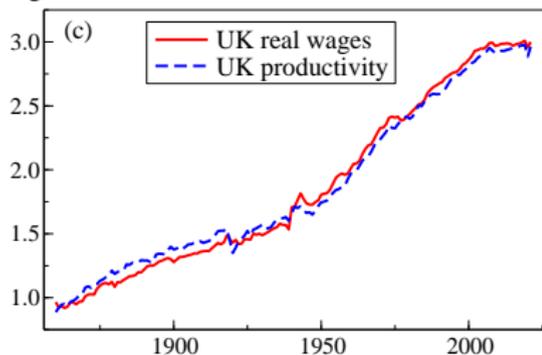
(c) Real wages and productivity.

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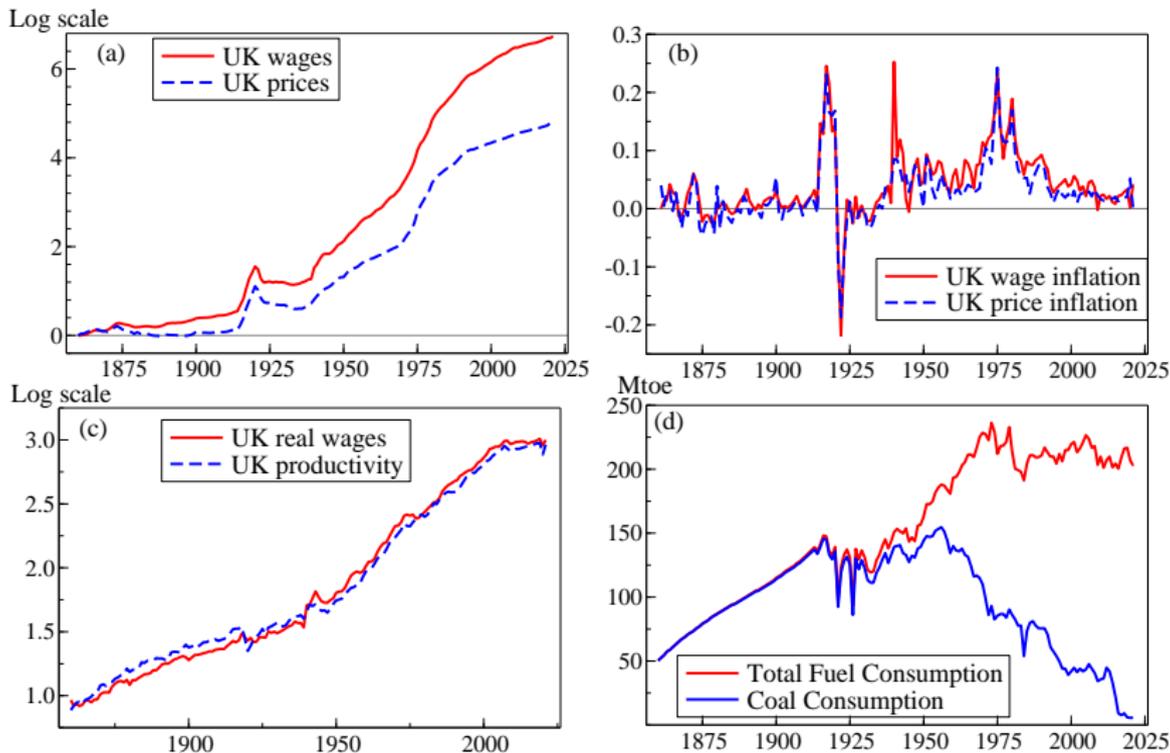
(d) UK energy use: coal replaced by oil, natural gas and renewables.
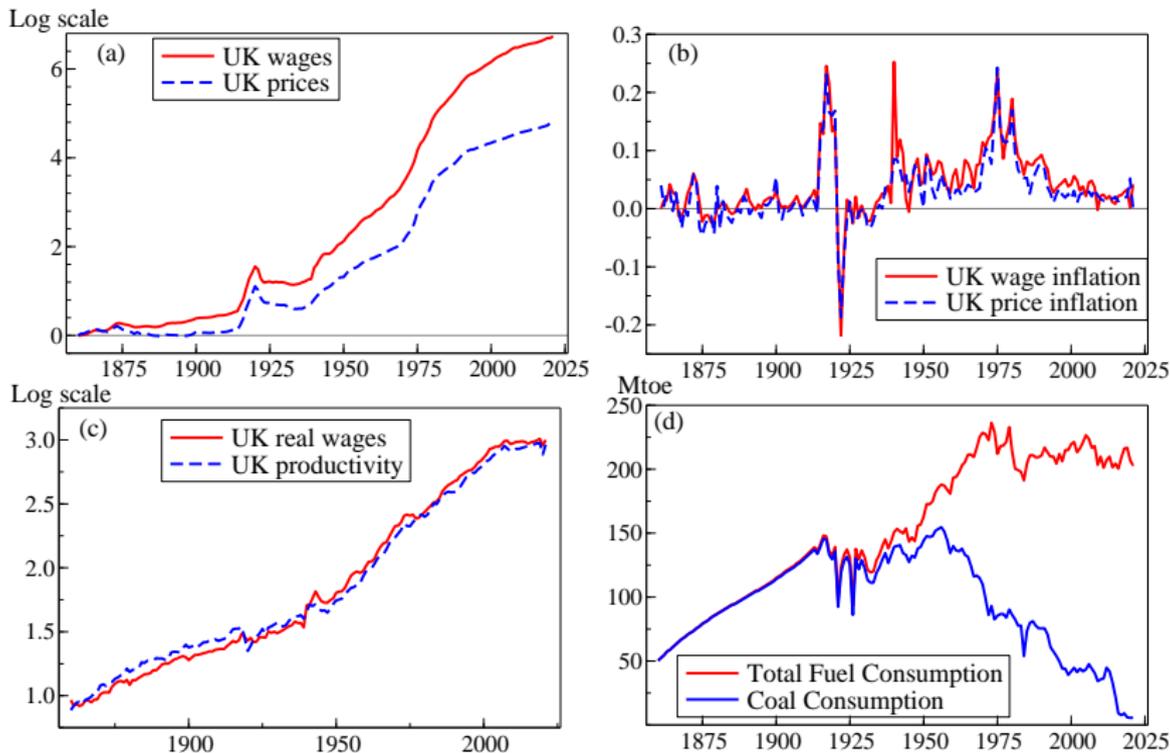
Use this variation to identify contributing factors to inflation

Must handle non-constancy of change, so use general-to-specific strategy with break detection and allow for non-linearities.

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General unrestricted models

Include intercept, lags of dependent variables and range of conditioning variables motivated by theory and past evidence.

Polynomials for possible non-linearities (double de-meaned to reduce collinearity).

Impulse, step and trend (for productivity) indicator saturation detecting outliers/location shifts.

Retain regressors & select indicators at 0.1% (Hendry and Johansen (2015)); select regressors at 1%.

Further transformations for cointegration to include long-run information.

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Super-exogeneity tested given single-equation conditional models.
Resulting equations are congruent encompassing models.

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Wage inflation

Real wage inflation driven by productivity and mark-up along with wage-price spiral and non-linear unemployment relation.

Unemployment

Unemployment explained by real borrowing costs over revenues proxied by output growth.

Productivity

Labour productivity driven by capital per worker and energy per unit of capital, with trend breaks capturing changes in technology.

Price inflation

Price inflation depends on short and long term interest rates, money growth, commodity prices, world inflation and unit labour costs.

Selected model over 1862–2015

$$\Delta(\widehat{w-p})_t = \underset{(0.04)}{0.38}\,\Delta(y-l)_t + \underset{(0.05)}{0.14}\,\Delta(y-l)_{t-1} - \underset{(0.03)}{0.15}\,\Delta^2 p_t$$

$$- \underset{(0.04)}{0.18}\,(U_{r,t} - 0.05) + \underset{(0.69)}{3.0}\,(U_{r,t} - 0.05)^2 - \underset{(0.06)}{0.22}\,\Delta_2 U_{r,t}$$

$$+ \underset{(0.07)}{0.42}\,(\widetilde{f\Delta p_t}) - \underset{(0.01)}{0.13}\,S_{1939} + \underset{(0.02)}{0.18}\,S_{1940} - \underset{(0.01)}{0.07}\,S_{1941}$$

$$- \underset{(0.01)}{0.05}\,I_{1916} - \underset{(0.01)}{0.05}\,I_{1977} + \underset{(0.01)}{0.03}\,IWWII$$

$$- \underset{(0.03)}{0.17}\,(w - p - y + l - \widehat{\mu})_{t-2} + \underset{(0.002)}{0.02}\,S_{2012}$$

$\widehat{\sigma} = 1.10\%$ $R^2 = 0.79$ $F_{ar}(2, 137) = 0.25$ $F_{arch}(1, 152) = 0.03$
$\chi^2_{nd}(2) = 0.62$ $F_{Het}(19, 130) = 2.50^{**}$ $F_{reset}(2, 137) = 3.35$
$F_{Chow}(6, 139) = 0.52$ $t(5)_{fcast} = 1.10$.

Notation: $\Delta(w - p)$: real wage growth; $\Delta(y - l)$ labour productivity; $\Delta^2 p$: change in inflation rate; U_r: unemployment rate; $(w - p - y + l - \widehat{\mu})$ mark-up of nominal output over wage bill; $\widetilde{f\Delta p}_t$ non-linear wage-price spiral; I impulse dummy; S step dummy; $IWWII$ composite dummy for WWII.

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- Short-run impact of changes in productivity on real wages ≈ 0.5, rapid incorporation of productivity increases into real wage increases but symmetric, also reflects dampening of real wages due to productivity slowdown since 2008.

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- Strong equilibrium correction from the labour share of income reflecting long-run feedback to real unit labour costs of about 20% p.a, or half life of just under 4 years.

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- Coefficient of $\widetilde{f}_t \Delta p_t$ highly significant reflecting importance of non-linearity.

Wage-price spirals

LSTAR specification with squared annualized inflation ($(\Delta p)^2$) (%) as transition variable with threshold (c) and speed of transition (γ) determined by grid search. Points calculated from observed data:

$$\widetilde{f} = \frac{1}{0.88}\left(\left[1 + \exp\left(-10\left(100\left(\Delta p\right)^2 - 0.2\right)\right)\right]^{-1} - 1\right) \quad (1)$$

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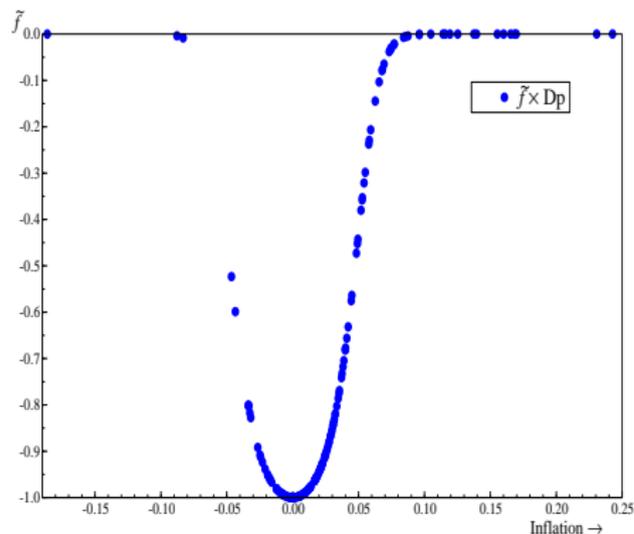
Little reaction of real wages to inflation when low, but workers become more attentive as price inflation rises and act to prevent further erosion of real wages. Suggests non-linearity induces wage-price spiral if inflation exceeds $6\% - 8\%$ p.a. UK CPI inflation currently at 8.7% (April 23) so wage-price spiral a key concern.

Selected model over 1862–2015

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- Non-linear unemployment effect initially has a negative effect on real wages, but the effect turns positive as unemployment rises (involuntary unemployment).
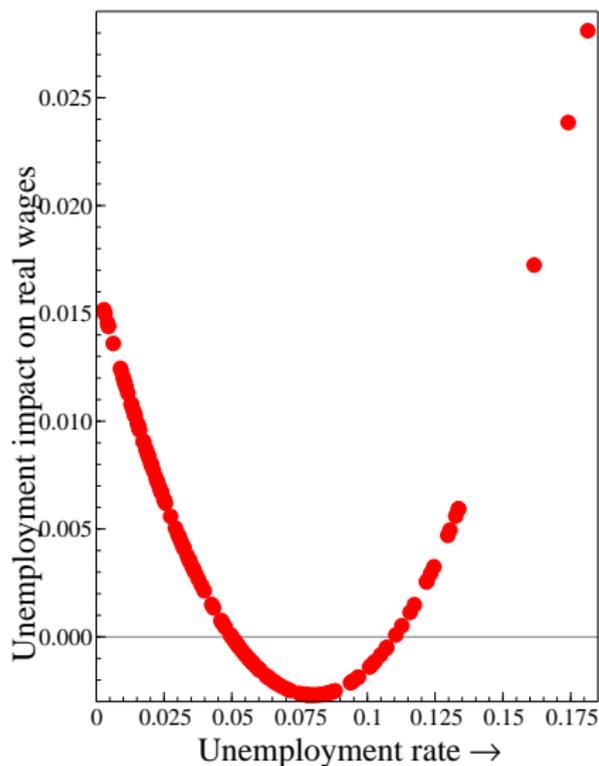
Non-linear unemployment term, $(U_{r,t} - 0.05)^2$, significant. Combined term positive at low rates with increasingly negative impact until unemployment rate exceeds approximately 8%, but then increases.
Initially loss of worker's bargaining power, but then movements along marginal product curve, raising real wages of those still employed both from more capital per worker and employed being the more productive workers.

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- Step indicators explain higher growth rate of real wages post WWII (1.7% p.a., versus 0.8% p.a. pre-1945), despite $\Delta(y-l)$ included. Spike in $\Delta(w-p)$ in 1940 induced permanent location shift not explained by variables in model, e.g. increase in female labour force participation & rapid up-skilling of labour force during war.

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- Step shift in 2012 reverses higher growth post WWII, real wage growth shifts down by 2% p.a. None of the economic variables in the model explain this step shift, but fundamental for forecast performance of model over 2016–2021.

Test if indicators can be ascribed economic interpretation as indicators are selected atheoretically, see Ericsson (2017).

UK has complicated history of wage and price controls:
1948-50 Cripps-TUC; 1956; 1961 BoP crisis; 1963; 1965-66 wage accord; 1966-67 forced wage freeze; 1968-69 relaxation; 1972-74 controls.

Define dummy variable taking the value 1 in years in which wage and price controls were in effect and include in our selected model, testing if the retained indicators are still significant.

All indicators remain significant and wage control dummy p-value$= 0.2$ so reject wage controls as interpretation of indicators.

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Testing exogeneity of conditioning variables using ISEs

Under H_0 parameters in conditional model invariant to shifts in marginal models of included regressors. Any indicators/step shifts in marginal models should not enter conditional model.

Test for super-exogeneity is not impugned by presence of common shifts in both marginal and conditional model. Understanding source of shifts is essential to determine whether common or idiosyncratic from marginal models due to failure of conditioning.

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VAR(2) in output per worker, price inflation and unemployment rate, retaining all regressors and selecting outliers and shifts using IIS+SIS at $\alpha = 0.001$.
10 impulse and 7 step indicators found.
Retained indicators included in conditional model and tested for their significance: $F_{SE,IIS+SIS}(15, 111) = 1.56$.
Conclude it is valid to condition on contemporaneous regressors.
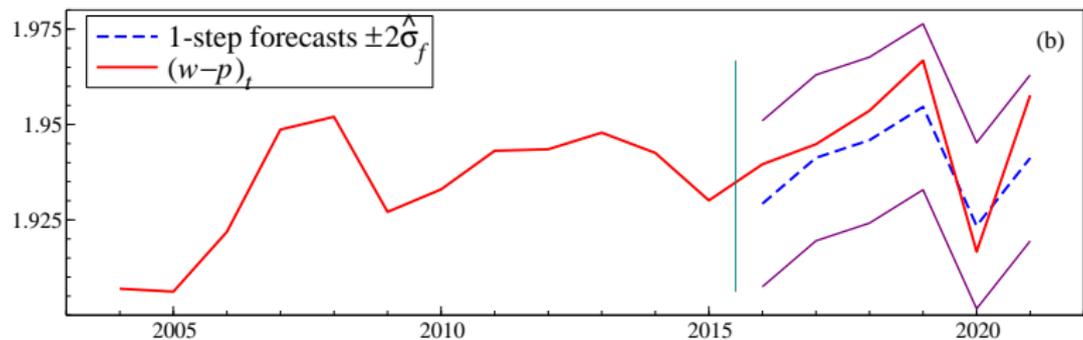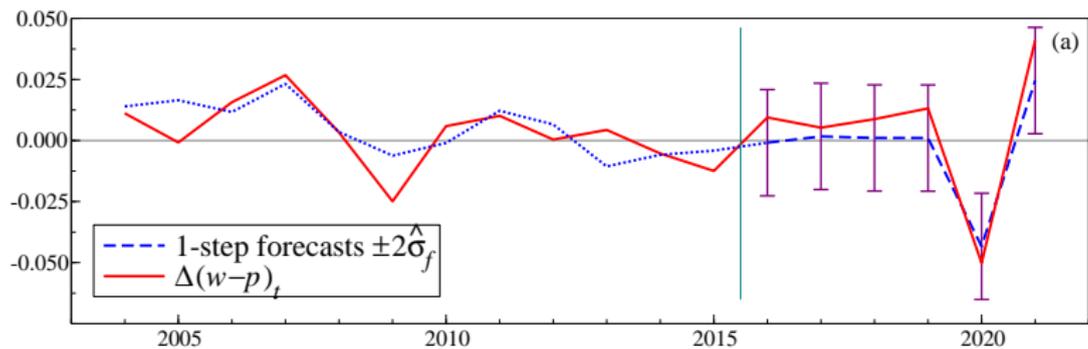
Model constant across pandemic crash and recovery

Employment increases if hiring profitable, proxy revenue minus costs.
Changes in revenues linked to changes in GDP, Δy_t ($(w-p)_t$ and $(y-l)_t$ suggest labour costs & revenues equilibrated).
Capital costs depend on real borrowing costs, $(R_L - \Delta p)_t$.
Combining gives profits proxy: $R_{r,t} = (R_L - \Delta p - \Delta y)_t$.
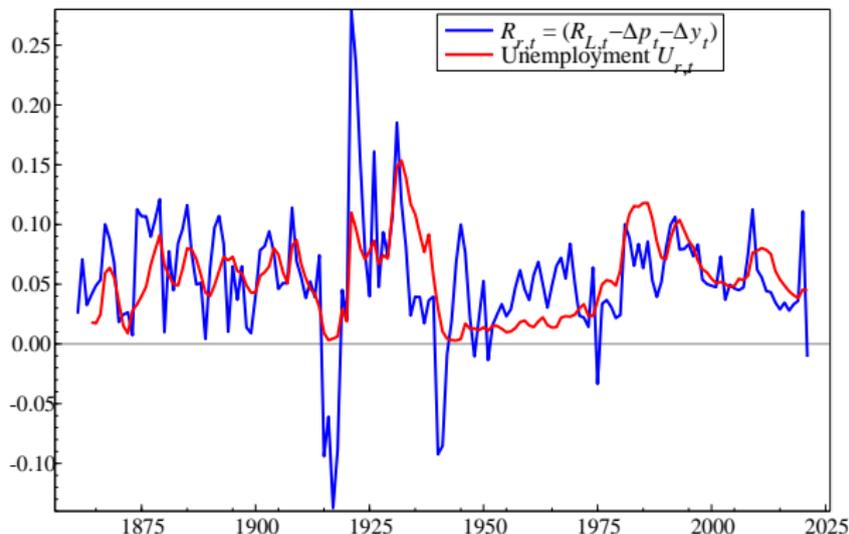
Selected model over 1863–2017

$$
\begin{aligned}
\widehat{U}_{r,t} = \quad & \underset{(0.07)}{1.26} \, U_{r,t-1} - \underset{(0.06)}{0.36} \, U_{r,t-2} + \underset{(0.002)}{0.006} \\
& + \underset{(0.02)}{0.15} \, \Delta R_{r,t} - \underset{(0.02)}{0.08} \, \Delta R_{r,t-1} \\
& + \underset{(0.001)}{0.052} \, \Delta 1_{1922} + \underset{(0.008)}{0.036} \, 1_{1930} - \underset{(0.008)}{0.035} \, 1_{1939}
\end{aligned}
$$

$\widehat{\sigma}_\epsilon = 0.83\%$ $R^{*2} = 0.94$ $F_{ar}(2, 146) = 1.93$
$\chi^2_{nd}(2) = 9.85^{**}$ $F_{Chow}(4, 147) = 0.44$
$F_{arch}(1, 153) = 0.08$ $F_{Het}(10, 142) = 1.16$
$F_{Reset}(2, 145) = 2.55$

Non-linear functions not included as no evidence of non-linear functional form at the 1% significance level: $\chi^2_{nl}(12) = 22.7^*$.

Transform to dynamic model in differences

$$\Delta \widehat{U}_{r,t} = \underset{(0.05)}{0.36} \ \Delta U_{r,t-1} + \underset{(0.016)}{0.15} \ \Delta R_{r,t} - \underset{(0.022)}{0.10} \ E_{Ur,t-1}$$

$$- \underset{(0.007)}{0.052} \ \Delta 1_{1922} + \underset{(0.008)}{0.036} \ 1_{1930} - \underset{(0.008)}{0.035} \ 1_{1939}$$

$$\widehat{\sigma}_{\epsilon} = 0.82\% \quad R^{*2} = 0.67 \quad F_{ar}(2, 147) = 1.9$$
$$\chi^2_{nd}(2) = 9.86^{**} \quad F_{Reset}(2, 147) = 0.85$$
$$F_{arch}(1, 153) = 0.08 \quad F_{Het}(8, 144) = 0.90$$

Long-run relation:

$$E_{Ur} = U_r - 0.054 - 0.72 R_r.$$

Constancy tested by 1-step *ex post* forecasts for 2018–2021:
$F_{Chow}(4, 149) = 0.45$, remarkable stability of simple model over
COVID-19 pandemic despite furlough interventions.

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When real long-term interest rate, $R_L - \Delta p$, equals Δy, then $R_r = 0$, and equilibrium U_r is about 5%, close to historical average.

Transform to dynamic model in differences

$$
\begin{aligned}
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Earlier in-sample period saw many key changes, including two world wars, unemployment benefits, and vast industrial changes, yet only 1 differenced and 2 impulse indicators are needed with just one explanatory variable and $\widehat{\sigma}_\epsilon$ under 1%.
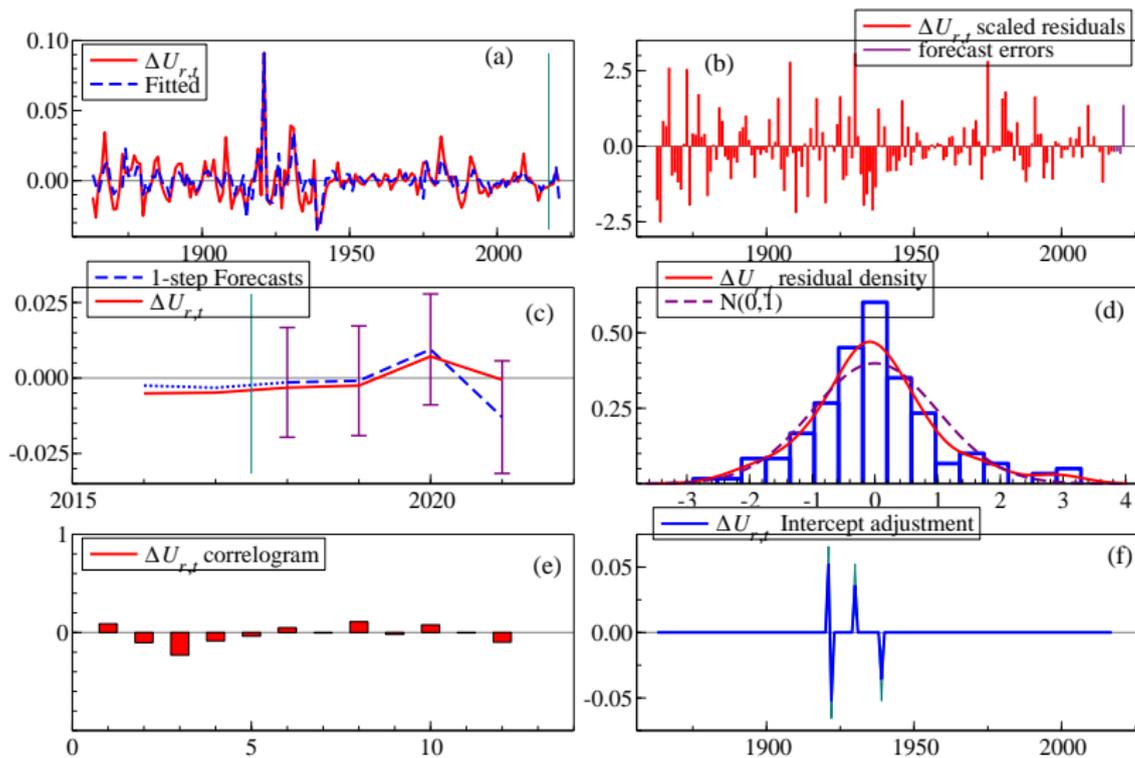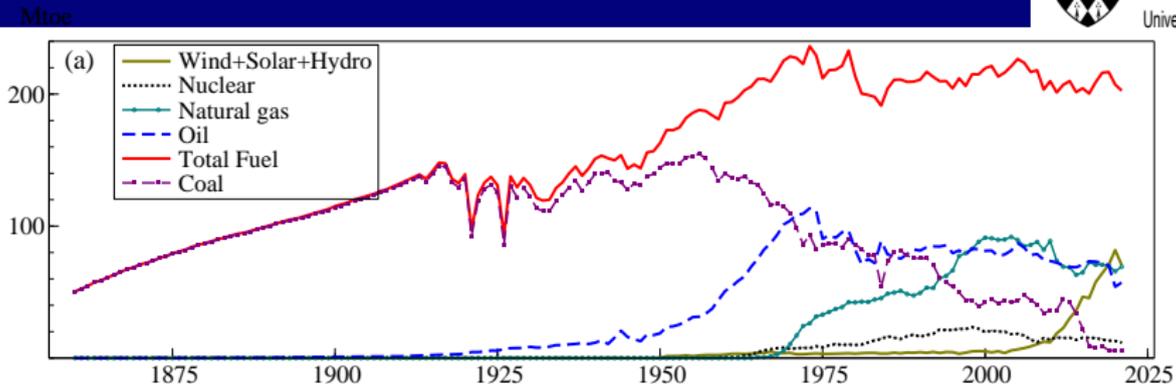
Increased efficiency from 1960s, end of coal, & rise of renewables

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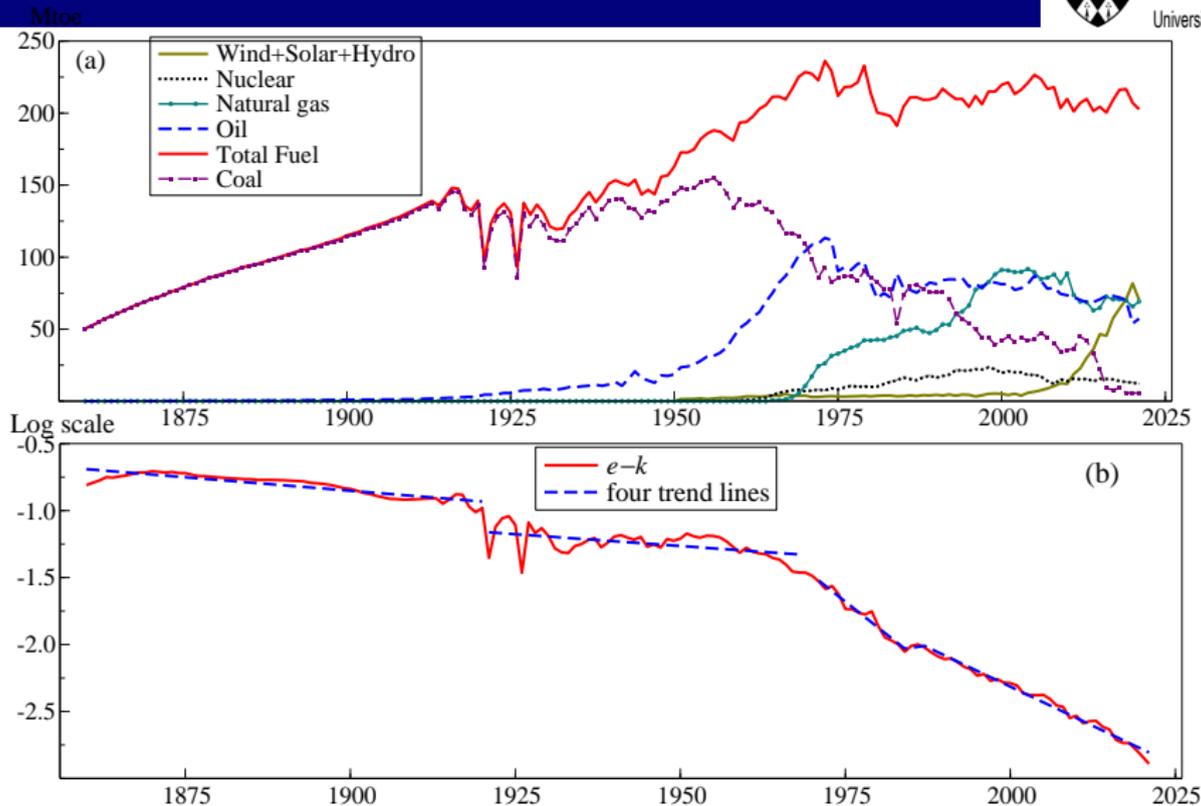
90% fall in ratio of energy use, e, to capital stock, k

Dynamic model of output on capital, labour and energy.
IIS+SIS+TIS at $\alpha = 0.0001$, see Ericsson (2011).
Homogeneity restriction $F(3, 140) = 1.95$ so reduction imposed.

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Equilibrium-correction model of productivity, $T = 1863 - 2021$

$$\widehat{\Delta(y-l)}_t = \underset{(0.0015)}{0.012} + \underset{(0.06)}{0.26}\,\Delta(k-l)_t + \underset{(0.02)}{0.15}\,\Delta(e-k)_t - \underset{(0.056)}{0.47}\,q_{t-1}$$

$$- \underset{(0.016)}{0.053}\,1_{1919} - \underset{(0.015)}{0.11}\,1_{1920} - \underset{(0.015)}{0.05}\,1_{1940} - \underset{(0.015)}{0.11}\,1_{2020}$$

$\widehat{\sigma} = 1.50\%$ $R^2 = 0.65$ $F_{ar}(2, 149) = 3.54^*$ $\chi^2_{nd}(2) = 0.22$
$F_{arch}(1, 157) = 1.65$ $F_{Het}(6, 148) = 1.47$ $F_{Reset}(2, 149) = 0.08$

Long-run relation: $q_t = (y-l) - \widetilde{(y-l)}_{LR}$

$$\widetilde{(y-l)}_{LR} = \underset{(0.044)}{0.41}\,(k-l) + \underset{(0.037)}{0.28}\,(e-k) + \underset{(0.003)}{0.017}\,t + \text{indicators}$$

Test of super exogeneity

To test the invariance of the coefficients, model $(e - k)_t$ as a function of $(y - l)_{t-1}$, $(k - l)_{t-1}$ and $(e - k)_{t-1}$ (retained) selecting SIS+TIS at $\alpha = 0.01\%$, then applied IIS at 0.1%.
Testing significance of indicators in conditional model yields $F(7, 82) = 1.4$, so super exogeneity not rejected.

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Testing significance of indicators in conditional model yields $F(7, 82) = 1.4$, so super exogeneity not rejected.

Solve for long-run to obtain production function (A_t collects deterministic functions):

$$\widetilde{Y}_{LR,t} = A_t L_t^{0.59} K_t^{0.13} E_t^{0.28}$$

Role of energy seems overly large, although increases in energy use were as crucial to industrial revolution as machinery to utilise it, and $(k-l)$ and $(e-k)$ are highly negatively correlated.
Intercept adjustment shows very few adjustments needed for model of change in output per worker.
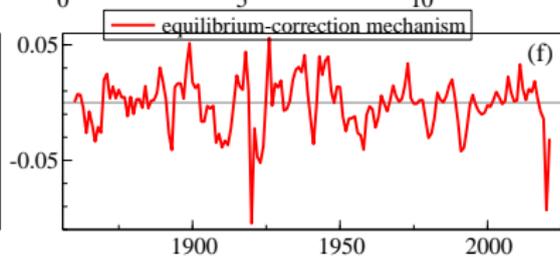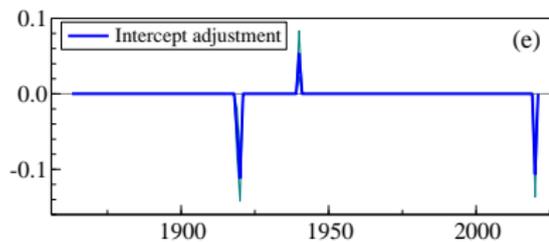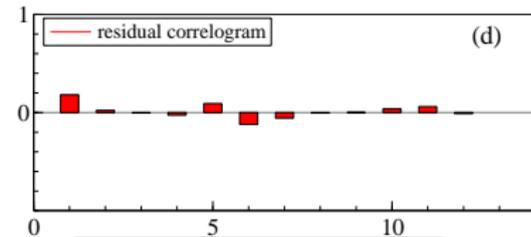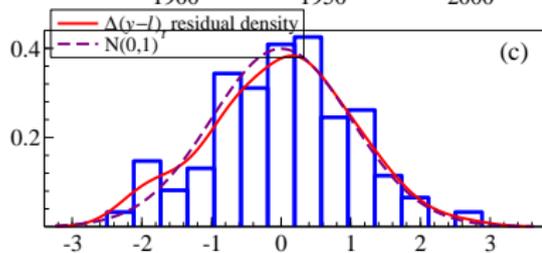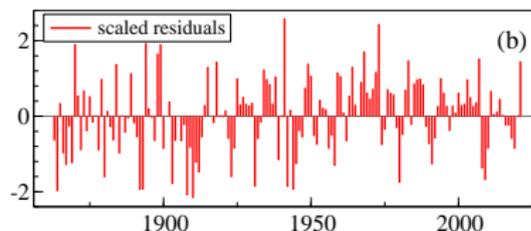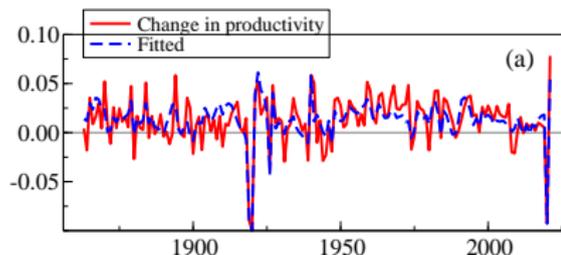
Dynamic model of GDP deflator applying saturation and selection over excess demand for output, money, and national debt; unemployment, exchange rate, unit labour costs, interest rates, wages, world and energy prices.

Dynamic model of GDP deflator applying saturation and selection over excess demand for output, money, and national debt; unemployment, exchange rate, unit labour costs, interest rates, wages, world and energy prices.

Price inflation model, 1965-2021

$$\widehat{\Delta p_t} = \underset{(0.040)}{0.29} \ \Delta p_{t-1} + \underset{(0.032)}{0.12} \ \Delta m_{t-1} + \underset{(0.095)}{0.17} \ \Delta R_{s,t} + \underset{(0.023)}{0.06} \ \Delta p_{w,t}$$

$$- \underset{(0.091)}{0.45} \ (R_s - R_l - \mu_R)_{t-1} + \underset{(0.005)}{0.007} \ \Delta p_{o,t-1} + \underset{(0.051)}{0.45} \ \Delta ulc_t$$

$$+ \underset{(0.004)}{0.03} \ I_{agg} - \underset{(0.002)}{0.005} \ \text{ChinaEffect}$$

$\widehat{\sigma} = 0.90\%$ $R^2 = 0.98$ $F_{ar}(2, 46) = 0.09$ $\chi^2_{nd}(2) = 0.25$
$F_{arch}(1, 55) = 0.70$ $F_{Het}(17, 39) = 1.08$ $F_{Reset}(2, 46) = 0.62$

Price inflation model, 1965-2021

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- Some inflation inertia

Price inflation model, 1965-2021

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- Δm: growth rate of broad money
- ΔR_s: change in short-term interest rate
- $R_{s,t} - R_{l,t} - \mu_R$: difference between short and long interest rates corrected for zero mean over full sample

Price inflation model, 1965-2021

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- Δp_o: growth rate of commodity price index linked to oil post 1997, measured in £.
  Energy prices, proxied by commodity prices, do not play a significant role but could test for non-constant parameters as more data accumulated.

Price inflation model, 1965-2021

$$\widehat{\Delta p_t} = \underset{(0.040)}{0.29} \ \Delta p_{t-1} + \underset{(0.032)}{0.12} \ \Delta m_{t-1} + \underset{(0.095)}{0.17} \ \Delta R_{s,t} + \underset{(0.023)}{0.06} \ \Delta p_{w,t}$$

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- Δulc: change in unit labour costs.
  Sizable coefficient on unit labour costs which shows important role
  for a wage price spiral effect in the price inflation equation.

Price inflation model, 1965-2021

$$
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$$

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- $\Delta p_{w,t}$: world inflation based on a trade-weighted world price index measured in £.

Price inflation model, 1965-2021

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- I_{agg}: an aggregated index of retained indicator variables weighting small, medium and large outliers into an index mostly covering wars and crises from earlier research

Price inflation model, 1965-2021

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- ChinaEffect is a step shift taking the value 0 to 1993 and 1 from 1994 onwards to represent the downward pressure from Chinese prices.

1 **Conditional models of wage inflation, unemployment, productivity and price inflation**

2 **Combining the conditional models**

3 **Assumptions for scenario projections**

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5 **Conclusions**

Close off non-linearity of unemployment in wage equation
(non-linearity kicks in when unemployment exceeds 8%); $\Delta_2 U_{r,t} \approx 0$.

Simplify estimated equations to:

$$
\begin{aligned}
\Delta w_t &= \left(1 + \gamma \widehat{f}\right) \Delta p_t - \zeta \left(\Delta p_t - \Delta p_{t-1}\right) + \kappa \Delta (y - l)_t \\
&\quad + \omega U_{r,t} + D_{\Delta w} \\
U_{r,t} &= \eta \Delta p + D_{Ur} \\
\Delta (y - l)_t &= D_{yl} \\
\Delta p_t &= \lambda \Delta w_t + \rho \Delta p_{t-1} - \lambda \Delta (y - l)_t + D_{\Delta p}
\end{aligned}
$$

where $D_{\Delta p_t}$, $D_{\Delta w_t}$, D_{yl} and D_{Ur} summarise other drivers in
conditional models.

Solve out for price inflation:

$$\Delta p_t \approx \widehat{\alpha}_{\widetilde{f}_j} \left[\lambda \left(\kappa - 1 \right) D_{yl} + \lambda \omega D_{Ur} + \lambda D_{\Delta w} + D_{\Delta p} \right]$$

where $\widehat{\alpha}_{\widetilde{f}_j} = \frac{1}{1 - \rho - \lambda \left(1 + \gamma \widetilde{f}_j + \omega \eta \right)}$.

Solve out for price inflation:

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where $\widehat{\alpha}_{\widetilde{f}_j} = \frac{1}{1 - \rho - \lambda \left(1 + \gamma \widetilde{f}_j + \omega \eta \right)}$.

Two cases for wage-price spiral parameter:

(a) $\widetilde{f}_j = 0$ workers demand 100% inflation compensation,

(b) $\widetilde{f}_j = -0.5$ workers demand 50% of inflation as compensation in wages (corresponds to inflation $\approx$ 3% historically).

Parameter	λ	ρ	γ	ω	ζ	η	κ	$\widehat{\alpha}_{\widetilde{f}=0}$	$\widehat{\alpha}_{\widetilde{f}=-0.5}$
Estimate	0.45	0.29	0.42	-0.18	-0.15	-0.72	0.52	5.0	3.4

Table: Estimates for the parameters in the solved out inflation model.

Using the production function:
$$\Delta(y - l)_t = 0.44\Delta(k - l) + 0.28\Delta(e - k) + 0.005$$

Substitute in other drivers of inflation:

$$\begin{aligned}
\Delta p_t \approx\ & 0.61\Delta m_t + 0.04\Delta p_{o,t} + 0.31\Delta p_{w,t} - 1.43R_{s,t} \\
& + 2.0R_{L,t} + 0.41\pi_{t-2} - 0.36\Delta(k - l)_t - 0.20\Delta(e - k)_t
\end{aligned}$$

for $f = 0$ (lower weights for $f = -0.5$)

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\end{aligned}$$

for $f = 0$ (lower weights for $f_j = -0.5$).

Increases in growth rate of broad money, energy prices, world prices, the markup and long-term interest rates all raise inflation.

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$$\Delta(y-l)_t = 0.44\Delta(k-l) + 0.28\Delta(e-k) + 0.005$$

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for $f = 0$ (lower weights for $f_j = -0.5$).

Increases in capital, energy and short-term interest rates reduce inflation.

A reduction in energy availability of 10% would simultaneously reduce output by 2.8% and exacerbate inflation by 2%.

Implications for current inflation:

- rapidly rising price inflation stems from commodity price inflation;
- tight labour market driving very low unemployment levels suggests the second non-linearity not likely to kick in;
- low productivity from recession likely to dampen effect on real wage growth;
- but, second round effects via wage-price spiral significantly exacerbate inflationary pressures and can dominate productivity effects.

1. **Conditional models of wage inflation, unemployment, productivity and price inflation**
2. **Combining the conditional models**
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Variable	Assumptions		
	Made in June 2022	Made in March 2023	
	2022	2023	2024
Δm_t	4%		
$\Delta p_{o,t}$	150%		
$\Delta p_{w,t}$	6%		
$R_{s,t}$	3%		
$R_{L,t}$	2%		
π_t	5%		
$\Delta(k-l)_t$	1%		
$\Delta(e-k)_t$	-10%		

Table: Baseline assumptions for inflation projections for 2022, 2023, 2024

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	Made in June 2022	Made in March 2023	
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Table: Baseline assumptions for inflation projections for 2022, 2023, 2024

Assumptions for 2022 made based on judgement.
For 2023 and 2024 use Cardt to obtain projections (Doornik, Castle, and Hendry, 2020).

Cardt

1. Estimate dampened trend (based on first differences removing large values) with seasonality,

2. estimate autoregressive model with seasonality, forcing unit root if estimates close,

3. estimate a trend-halved integrated moving average model (dampened trend, halved together with intercept correction estimated by moving average model),

4. average three forecasts (simple arithmetic mean),

5. calibrate by treating forecasts as if observed and estimate richer autoregressive model,

6. Fitted values are final forecasts, undoing any transformations (e.g. logs).

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Broad Money forecasts

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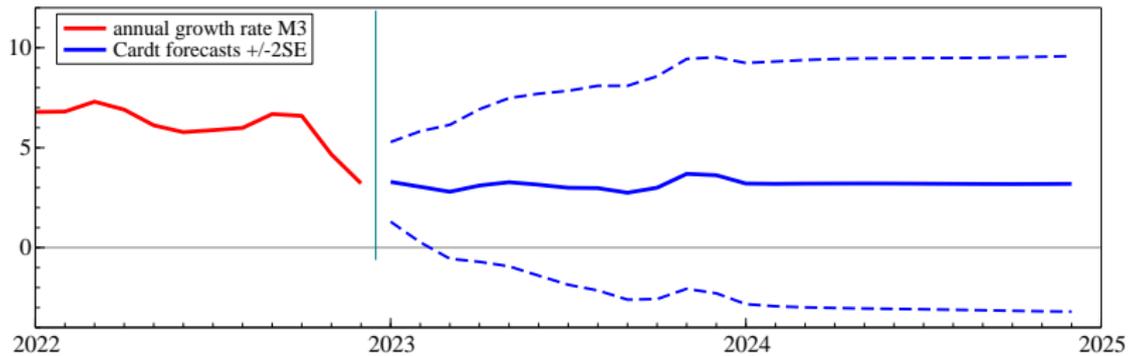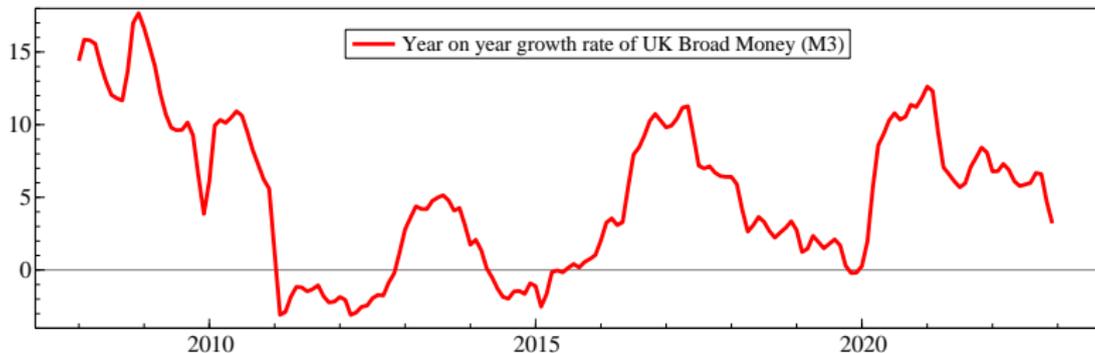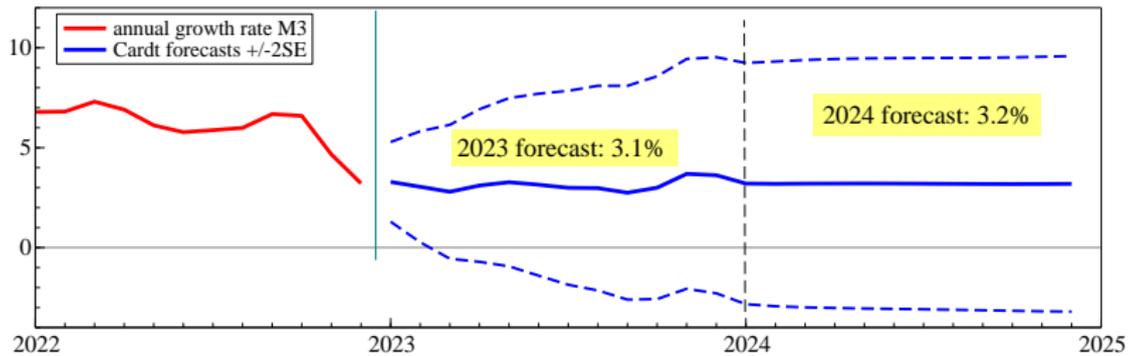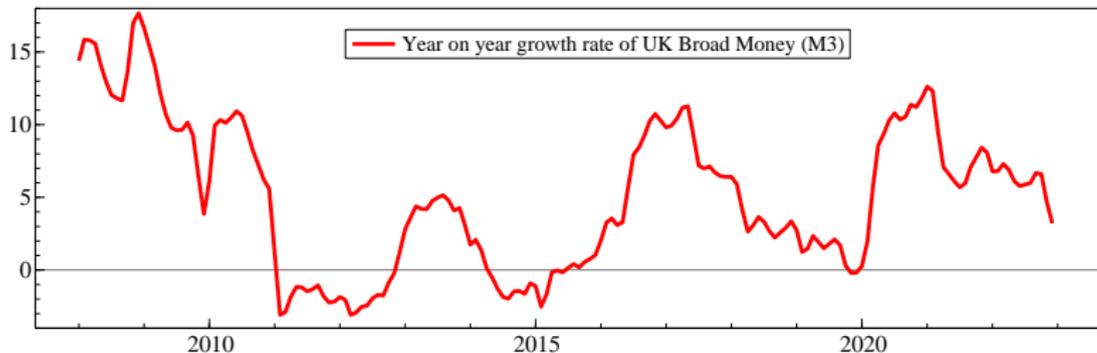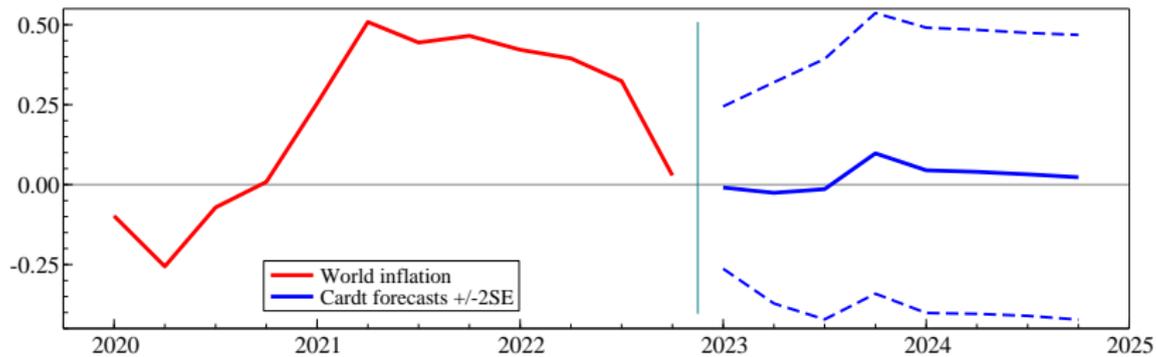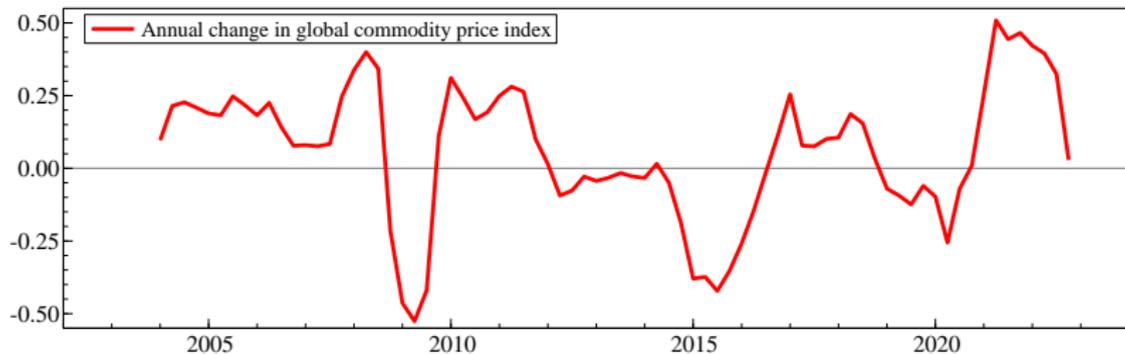
World commodity prices

World commodity prices
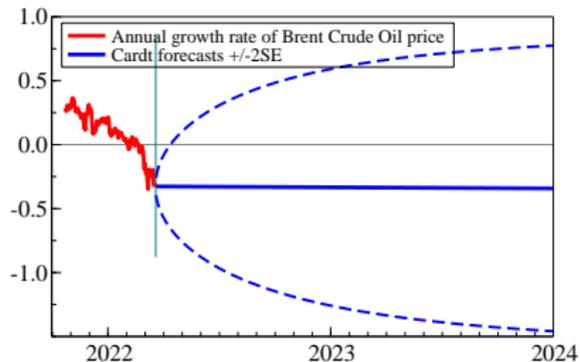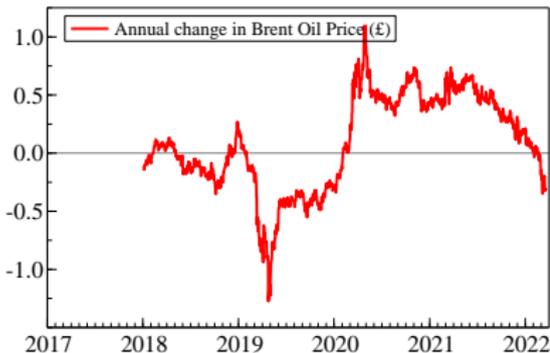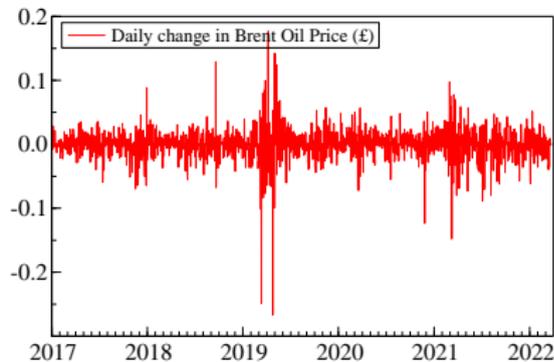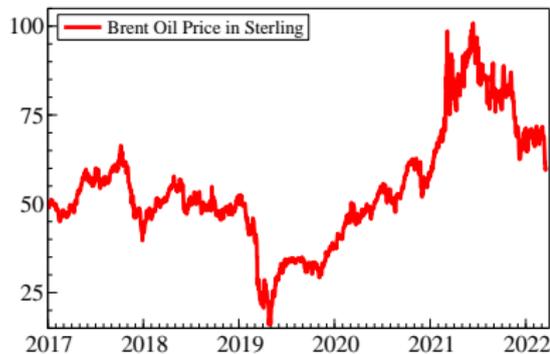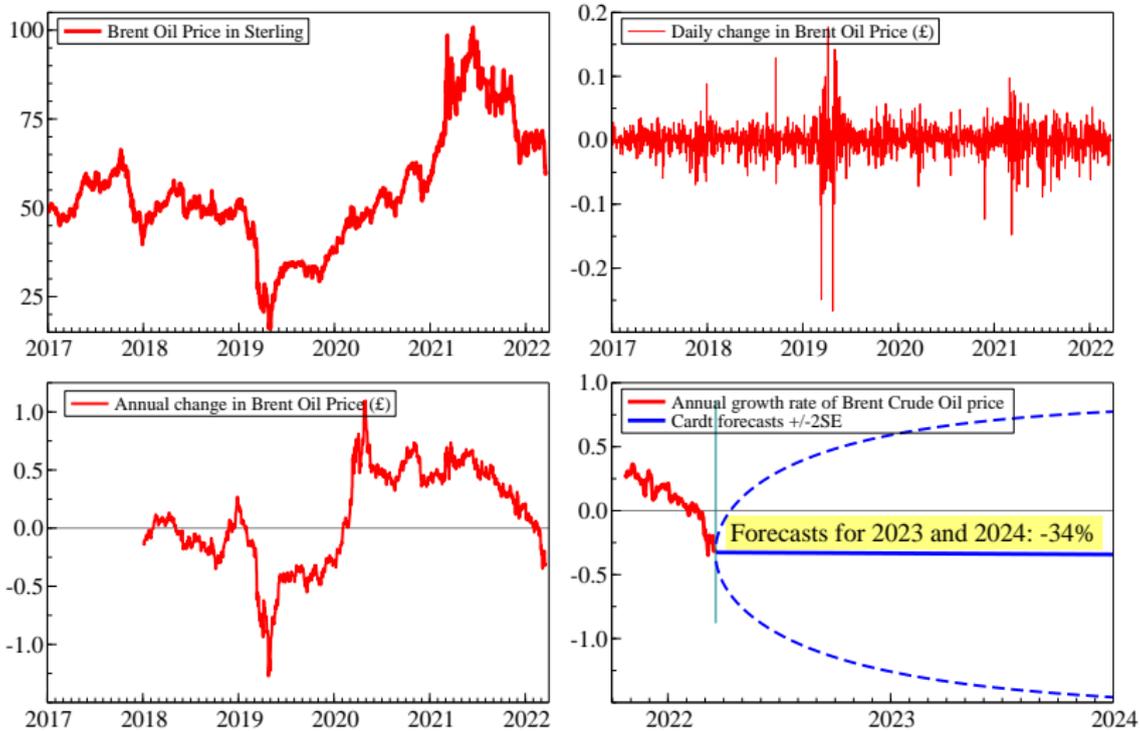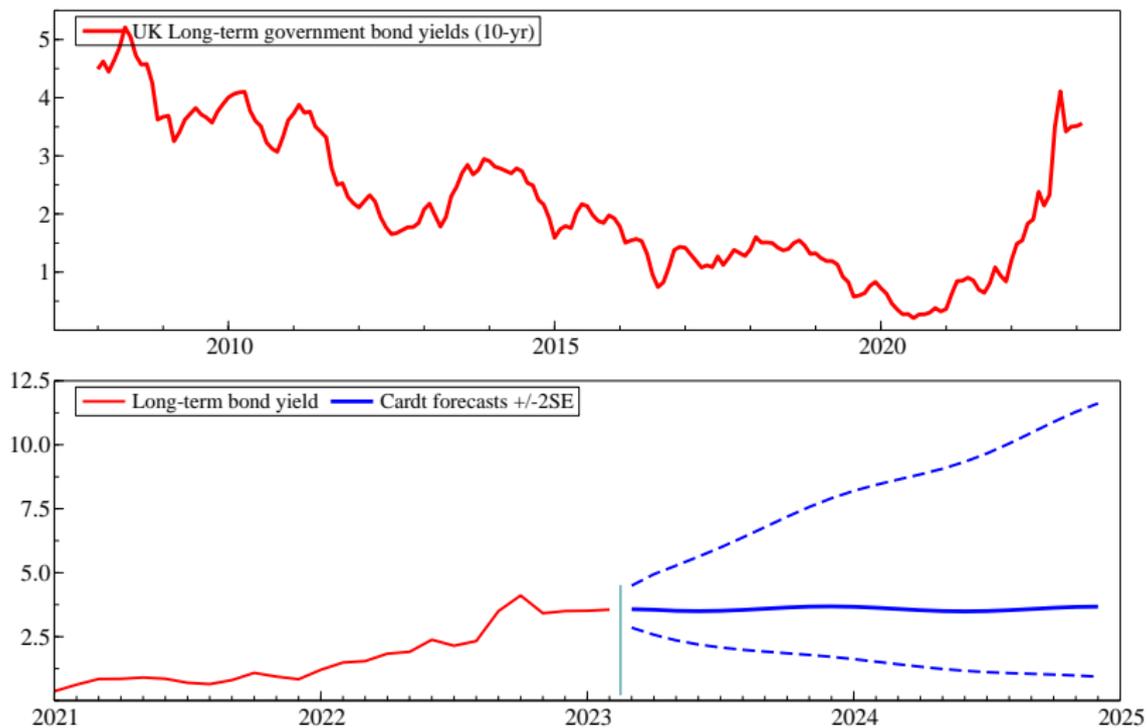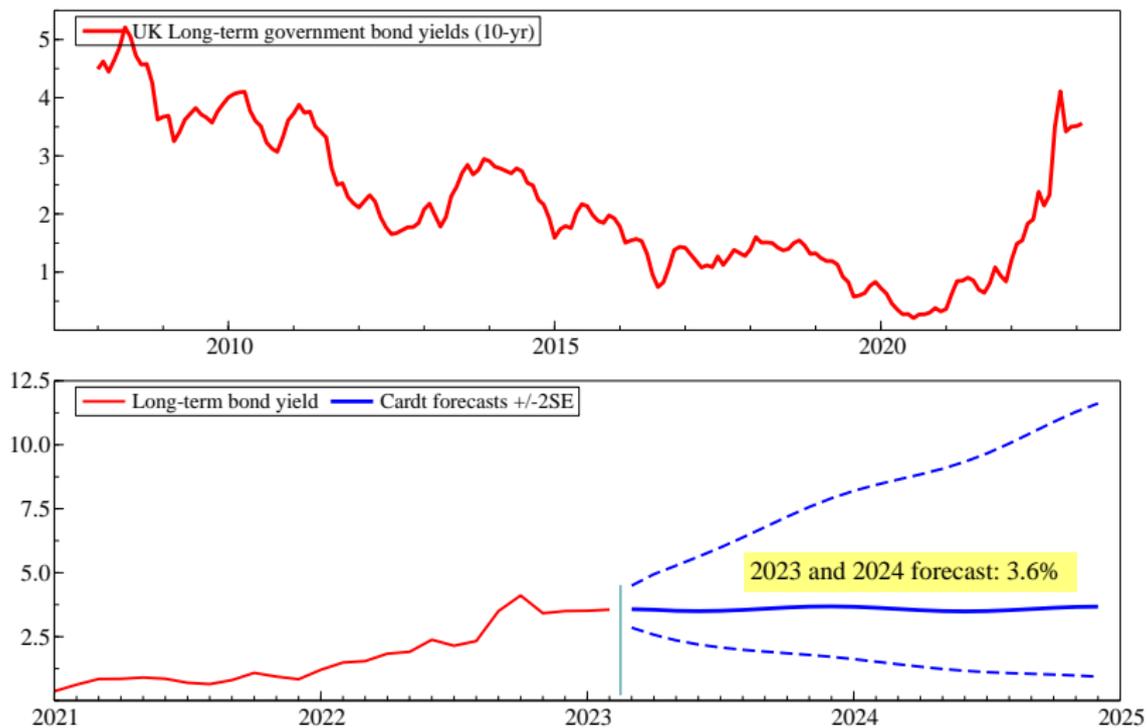
UK commodity prices

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Data on capital stock only available at annual frequency

- ln(UK Gross Capital Stock, total assets)
- ln(UK Total Employment)

matched by means and ranges

- Growth rate of capital per worker

- Growth rate of capital per worker
- Cardt forecasts +/-2SE

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ln(UK Gross Capital Stock, total assets)
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Growth rate of capital per worker

Growth rate of capital per worker
Cardt forecasts +/-2SE

2024 forecast: 1.2%

2023 forecast: 1%

ln(Energy consumption of primary fuels and equivalent (Mtoe)
ln(UK Gross Capital Stock, total assets)

Growth in energy per unit of capital stock

Growth in energy per unit of capital
Cardt forecasts +/-2SE

forecasts for 2023 & 2024: -8%
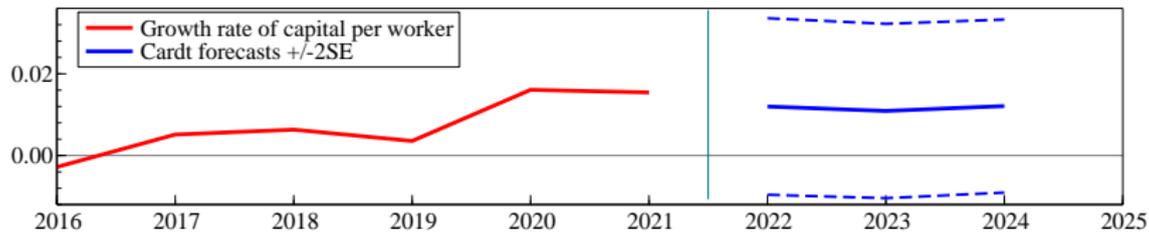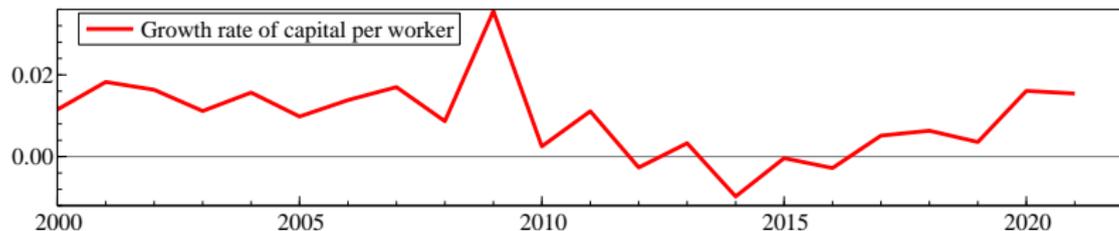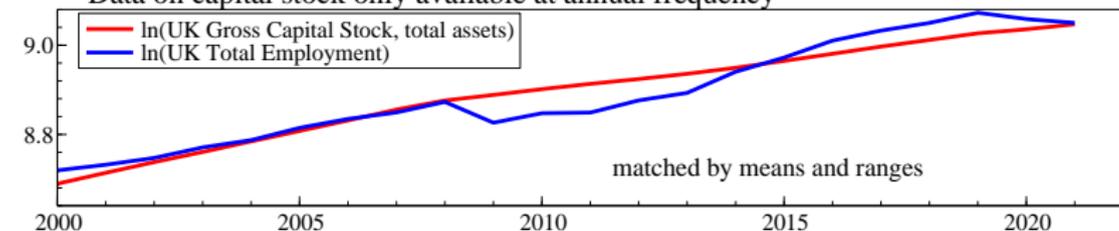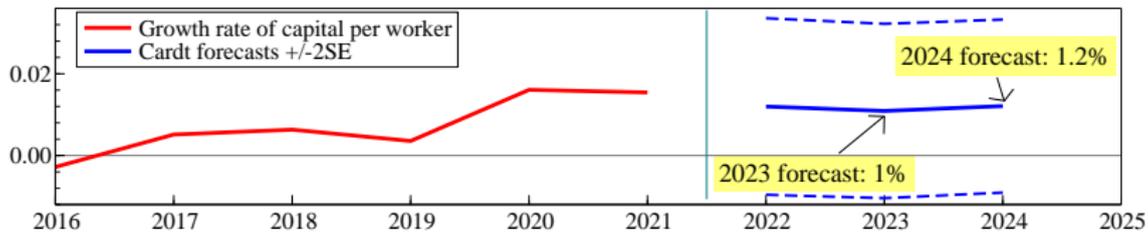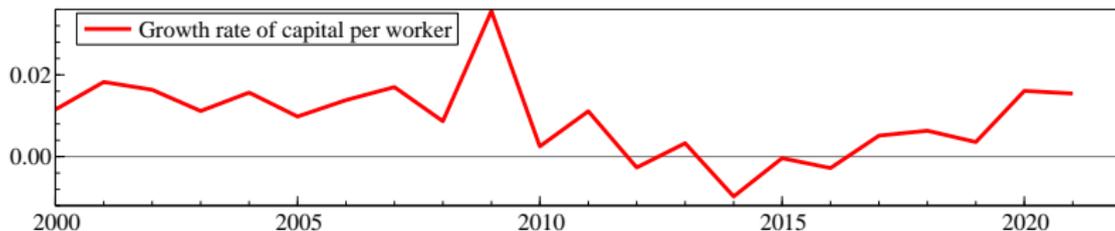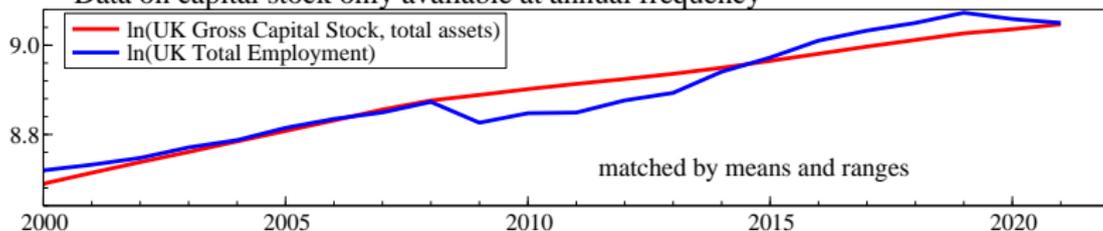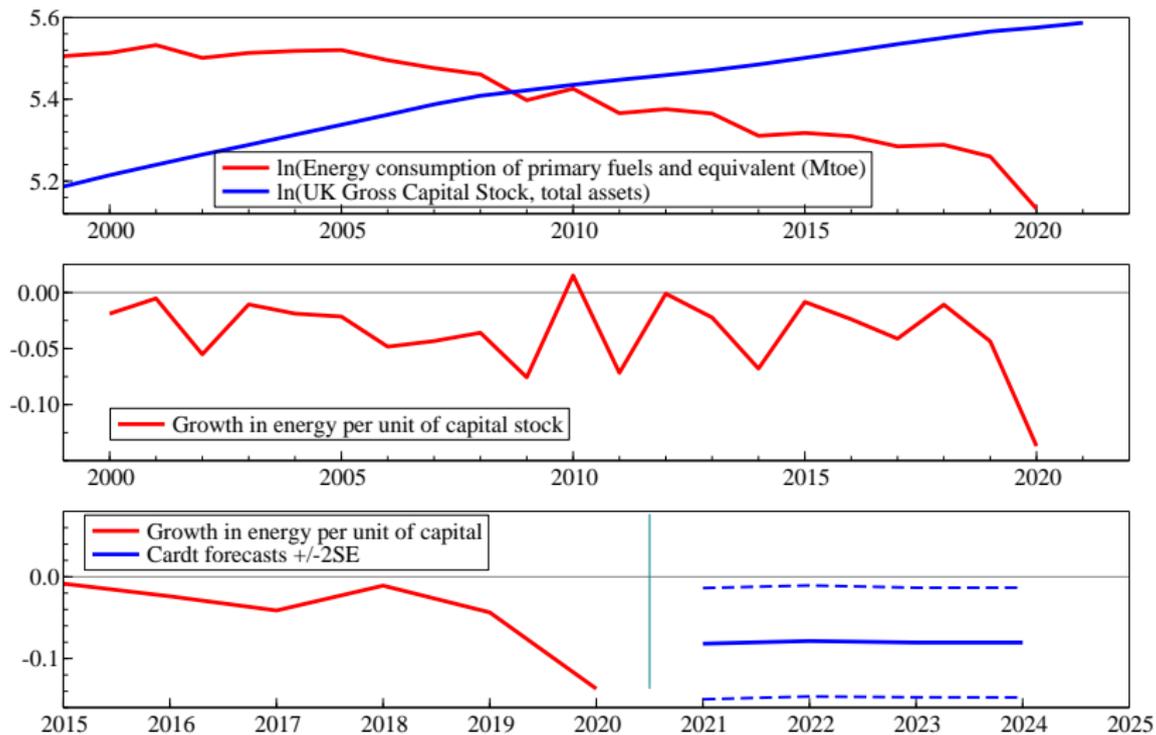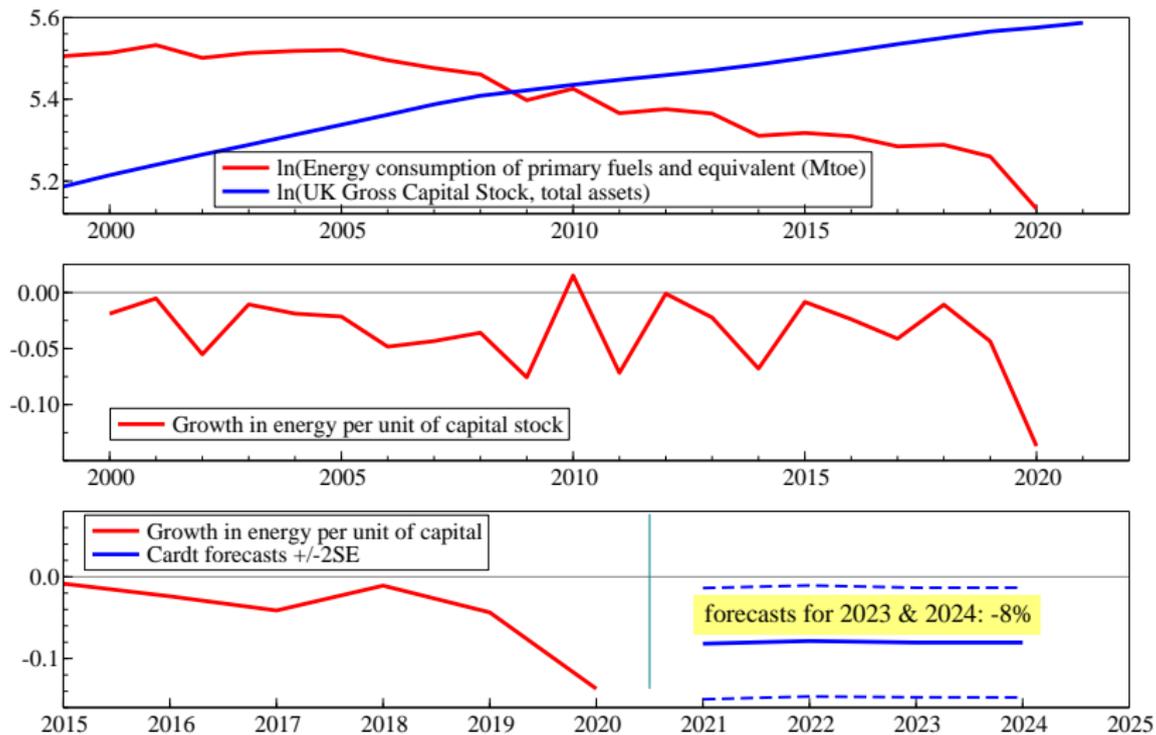
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π_t	5%	3%	3%
$\Delta(k-l)_t$	1%	1%	1.2%
$\Delta(e-k)_t$	-10%	-8%	-8%

Table: Baseline assumptions for inflation projections for 2022, 2023, 2024

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Projected inflation for 2023 with full pass through

Increase ■ Decrease ■ Total

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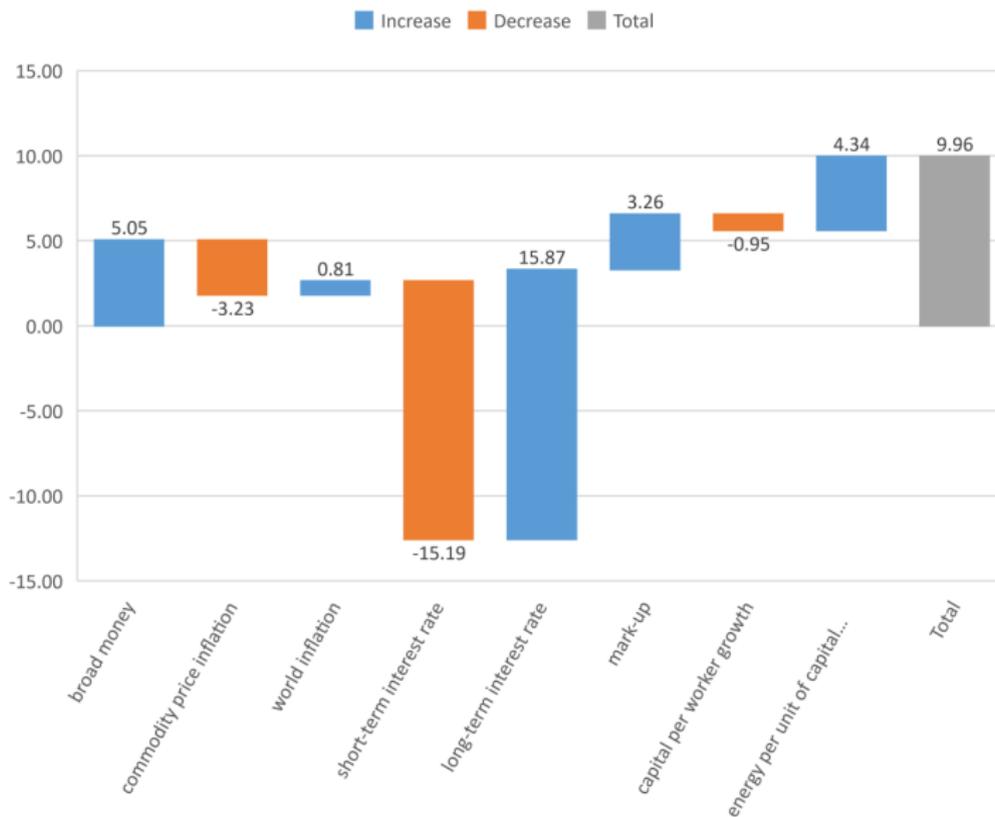
Projected inflation for 2023 with 50% pass through
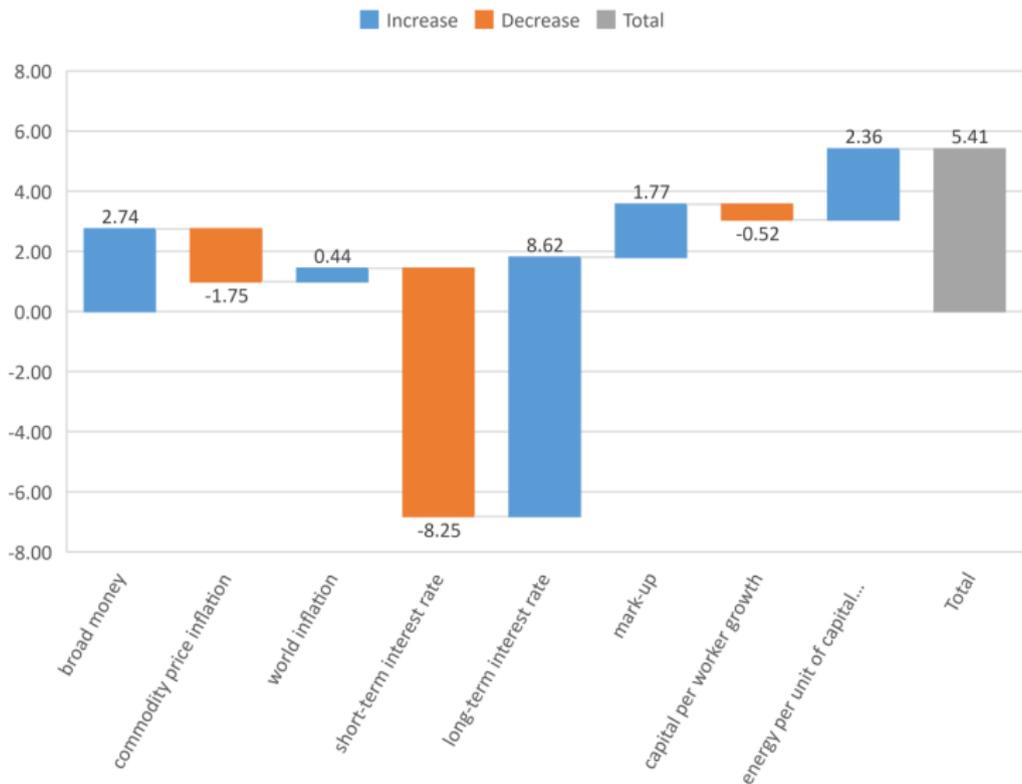
	Annual inflation (%)	
	Full wage pass-through	50% wage pass-through
2022	12.5	6.6
2023	10	5.4
2024	7.4	4.0

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Scenario: $\Delta p_{o,t}$ uses equally-weighted average of 50% increase for oil and 250% increase for natural gas, resulting in 150% increase in commodity prices. **[Outturn: 170%]**

UK inflation nowcast for 2022: Scenario 1
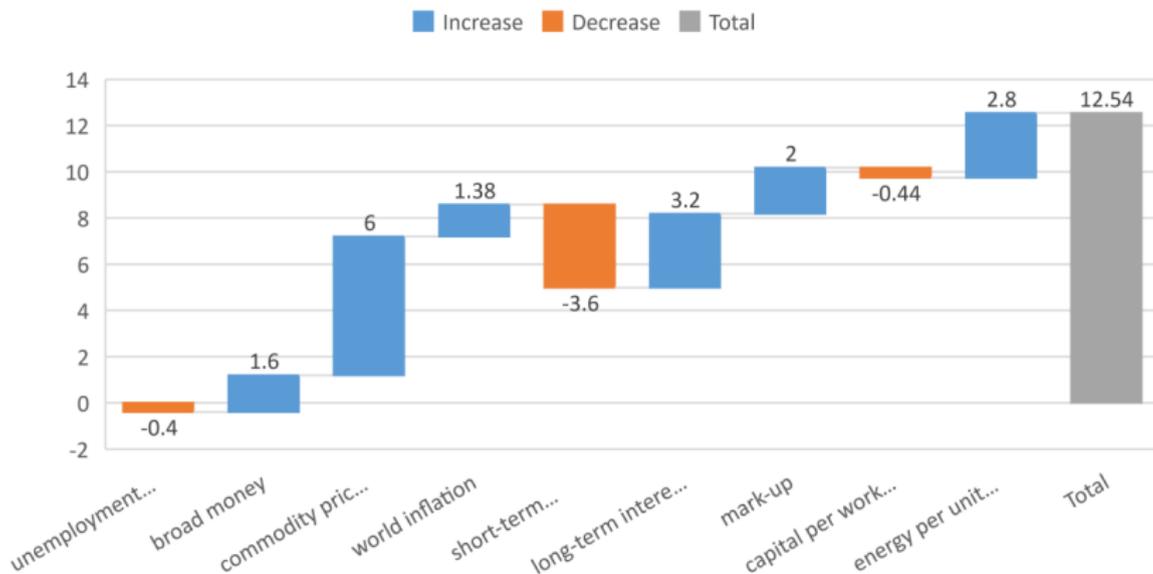
Impact of energy price increase contributes almost half projected contribution to price inflation. Short term interest rates would need to rise to 5% to offset direct contributions of energy price rises.

UK inflation nowcast for 2022: Scenario 1

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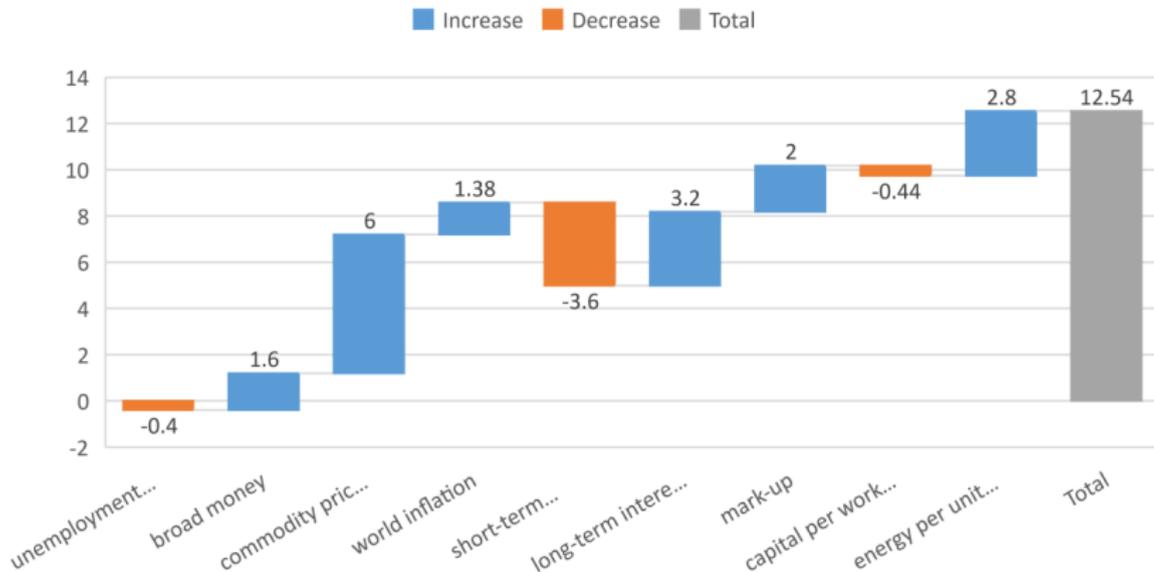
Actual inflation reached 11.1% in October 2022.
Average annualised inflation rate over 2022 was 9%.

UK inflation nowcast for 2022: Scenario 1

■ Increase ■ Decrease ■ Total
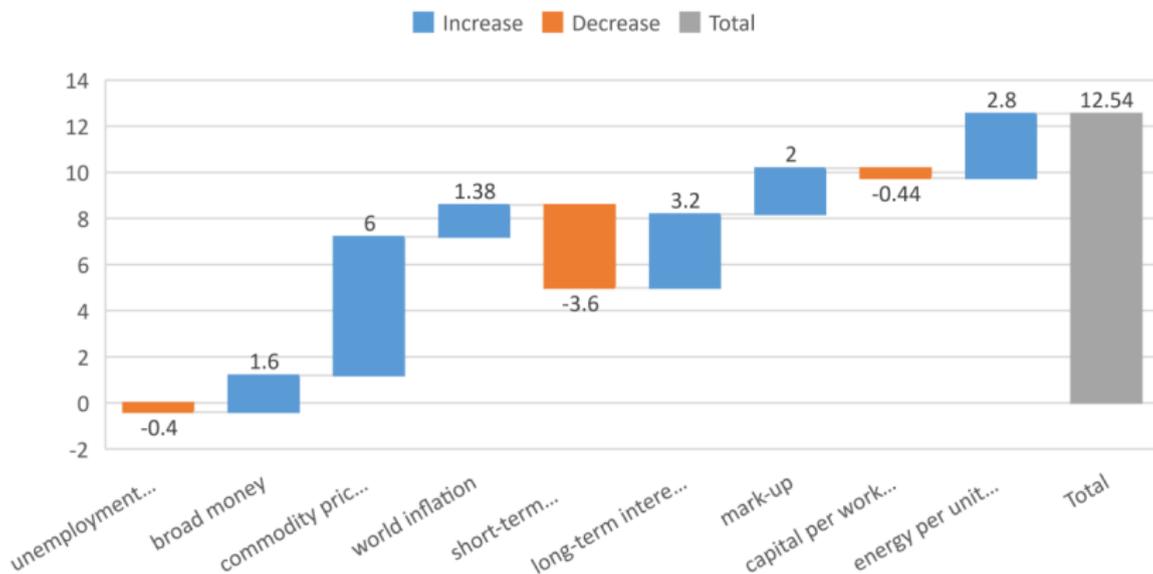
Realised values for 2022:
$\Delta p_o = 170\%$; $\Delta p_w = 8.3\%$;
$R_s \approx 2\%$ [0.25% (Jan 2022) to
3.5% (Dec 2022)]; $R_L \approx 2.5\%$
$\Delta m \approx 6\%$. $\Delta(k-l)$, $\Delta(e-k)$
actual data not available so use
projections.

**Given these values, inflation of
9% implies a pass-through of
0.8.**

Non-linear reaction of -0.2 suggests inflation of approx 7%
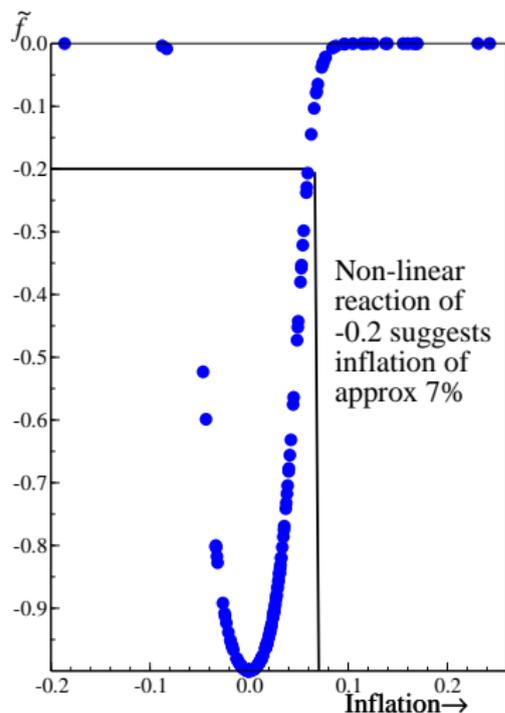
	Annual inflation (%)		
	Full wage pass-through	50% wage pass-through	80% wage pass-through
2022	12.5	6.6	9.4
2023	10	5.4	7.5
2024	7.4	4.0	5.6

Bank of England forecast inflation of 8.3% (2023Q2) 3.4% (2024Q2) and 1.1% (2025Q2) $\Rightarrow$ annualised inflation forecast of 7.7% for 2023 and 3.3% for 2024.

2022 projections suggest pass-through from wages to prices of 0.8 **which would result in inflation of** 7.5% **in 2023 given assumptions.**

Forecasts hinge on non-linearity of wage-price spiral. Degree of pass through from prices to wages critical to scenario predictions.

1 **Conditional models of wage inflation, unemployment, productivity and price inflation**
2 **Combining the conditional models**
3 **Assumptions for scenario projections**
4 **UK inflation nowcasts for 2023 and forecasts for 2024**
5 **Conclusions**

Recent rise in UK price inflation was unanticipated but not new; history can shed light on current inflationary climate.

Long-run time-series data contain lots of variation helping to identify explanatory factors and non-linearities, but also highly non-stationary with structural breaks and distributional shifts.

Conditional empirical models jointly model dynamics, location shifts, relevant variables and non-linearities.

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Price and wage equations combined with non-linearities to obtain projections for contributions to current inflation.

Energy costs along with unit labour costs are fundamental to explaining past inflation episodes, and hence understanding current inflationary pressures.

Thank you!

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