What Can Time-Series Regressions Tell Us About Policy Counterfactuals?[†]

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Abstract: We show that, in a general family of linearized structural macroeconomic models, knowledge of the empirically estimable causal effects of contemporaneous and news shocks to the prevailing policy rule is sufficient to construct counterfactuals under alternative policy rules. If the researcher is willing to postulate a loss function, our results furthermore allow her to recover an optimal policy rule for that loss. Under our assumptions, the derived counterfactuals and optimal policies are robust to the Lucas critique. We then discuss strategies for applying these insights when only a limited amount of empirical causal evidence on policy shock transmission is available.

Keywords: Lucas critique, policy counterfactuals, macroeconomic modeling, business cycles, monetary policy, policy shocks. *JEL codes:* E32, E61.

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1 Introduction

An important function of macroeconomics is to predict the consequences of changes in policy. In this paper we revisit the role that evidence on policy *shocks*—that is, surprise deviations from a prevailing rule—can play in helping macroeconomists learn about policy *rule* counterfactuals. Existing work mainly uses such policy shocks in two ways. First, in what Christiano et al. (1999) call the "Lucas program", researchers begin by estimating the causal effects of a policy shock in the data, then construct a micro-founded structural model that matches these effects, and finally trust the model as a laboratory for predicting the effects of changes in policy rules. By design, this approach yields counterfactuals that are robust to the Lucas (1976) critique; on the other hand, the researcher needs to commit to a particular parametric model, thus introducing concerns about model misspecification. An alternative approach, proposed by Sims & Zha (1995), instead relies *only* on the estimated policy shock: in their procedure, the economy is subjected to a new policy shock at each date t, with the shocks chosen so that, t-by-t, the counterfactual policy rule holds.[1] This strategy does not require the researcher to commit to a particular model, but it is subject to the Lucas critique: a rule change announced at date 0 will in general have different effects on private-sector decisions than a sequence of surprise policy shocks at $t = 0, 1, \ldots$.

The contribution of this paper is to propose a method that constructs policy counterfactuals using empirical evidence on *multiple distinct* policy shocks, rather than just a single one. Like Sims & Zha, the method does not rely on a particular parametric structural model; at the same time, for a family of models that nests many of those popular in the Lucas program, it yields counterfactuals that are robust to the Lucas critique. At the heart of our methodology lies an identification result. We prove that, for a relatively general family of macro models, the causal effects of contemporaneous as well as news shocks to a given policy rule are sufficient to construct Lucas critique-robust counterfactuals for alternative policy rules. The core intuition is that, by subjecting the economy to multiple distinct policy shocks at date 0 (rather than a new value of a single shock at $t = 0, 1, \ldots$, as done in Sims & Zha), we are able to enforce the contemplated counterfactual policy rule not just *ex post* along the equilibrium path, but also *ex ante* in private-sector expectations. Under our assumptions, doing so is enough to fully sidestep the Lucas critique. While our exact identification result

[1] See for example Ramey (1993), Bernanke et al. (1997), Leeper & Zha (2003), Hamilton & Herrera (2004), Uribe & Yue (2006), Degasperi et al. (2020), Eberly et al. (2020), Brunnermeier et al. (2021), and Antolin-Diaz et al. (2021) for important applications and extensions of this method.

requires knowledge of the causal effects of a very large number of policy shocks, our proposed empirical method can be applied in the empirically relevant case of a researcher with access to only a couple of distinct shocks. We demonstrate the usefulness of the proposed approach with several applications to monetary policy rule counterfactuals.

Identification result. The first part of the paper establishes the identification result. Our analysis builds on a general linear data-generating process, with one key added restriction: policy is allowed to affect private-sector behavior *only* through the current and future expected path of the policy instrument.[2] For example, for monetary policy, the private sector only cares about the expected future path of the nominal interest rate, and not whether this path is the result of the systematic component of policy—i.e., the policy *rule*—or due to *shocks* to a given rule. We consider an econometrician that lives in this economy and observes data generated under some baseline policy rule, where that baseline rule is subject to shocks. The econometrician then wishes to predict the effects of a switch to some alternative policy rule. Using standard time-series methods, she can estimate the causal effects of shocks to the prevailing policy rule (e.g., Ramey, 2016; Stock & Watson, 2018). Our identification result states that, if the econometrician has successfully estimated the effects of contemporaneous shocks to the prevailing rule as well as the effects of news about deviations from the rule *at all future horizons*, then those estimates contain all the information she needs to construct the counterfactual. Key to the proof is our assumption on how policy rules are allowed to shape private-sector behavior. Since only the expected future path of the policy instrument matters, any given *rule*—characterized by the instrument path that it implies—can equivalently be synthesized by adding well-chosen *shocks* to the baseline rule. All that is required is that those policy shocks imply the same expected instrument path from date-0 onwards as the counterfactual rule. Finally we show that, given a loss function, our econometrician can furthermore leverage the same logic to also characterize *optimal* policy.[3]

How general is the setting of this identification result? Our two key model restrictions are (i) linearity and (ii) the way that the policy instrument is allowed to shape private-sector behavior. We show that property (ii) is a feature shared by many standard business-cycle

[2]More precisely, the policy rule is allowed to matter only through (a) the expected path of the instrument and (b) equilibrium selection. Our assumptions on equilibrium existence and uniqueness for the different rules that we consider address equilibrium selection.

[3]To be clear, our results are silent on the mapping from observables to welfare, and so on the shape of loss functions. Structural models are one way to arrive at such objectives. However, given that objective functions in practice are often derived from a legislated mandate rather than economic theory (e.g., dual mandate), we believe it is useful to have a method of calculating optimal policy for a given objective.

models, including those with many frictions (e.g., Christiano et al., 2005), shocks (e.g., Smets & Wouters, 2007), and even rich micro-level heterogeneity (e.g., Kaplan et al., 2018; Ottonello & Winberry, 2020). Perhaps the most popular class of models violating our restriction is those with an asymmetry of information between the policymaker and private sector, as in Lucas (1972). In such models, private-sector agents solve a filtering problem, and the policy rule affects both the dynamics of the policy instrument as well as the information contained in that policy choice; as a result, the policy instrument itself does not afford a full characterization of the policy stance. The linearity assumption (i), on the other hand, is not a conceptual necessity, but rather a practical one. Linearity implies that the effects of policy changes are invariant to their size, their sign, and the state of the economy. Given certainty equivalence, we can thus simply focus on expected values. As we will see, these simplifications are crucial to connect our theory to empirical time series evidence. Linearity does, of course, also impose costs: the empirical methodology that we propose can be used to compare different cyclical stabilization policies (e.g., Taylor rules), but is less well-suited to study policies that alter the steady state (e.g., changes in the inflation target).

Empirical strategy. The main challenge to operationalizing our identification result is that empirical evidence on the effects of policy shocks is limited. Our theory says that we need to select a linear combination of policy shocks at date-0 that perturbs the current and expected future path of the policy instrument exactly like the contemplated counterfactual rule. This is a daunting informational requirement: in general, to synthesize the effects of any possible expected policy instrument path of some (in practice large) length T, we would need access to T distinct policy shocks. While existing empirical evidence falls short of this ideal, recent research has however made progress on identifying the effects of at least *some* distinct policy shocks with rather different implications for future expected policy paths.[4] How much can be done with this available evidence?

The idea of our empirical method is to use the available evidence on policy shock transmission to provide a *best Lucas critique-robust approximation* to the desired systematic policy rule counterfactual. Given estimates of the dynamic causal effects of a small number n_s of

[4]For monetary policy, many of the different popular shock series (e.g., Romer & Romer, 2004; Gertler & Karadi, 2015; Antolín-Díaz & Rubio-Ramírez, 2018; Bauer & Swanson, 2022) are well-known to lead to rather different responses of short-term rates. Other identification strategies explicitly aim to identify shocks at different parts of the yield curve (e.g., Gürkaynak et al., 2005; Antolin-Diaz et al., 2021; Inoue & Rossi, 2021), as required by our theory. For fiscal policy, Ramey (2011) and Ramey & Zubairy (2018) estimate the effects of short-lived as well as more persistent shocks. Mertens & Ravn (2010) and Leeper et al. (2013) are similarly focussed on disentangling shocks with different policy dynamics.

policy shocks and their associated instrument paths, we face the challenge that our population identification result cannot be applied immediately: the counterfactual policy rule needs to hold in ex post equilibrium and ex ante expectation for a large number T of periods, but we only have access to $n_s \ll T$ shocks—more equations than unknowns. Our proposal is simply to choose the linear combination of date-0 shocks that enforces the desired counterfactual rule as well as possible, in a standard least-squares sense. Crucially, since this approach involves no ex post surprises dated $t = 1, 2, \ldots$, it is—under our assumptions—fully robust to Lucas critique concerns. Whether or not this best approximation is then in fact a sufficiently accurate representation of the desired counterfactual rule is invariably an application-dependent question.

Applications. We demonstrate the uses and limitations of our empirical method through several examples. Our object of interest is the propagation of a contractionary investment-specific technology shock under different monetary policy rules. As the inputs to our method, we consider the two most popular monetary policy shock series: those of Romer & Romer (2004) and Gertler & Karadi (2015).[5] Importantly, these two shocks reflect different kinds of monetary news—a relatively transitory innovation for Romer & Romer, and a much more gradual rate change for Gertler & Karadi.

Armed with the causal effects associated with those two distinct nominal interest rate paths, we then apply our empirical method to construct counterfactuals for alternative policy rules that: target the output gap, enforce a Taylor-type rule, peg the nominal rate of interest, target nominal GDP, and minimize a simple dual-mandate loss function. We find that, with the exception of the nominal rate peg, the counterfactual rules can be enforced to quite a high degree of accuracy. The conclusion is that, at least for our investment shock, several rather different monetary policy counterfactuals can already be characterized quite sharply simply by combining existing pieces of empirical evidence on monetary policy shock transmission, without commitment to any particular parametric structural model.

Literature. Our identification result provides a bridge between the micro-founded models of the "Lucas program" (as discussed in Christiano et al., 1999) and the empirical strategy proposed by Sims & Zha (1995). Our results reveal that, in the structural models typically used in the Lucas program, the estimand of the econometric strategy of Sims & Zha is not equal to the true policy rule counterfactual *only* because of expectational effects related to

[5]For robustness, we also repeat our exercise using two recent refinements of those canonical shock series.

the future conduct of policy. In theory, using multiple distinct policy shocks at date 0 (rather than a single one at each $t \geq 0$) circumvents this problem; in practice, doing so is feasible because a growing literature on the semi-structural identification of policy shocks provides us with a fairly rich body of empirical evidence (see the references in Footnote 4).

Our work also relates to other more recent contributions to counterfactual policy analysis. Beraja (2020) similarly forms policy counterfactuals without relying on particular parametric models. His approach relies on stronger exclusion restrictions in the non-policy block of the economy, but given those restrictions requires less evidence on policy news shocks. Barnichon & Mesters (2021) use policy shock impulse responses to evaluate whether a given policy decision is optimal, and if not how to improve upon it. While their focus is on evaluating a single policy decision, we instead study systematic changes in the policy rule, requiring additional assumptions on the economic environment—our two assumptions discussed above.[6] More broadly, our work relates to the increasing popularity of a "sufficient statistics" logic for counterfactual analysis (e.g., Chetty, 2009; Arkolakis et al., 2012; Nakamura & Steinsson, 2018). Our identification result reveals that, across a broad class of structural models, the empirically estimable causal effects of policy shocks are precisely such sufficient statistics.

Finally, to prove our identification result, we build on recent advances in solution methods for structural macroeconomic models. At the heart of our analysis lies the fact that equilibria in such models can be characterized by matrices of impulse response functions. As in Guren et al. (2021) and Wolf (2020), we connect this sequence-space representation to empirically estimable objects. In contemporaneous and independent work, De Groot et al. (2021) and Hebden & Winker (2021) show how to use similar arguments to efficiently *compute* policy counterfactuals by generating impulse responses to policy shocks from a structural model. Our focus is not computational—we aim to calculate policy counterfactuals directly from empirical evidence, forcing us to confront the fact that such evidence is limited.

Outline. Section 2 presents our identification result, mapping the effects of policy *shocks* to counterfactuals for policy *rules*. Section 3 introduces our empirical methodology, and Section 4 provides applications to monetary policy rule counterfactuals. Section 5 concludes.

[6] In a newer version of their paper, Barnichon & Mesters (2022) assume a model environment as restrictive as ours as their baseline and consider the more general case as an extension. Another related contribution is Kocherlakota (2019), who presents a dynamic policy game in which the policymaker can select the optimal action via regression analysis. In his setting, the policy action does not cause the private sector to update its beliefs about the future strategy of the policymaker. Therefore policymaker payoffs only depend on the current policy choice and not on the future expected instrument paths that we emphasize in our analysis.

2 From policy shocks to policy rule counterfactuals

We begin in Section 2.1 by presenting a stylized version of our identification argument in a particular, familiar environment: the canonical three-equation New Keynesian model. We then in Sections 2.2 to 2.5 extend the argument to a general class of infinite-horizon models and discuss its scope and limitations.

The main identification result will be presented for a linearized perfect-foresight economy. Due to certainty equivalence, the equilibrium dynamics of a linear model with uncertainty coincide with the solution to such a linearized perfect-foresight environment. We thus emphasize that all results presented below extend without any change to models with aggregate risk solved using first-order perturbation techniques.[7] In particular, the perfect-foresight transition paths that we characterize will correspond to *expected* transition paths—or impulse response functions—in the analogous linearized economy with aggregate risk.

2.1 A simple example

We begin with a discussion of our identification argument in the context of a simple and familiar model environment: the canonical three-equation New Keynesian model (Galí, 2015; Woodford, 2003). We also use this model to explain the relationship between our approach to constructing policy counterfactuals and that of Sims & Zha (1995).

MODEL. The variables of the economy are two private-sector aggregates—output y_t and inflation π_t—and a policy instrument—the nominal rate i_t. They are related through three equations: an Euler equation and a Phillips curve as the private-sector block,

$$y_t = y_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}), \tag{1}$$

$$\pi_t = \kappa y_t + \beta \pi_{t+1} + (\varepsilon_t + \theta \varepsilon_{t-1}), \tag{2}$$

and a simple Taylor rule as the policy rule,

$$i_t = \phi \pi_t + \underbrace{\nu_{0,t}}_{\text{contemp. shock}} + \underbrace{\sum_{\ell=1}^{\infty} \nu_{\ell,t-\ell}}_{\text{news shocks}}. \tag{3}$$

[7] For example see Fernández-Villaverde et al. (2016), Boppart et al. (2018) or Auclert et al. (2021) for a detailed discussion of this point.

In our perfect-foresight set-up, the two private-sector equations as well as the policy rule hold for $t = 0, 1, 2, \ldots$. These equations feature two kinds of disturbances. First, ε_t is a cost-push shock; for the illustrative analysis in this section, we will find it useful to assume that it induces a first-order moving average wedge in the Phillips curve (2). Second, there are the policy shocks $\nu_{\ell,t-\ell}$; here, $\nu_{0,t}$ is a conventional contemporaneous policy shock, while $\nu_{\ell,t-\ell}$ for $\ell > 0$ denotes a deviation from the policy rule at time t announced at $t - \ell$—an ℓ-period "news" shock. These policy shocks will turn out to be crucial for our identification result. As usual, given a vector of time-0 cost-push as well as policy (news) shocks $\{\varepsilon_0, \nu_{0,0}, \nu_{1,0}, \ldots\}$, a perfect-foresight transition path—or impulse response function—are paths of $\{y_t, \pi_t, i_t\}$ such that (1) - (3) all hold at all t.

For the subsequent analysis, the key property of this model economy will turn out to be that the coefficients in the private-sector equations (1) - (2) are independent of the policy rule—i.e., γ, κ and β are unaffected by changes in ϕ. Equivalently, private-sector behavior is affected by policy only through the current and future values of the policy instrument i_t. Our general identification analysis in Sections 2.2 to 2.5 will discuss the generality and limitations of this crucial assumption.

Object of interest. Under the baseline policy rule, the impulse response of the economy to a cost-push shock is given as the solution of (1) - (3) for some cost-push shock ε_0 together with $\nu_{\ell,0} = 0$ for all ℓ. We wish to instead characterize the behavior of this economy in response to ε_0 not under the baseline policy rule (3), but instead under some counterfactual policy rule of the form

$$i_t = \tilde{\phi}\pi_t \tag{4}$$

where $\tilde{\phi} \neq \phi$. Note that this thought experiment supposes that the private sector perfectly understands the change in rule: the new relation between i and π holds at $t = 0, 1, 2, \ldots$. Our identification result characterizes the information required to construct this counterfactual.

The identification argument. We consider an econometrician living in our simple three-equation economy (1) - (3). Using conventional semi-structural time series methods (Ramey, 2016), and with access to suitable identifying assumptions or instruments, that econometrician can in principle estimate the dynamic causal effects of the cost-push shock ε_t as well as the policy shocks $\{\nu_{\ell,t-\ell}\}_{\ell=0}^{\infty}$ under the baseline rule (3). Our main identification result states that this knowledge is sufficient to predict the counterfactual propagation of the shock ε_t under the alternative rule (4). While our formal result is stated and proved for

a more general class of models in Sections 2.2 and 2.3, we here provide the core intuition using our simple three-equation model structure.

The key idea is to choose *time-0* policy shocks $\nu_{\ell,0}$ to the baseline rule in order to mimic the desired counterfactual policy rule. To develop the argument, note first that, because our model has no endogenous state variables, the impulse response to a time-0 cost-push shock will die out after $t = 1$, by our assumption on shock persistence. We collect the 2×1 transition paths of $\{y_t, \pi_t, i_t\}$ in response to a cost-push shock ε_0 under the baseline rule as the vectors $\{\boldsymbol{y}_\phi(\varepsilon_0), \boldsymbol{\pi}_\phi(\varepsilon_0), \boldsymbol{i}_\phi(\varepsilon_0)\}$. Similarly, contemporaneous and one-period-ahead policy shocks also have no effects after $t = 1$. For $\ell \in \{0, 1\}$, we collect the corresponding 2×1 impulse responses under the baseline rule to a policy shock $\nu_{\ell,0}$ as the vectors $\{\Theta_{y,\nu_\ell,\phi}, \Theta_{\pi,\nu_\ell,\phi}, \Theta_{i,\nu_\ell,\phi}\} \times \nu_{\ell,0}$; e.g., $\Theta_{y,\nu_\ell,\phi}$ is the 2×1 impulse response path of y to an ℓ-period ahead shock to the baseline ϕ-rule (3). Now consider setting the two policy shocks to values $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ so that, under the baseline rule (3) and in response to the shock tuple $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$, the counterfactual rule (4) holds at both $t = 0$ and $t = 1$ along the perfect foresight transition path; that is, we solve the following two equations in the two unknowns $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$:

$$\boldsymbol{i}_\phi(\varepsilon_0) + \Theta_{i,\nu_0,\phi}\tilde{\nu}_{0,0} + \Theta_{i,\nu_1,\phi}\tilde{\nu}_{1,0} = \tilde{\phi} \times [\boldsymbol{\pi}_\phi(\varepsilon_0) + \Theta_{\pi,\nu_0,\phi}\tilde{\nu}_{0,0} + \Theta_{\pi,\nu_1,\phi}\tilde{\nu}_{1,0}]\,. \tag{5}$$

The left-hand side of this equation gives us the impulse response of the interest rate following our combination of three shocks $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ under the baseline rule (3), while the right-hand side does the same for inflation, just scaled by $\tilde{\phi}$. By our informational assumptions, the econometrician can evaluate the system of equations (5) given ε_0 and for any candidate set of the two policy shocks $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$. Now suppose a solution $\{\tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ to (5) exists, and then compute the responses of $\{y_t, \pi_t, i_t\}$ to $\{\varepsilon_0, \tilde{\nu}_{0,0}, \tilde{\nu}_{1,0}\}$ under the baseline policy rule.[8] The content of our identification result is that those impulse responses are in fact identical to the impulse responses to ε_0 alone under the counterfactual rule (4). Crucially, this alternative computation uses *only* impulse responses under the baseline rule, and so in particular does not require direct knowledge of the structural equations (1)-(3).

The intuition underlying the identification result is straightforward. Since the private sector's decisions only depend on the expected path of the policy instrument $\{i_0, i_1, \dots\}$, it follows that it does not matter whether this path comes about due to the systematic conduct of policy or due to policy shocks. Equation (5) leverages this logic, looking for a combination

[8] Our general discussion in Sections 2.2 and 2.3 will in detail address the question of when solutions to equations like (5) actually exist.

of date-0 policy shocks that results in the counterfactual policy rule (4) holding both at $t = 0$ *and in expectation* at $t = 1$.

We emphasize that this argument inherently relies on knowledge of the causal effects of both the contemporaneous policy shock $\tilde{\nu}_{0,0}$ *as well as the policy news shock* $\tilde{\nu}_{1,0}$: it is only with those two that we can enforce the counterfactual rule along the entire transition path (which here consists of two time periods). With access only to the contemporaneous policy shock $\tilde{\nu}_{0,0}$, on the other hand, the researcher could only impose the counterfactual rule at $t = 0$, but not at $t = 1$. The proposal of Sims & Zha (1995) is then to subject the economy to another *surprise* contemporaneous policy shock $\tilde{\nu}_{0,1}$ at $t = 1$, chosen to also enforce the counterfactual policy rule at $t = 1$. The key difference relative to our construction is that the private-sector block did not at $t = 0$ *expect* the counterfactual policy rule to hold at $t = 1$; rather, the rule only holds at $t = 1$ because of yet another surprise. In other words, under the approach of Sims & Zha, the counterfactual policy rule only holds ex post along the equilibrium transition path, but not in ex ante expectation. As a result, as long as policy at $t = 1$ matters for $t = 0$ decisions, the constructed counterfactual will differ from the true counterfactual $\{\boldsymbol{y}_{\tilde{\phi}}(\varepsilon_0), \boldsymbol{\pi}_{\tilde{\phi}}(\varepsilon_0), \boldsymbol{i}_{\tilde{\phi}}(\varepsilon_0)\}$. We will further elaborate on this connection between our identification result and the empirical methodology of Sims & Zha in Section 2.4.

Discussion & outlook. The identification result sketched in this section is special in two respects: first, it is presented within the context of a particular explicit structural model; and second, since impulse responses to ε_0 are non-zero only for two periods, the result required knowledge of the effects of two policy shocks. The remainder of this section will state and prove our main identification result in the context of a general class of infinite-horizon models. In terms of our informational requirements, the key change will be that the econometrician now needs to know the causal effects of *all* policy shocks $\{\nu_{\ell,0}\}_{\ell=0}^{\infty}$, rather than just the first two. The economic intuition on the other hand will be exactly the same: the argument will work as the long as the private-sector block of the model depends on the policy rule only through the path of the policy instrument, as was the case here.

2.2 General model & objects of interest

We consider a linearized perfect-foresight, infinite-horizon model economy. Throughout, boldface denotes time paths for $t = 0, 1, 2, \ldots$, and all variables are expressed in deviations from the model's deterministic steady state.

The economy is summarized by the system

$$\mathcal{H}_w \boldsymbol{w} + \mathcal{H}_x \boldsymbol{x} + \mathcal{H}_z \boldsymbol{z} + \mathcal{H}_\varepsilon \boldsymbol{\varepsilon} = \mathbf{0} \tag{6}$$

$$\mathcal{A}_x \boldsymbol{x} + \mathcal{A}_z \boldsymbol{z} + \boldsymbol{\nu} = \mathbf{0} \tag{7}$$

w_t and x_t are n_w- and n_x-dimensional vectors of endogenous variables, z_t is a n_z-dimensional vector of policy instruments, ε_t is a n_ε-dimensional vector of exogenous structural shocks, and ν_t is an n_z-dimensional vector of policy shocks.[9] The distinction between w and x is that the variables in x are observable while those in w are not; in particular, x contains the outcomes of interest for our econometrician as well as the arguments of the counterfactual policy rule that she contemplates.[10] The linear maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ summarize the non-policy block of the economy, yielding $n_w + n_x$ restrictions for each t. Our key assumption—echoing the model of Section 2.1—is that the maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ do not depend on the coefficients of the policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$; instead, policy only matters through the path of the instrument z, with the rule (7) giving n_z restrictions on the policy instruments for each t. As in our simple example, entries of the shock vectors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\nu}$ for $t > 0$ should again be interpreted as news shocks. In particular, the policy shock vector $\boldsymbol{\nu}$ collects the full menu of contemporaneous and news shocks to the prevailing policy rule at all horizons, thus generalizing the two-shock set-up that was our focus in the simple three-equation model.

Given bounded $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$, an equilibrium is a set of bounded transition paths $\{\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}\}$ that solves (6) - (7). We assume that the baseline rule $\{\mathcal{A}_x, \mathcal{A}_z\}$ is such that an equilibrium exists and is unique for any $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$.

Assumption 1. *The policy rule in* (7) *induces a unique equilibrium. That is, the infinite-dimensional linear map*

$$\mathcal{B} \equiv \begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \mathcal{A}_x & \mathcal{A}_z \end{pmatrix}$$

is invertible.

Given $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$, we write that unique solution as $\{\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu}), \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \boldsymbol{\nu})\}$. As in the simple example, we often focus on impulse responses to exogenous shocks $\boldsymbol{\varepsilon}$ when the policy

[9] The boldface vectors $\{\boldsymbol{w}, \boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\varepsilon}, \boldsymbol{\nu}\}$ stack the time paths for all variables (e.g., $\boldsymbol{x} = (\boldsymbol{x}_1', \ldots, \boldsymbol{x}_{n_x}')'$), and the linear maps $\{\mathcal{H}_w, \mathcal{H}_x, \mathcal{H}_z, \mathcal{H}_\varepsilon\}$ are conformable.

[10] For expositional simplicity, we do not include $\boldsymbol{w}$ as an argument of the *baseline* policy rule (7), though doing so would not pose a problem. The key restriction is that the *counterfactual* policy rule only features variables observable to the econometrician.

rule is followed perfectly ($\boldsymbol{\nu} = \mathbf{0}$); with some slight abuse of notation we will simply write those impulse responses as $\{\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$.

Discussion & scope. Our identification results in Section 2.3 and the empirical analysis in Section 3 will apply to *any* structural model that can be written in the general form (6) - (7). As emphasized before, in addition to linearity, the key property of the model for our purposes is that policy enters the non-policy block of the economy *only* through the realized path of the policy variables $\boldsymbol{z}$; equivalently, in the linearized economy with aggregate risk, policy matters only through its effects on the expected future path of the instrument z. How restrictive are those assumptions?

Our first observation is that many of the explicit, parametric structural models used for counterfactual and optimal policy analysis in the classical Lucas program literature fit into our framework. Such models are routinely linearized, and their linear representation features the separation between policy rule and non-policy block that our results require. We here illustrate this point by giving several examples of well-known models that are consistent with our assumptions. The simple model in Section 2.1 has already illustrated that one particular canonical model environment—the textbook three-equation New Keynesian model—fits into our framework.[11] By the exact same line of reasoning, even workhorse estimated business-cycle models (e.g., Christiano et al., 2005; Smets & Wouters, 2007) as well as recent quantitative HANK models (e.g., Auclert et al., 2020; McKay & Wieland, 2021) fit into our structure. For example, in standard HANK-type models, the standard Euler equation of the representative household is simply replaced by a more general "aggregate consumption function" (e.g., Auclert et al., 2018; Wolf, 2021):

$$\boldsymbol{c} = \mathcal{C}(\boldsymbol{y}, \boldsymbol{\pi}, \boldsymbol{i}, \boldsymbol{\varepsilon}^d) = \mathcal{C}_y \boldsymbol{y} + \mathcal{C}_\pi \boldsymbol{\pi} + \mathcal{C}_i \boldsymbol{i} + \boldsymbol{\varepsilon}^d$$

where c is consumption, y is income, π is inflation, i is the nominal rate, and ε^d is a demand shock. Such models continue to fit into our framework precisely because the derivative matrices $\mathcal{C}_\bullet$ depend only on the model's deterministic steady state, and not on policy rules that influence the economy's fluctuations *around* that steady state (e.g., a Taylor rule for nominal rates). We will give a concrete numerical illustration of our identification result in the context of a quantitative HANK-type model in Section 2.4. Finally, as we discuss further

[11] For reference, we in Appendix A.1 explicitly write down the model (1) - (3) in the form of our general matrix system (6) - (7).

in Appendix A.1, several interesting behavioral models (such as those of Gabaix (2020) or Mankiw & Reis (2002)) are also consistent with our assumptions.

While thus clearly quite general, our framework also has some important limitations. Recall that our two key restrictions are (i) linearity and (ii) the way the policy instrument is allowed to shape private-sector behavior. The separation between policy and non-policy block embedded in (ii) is violated in some structural models. Important examples are environments that feature an asymmetry of information between the policymaker and the private sector (like Lucas, 1972). In such models, private-sector agents solve a filtering problem, and in general the coefficients of the policy rule will matter for this filtering problem both through the induced movements of the policy instrument *and* through the information contained in those movements. The separation between the private-sector and policy blocks of the model at the heart of our results thus breaks down—that is, the coefficients in $\mathcal{H}_x$ depend directly on the policy rule (see Appendix A.2 for a formal derivation).

As we discuss in detail in Appendix A.7, the linearity restriction (i) on the other hand is not conceptual, but instead practical. By linearity, the effects of the policy instrument are sign-, size-, and state-invariant. Given certainty equivalence, we can thus focus on expected policy instrument paths, reducing the informational requirements of our identification results. The costs of linearity are twofold. First, our identification results will generally not yield *globally* valid policy counterfactuals. Second, we will be able to construct counterfactuals for alternative policy rules that change the policymaker's response to aggregate perturbations of the macro-economy (such as Taylor rules), but our results are unlikely to apply to policies that change the model's steady state (such as changes in the long-run inflation target).

Objects of interest. As in our simple model, we wish to learn about systematic policy rule counterfactuals. Specifically, we consider an alternative policy rule

$$\tilde{\mathcal{A}}_x \boldsymbol{x} + \tilde{\mathcal{A}}_z \boldsymbol{z} = \mathbf{0} \tag{8}$$

This alternative policy rule is also assumed to induce a unique equilibrium.

Assumption 2. *The policy rule in* (8) *induces a unique equilibrium. That is, the infinite-dimensional linear map*

$$\tilde{\mathcal{B}} \equiv \begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \tilde{\mathcal{A}}_x & \tilde{\mathcal{A}}_z \end{pmatrix}$$

is invertible.

Given this alternative rule $\tilde{\mathcal{A}}$, we ask: what are the dynamic response paths $\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ to some given exogenous non-policy shock path $\boldsymbol{\varepsilon}$?

As a special case of the general counterfactual rule (8), we will study *optimal* policy rules corresponding to a given loss function. Specifically, we consider a policymaker with a simple exogenously given quadratic loss function of the form

$$\mathcal{L} = \sum_{i=1}^{n_x} \lambda_i \boldsymbol{x}_i' W \boldsymbol{x}_i \tag{9}$$

where i indexes the n_x distinct (observable) aggregates collected in x, λ_i denotes policy weights, and $W = \text{diag}(1, \beta, \beta^2, \cdots)$ allows for discounting.[12] As for our general counterfactual rule, we assume that the optimal policy problem has a unique solution.

Assumption 3. *Given any vector of exogenous shocks $\boldsymbol{\varepsilon}$, the problem of choosing the policy variable $\boldsymbol{z}$ to minimize the loss function (9) subject to the non-policy constraint (6) has a unique solution.*

We are then interested in two questions. First, what policy rule is optimal for such a policymaker? Second, given that optimal rule $(\mathcal{A}_x^*, \mathcal{A}_z^*)$, what are the corresponding dynamic response paths $\boldsymbol{x}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ for a given non-policy shock path $\boldsymbol{\varepsilon}$?

Finally, for both general as well as optimal counterfactual policy rules, we would like to go beyond counterfactuals conditional on particular non-policy shock paths $\boldsymbol{\varepsilon}$, and instead also predict the effects of a rule change on *unconditional* macroeconomic dynamics. In particular, we would like to predict how the change in policy rule would affect the unconditional second-moment properties of the observed macroeconomic aggregates x.

The objective of the remainder of this section is to characterize the information required to recover these desired policy counterfactuals. The key insight is that, exactly as in our simple model, all of the required information can in principle be recovered from data generated under the baseline policy rule.

[12] We emphasize that our results are completely silent on the *shape* of the loss function, with structural modeling still the most natural way of obtaining a mapping from observables to welfare. We instead take as given the loss function and ask what kind of empirical evidence would be most useful to figure out how to minimize the loss. We furthermore note that our focus on a separable quadratic loss functions is in line with standard optimal policy analysis, but not essential. As shown in Appendix A.3, our results extend to the non-separable quadratic case, where the loss is now given by $\boldsymbol{x}'Q\boldsymbol{x}$ for a weighting matrix Q. While our approach in principle also applies to even richer loss functions, the resulting optimal policy rule will generally not fit into the form (8).

2.3 Identification results

We begin by defining the dynamic causal effects that lie at the heart of our identification results. By Assumption 1, we can write the solution to the system (6) - (7) as

$$\begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \underbrace{-\mathcal{B}^{-1} \times \begin{pmatrix} \mathcal{H}_{\varepsilon} & \mathbf{0} \\ \mathbf{0} & I \end{pmatrix}}_{\equiv \Theta_{\mathcal{A}}} \times \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\nu} \end{pmatrix}$$

The linear map $\Theta_{\mathcal{A}}$ collects the impulse responses of $\boldsymbol{w}$, $\boldsymbol{x}$ and $\boldsymbol{z}$ to the non-policy and policy shocks $(\boldsymbol{\varepsilon}, \boldsymbol{\nu})$ under the prevailing baseline policy rule (7) with parameters $\mathcal{A}$. We will partition it as

$$\Theta_{\mathcal{A}} \equiv \begin{pmatrix} \Theta_{w,\varepsilon,\mathcal{A}} & \Theta_{w,\nu,\mathcal{A}} \\ \Theta_{x,\varepsilon,\mathcal{A}} & \Theta_{x,\nu,\mathcal{A}} \\ \Theta_{z,\varepsilon,\mathcal{A}} & \Theta_{z,\nu,\mathcal{A}} \end{pmatrix}. \tag{10}$$

All of our identification results will require knowledge of $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$—the impulse responses of the policy instruments z and macroeconomic observables x to contemporaneous as well as all possible future shocks $\boldsymbol{\nu}$ to the prevailing policy rule. Furthermore, to construct counterfactual paths that correspond to a given non-policy shock sequence $\boldsymbol{\varepsilon}$, we also require knowledge of the dynamic causal effects of that particular shock sequence under the baseline policy rule, $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$. We emphasize that, in principle, all of these objects are estimable using data generated under the baseline policy rule: for example, given valid instrumental variables for all the distinct policy shocks $\boldsymbol{\nu}$ as well as a single instrument for the non-policy shock path $\boldsymbol{\varepsilon}$, the required entries of the Θ's can be estimated using semi-structural time-series methods (e.g., see Ramey, 2016, for a review).

These informational requirements are the natural generalization of those for the simple model in Section 2.1. First, since we are now considering an infinite-horizon economy, any given shock generates entire *paths* of impulse responses, corresponding to the rows of the Θ's. Second, rather than two policy shocks, we now need to know causal effects corresponding to the full menu of possible contemporaneous and news shocks $\boldsymbol{\nu}$—all columns of the Θ_{ν}'s.

General counterfactual rule. We begin with the main object of interest—policy counterfactuals after a non-policy shock sequence $\boldsymbol{\varepsilon}$ under an alternative policy rule.

Proposition 1. *For any alternative policy rule* $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$ *that induces a unique equilibrium,*

we can recover the policy counterfactuals $\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ *and* $\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$ *as*

$$\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) = \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}) \equiv \boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{x,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}} \tag{11}$$

$$\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) = \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}) \equiv \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{z,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}} \tag{12}$$

where $\tilde{\boldsymbol{\nu}}$ *is the unique solution of the system*

$$\tilde{\mathcal{A}}_x \left[\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{x,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}\right] + \tilde{\mathcal{A}}_z \left[\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Theta_{z,\nu,\mathcal{A}} \times \tilde{\boldsymbol{\nu}}\right] = \mathbf{0}. \tag{13}$$

Proof. The equilibrium system under the new policy rule can be written as

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \tilde{\mathcal{A}}_x & \tilde{\mathcal{A}}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \begin{pmatrix} -\mathcal{H}_\varepsilon \\ \mathbf{0} \end{pmatrix} \boldsymbol{\varepsilon} \tag{14}$$

By Assumption 2 we know that (14) has a unique solution $\{\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$. To characterize this solution as a function of observables, suppose instead that we could find a $\tilde{\boldsymbol{\nu}}$ that solves (13). Since (6) also holds under the baseline policy rule, and since (13) imposes the new policy rule, it follows that any $(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}, \tilde{\boldsymbol{\nu}}))$ with $\tilde{\boldsymbol{\nu}}$ solving (13) are also part of a solution of (14). Since by assumption (14) has a unique solution, it follows that the system (13) is solved by at most one $\tilde{\boldsymbol{\nu}}$.

It remains to establish that the system (13) has a solution. For this consider the candidate $\tilde{\boldsymbol{\nu}} = (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})$. Since the paths $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ solve (14), it follows that they are also a solution to the system

$$\begin{pmatrix} \mathcal{H}_w & \mathcal{H}_x & \mathcal{H}_z \\ \mathbf{0} & \mathcal{A}_x & \mathcal{A}_z \end{pmatrix} \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = - \begin{pmatrix} \mathcal{H}_\varepsilon \boldsymbol{\varepsilon} \\ (\tilde{\mathcal{A}}_x - \mathcal{A}_x)\boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) + (\tilde{\mathcal{A}}_z - \mathcal{A}_z)\boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}) \end{pmatrix} \tag{15}$$

But by Assumption 1 we know that the system (15) has a unique solution, so indeed the paths $\{\boldsymbol{w}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{x}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\tilde{\mathcal{A}}}(\boldsymbol{\varepsilon})\}$ are that solution. It then follows from the definition of $\Theta_{\mathcal{A}}$ in (10) that the candidate $\tilde{\boldsymbol{\nu}}$ also solves (13), completing the argument. □

It follows from Proposition 1 that we can recover the desired counterfactual as a function of $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$ and $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$ alone.[13] The key building block equation (13) is the

[13]While Proposition 1 applies to a particular shock path $\boldsymbol{\varepsilon}$, it is immediate that the exact same argument also applies to a particular *historical scenario* (as studied in Antolin-Diaz et al., 2021): a historical scenario is simply an observed set of paths $\boldsymbol{x}_{\mathcal{A}}$ and $\boldsymbol{z}_{\mathcal{A}}$ for a given time period in history, and we can use the logic of

infinite-horizon analogue of the bivariate system (5) from our simple two-period example in Section 2.1. The intuition is exactly the same: since we know the effects of all possible perturbations $\boldsymbol{\nu}$ of the baseline rule, we can always construct a date-0 shock vector $\tilde{\boldsymbol{\nu}}$ that mimics the equilibrium instrument path under the new instrument rule. But since the first model block (6) depends on the policy rule *only* via the expected instrument path, the equilibrium allocations under the new counterfactual rule and the perturbed prevailing rule are the same.[14] The only difference relative to the simple model is that, because we now consider an infinite-horizon setting, we in general require evidence on contemporaneous and all possible future news shocks to the baseline rule in order to be able to mimic an arbitrary alternative policy rule.

Optimal policy. A very similar argument allows us to recover optimal policy rules corresponding to a given loss function.

Proposition 2. *Consider a policymaker with loss function (9). For any $\boldsymbol{\varepsilon}$, the solution to the optimal policy problem is uniquely implemented by the rule $\{\mathcal{A}_x^*, \mathcal{A}_z^*\}$ with*

$$\mathcal{A}_x^* = \left(\lambda_1 \Theta'_{x_1,\nu,\mathcal{A}} W, \lambda_2 \Theta'_{x_2,\nu,\mathcal{A}} W, \dots, \lambda_{n_x} \Theta'_{x_{n_x},\nu,\mathcal{A}} W\right), \tag{16}$$

$$\mathcal{A}_z^* = \mathbf{0}. \tag{17}$$

Given $\{\mathcal{A}_x^*, \mathcal{A}_z^*\}$, the corresponding counterfactual paths under the optimal policy rule, $\boldsymbol{x}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$ and $\boldsymbol{z}_{\mathcal{A}^*}(\boldsymbol{\varepsilon})$, are characterized as in Proposition 1.

Proof. The solution to the optimal policy problem is characterized by the following first-order conditions:

$$\mathcal{H}'_w (I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{18}$$

$$(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}'_x (I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{19}$$

$$\mathcal{H}'_z (I \otimes W)\boldsymbol{\varphi} = \mathbf{0} \tag{20}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots)$ and φ is the multiplier on (6). By Assumption 3 we know that the system (18) - (20) together with (6) has a unique solution $\{\boldsymbol{x}^*(\boldsymbol{\varepsilon}), \boldsymbol{z}^*(\boldsymbol{\varepsilon}), \boldsymbol{w}^*(\boldsymbol{\varepsilon}), \boldsymbol{\varphi}^*(\boldsymbol{\varepsilon})\}$.

Proposition 1 to recover the analogous counterfactual historical scenario $\boldsymbol{x}_{\bar{\mathcal{A}}}$ and $\boldsymbol{z}_{\bar{\mathcal{A}}}$.

[14] Assumption 2 is important for this argument to work. First, if there is no unique equilibrium under the counterfactual rule, then the construction in Proposition 1 will only recover one possible equilibrium; more specifically, it will recover the fundamental (minimum state variable, or MSV) equilibrium. Second, if the counterfactual rule is such that no equilibrium exists, then (13) will not have a solution.

Now consider the alternative problem of choosing deviations $\boldsymbol{\nu}^*$ from the prevailing rule to minimize (9) subject to (6) - (7). This second problem gives the first-order conditions

$$\begin{aligned}
\mathcal{H}'_w(I \otimes W)\boldsymbol{\varphi} &= \mathbf{0} && (21)\\
(\Lambda \otimes W)\boldsymbol{x} + \mathcal{H}'_x(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}'_x W\boldsymbol{\varphi}_z &= \mathbf{0} && (22)\\
\mathcal{H}'_z(I \otimes W)\boldsymbol{\varphi} + \mathcal{A}'_z W\boldsymbol{\varphi}_z &= \mathbf{0} && (23)\\
W\boldsymbol{\varphi}_z &= \mathbf{0} && (24)
\end{aligned}$$

where φ_z is the multiplier on (7). It now follows from (24) that $\boldsymbol{\varphi}_z = \mathbf{0}$. But then (21) - (23) together with (6) determine the same unique solution for $\{\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{w}\}$ as before, and $\boldsymbol{\nu}^*$ adjusts residually to satisfy (7). The original problem and the alternative problem are thus equivalent. Next note that, by Assumption 1, we can re-write the alternative problem's constraint set as

$$\begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{x} \\ \boldsymbol{z} \end{pmatrix} = \Theta_{\mathcal{A}} \times \begin{pmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\nu}^* \end{pmatrix} \tag{25}$$

The problem of minimizing (9) subject to (25) gives the optimality condition

$$\sum_{i=1}^{n_x} \lambda_i \Theta'_{x_i,\nu,\mathcal{A}} W \boldsymbol{x}_i = 0 \tag{26}$$

By the equivalence of the policy problems, it follows that (26) is an optimal policy *rule*, taking the form (16) - (17). Finally, the second part of the result follows from Proposition 1 since (26) is just a special example of a policy rule $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$.

□

Proposition 2 reveals that, in conjunction with a given policymaker loss function, the information required to construct valid counterfactuals for arbitrary policy rules also suffices to characterize *optimal* policy rules.[15] The intuition is exactly as before: since we know the

[15]Note that, by mapping our perfect foresight economy to a linearized economy with aggregate risk, we can re-write that optimal policy rule as a forecasting targeting rule (Svensson, 1997):

$$\sum_{i=1}^{n_x} \lambda_i \Theta'_{x_i,\nu,\mathcal{A}} W \mathbb{E}_t [\boldsymbol{x}_i] = \mathbf{0}, \tag{27}$$

where now $\boldsymbol{x}_i = (x_{it}, x_{it+1}, \dots)'$. In words, expectations of future targets must always minimize the policymaker loss within the space of (expected) allocations that are implementable via changes in the policy

causal effects of every possible policy perturbation $\boldsymbol{\nu}$ on the policymaker targets $\boldsymbol{x}$, we in particular know the space of those targets that is implementable through policy actions. At an optimum, we must be at the point of this space that minimizes the policymaker loss. As before, it does not matter whether this optimum is attained through some systematic policy rule or through shocks to an alternative rule.

Unconditional second-moment properties. While Propositions 1 and 2 predict counterfactual dynamics *conditional* on particular non-policy shock paths $\boldsymbol{\varepsilon}$, researchers may also be interested in the *unconditional* second-moment properties of macroeconomic aggregates following a change in policy rule. Of course, if researchers have estimated the effects of all distinct non-policy shocks hitting the economy, then such unconditional analysis is simple: apply Propositions 1 and 2 for each such shock and then collect the results in the form of a vector moving average representation.

In practice, however, researchers may not be able to isolate all distinct aggregate non-policy shocks. Our third identification result states that, in some cases, it is nevertheless possible to recover the desired counterfactual second-moment properties. Since the result requires some investment in additional notation, we only state the main idea here and relegate all details to Appendix A.4. The key assumption allowing us to make progress is that of "invertibility": we need to assume that the structural vector moving average representation of the observable data x and z under the baseline policy rule is invertible with respect to the structural shocks driving the economy. This assumption, while restrictive (Plagborg-Møller & Wolf, 2021a), is routinely imposed in conventional Structural Vector Autoregression analysis (Fernández-Villaverde et al., 2007). Under this assumption, researchers need not be able to separately observe all of the individual structural shocks; instead, it suffices to simply apply our counterfactual prediction results in Propositions 1 and 2 to the Wold innovations and then again collect the results in the form of a counterfactual vector moving average. Appendix A.4 also discusses why this argument fails in the non-invertible case.

Discussion. The identification results in Propositions 1 and 2 offer a novel bridge between the "Lucas program" (see Christiano et al., 1999)—a strategy that relies on micro-founded structural models to form policy counterfactuals—and the purely empirical approach of Sims & Zha (1995). The propositions reveal that, under our assumptions, impulse responses to contemporaneous and news policy *shocks*—objects that are estimable using semi-structural

stance. For a timeless perspective, (27) must apply to *revisions* of policymaker expectations at each t.

empirical techniques—suffice to predict the effects of changes in systematic policy *rules*. Key to our argument is the use of *multiple distinct* policy shocks. By using many such shocks (all at date 0), counterfactual rules can be imposed not just ex post but also in ex ante expectation, and this turns out to be enough to fully sidestep the Lucas critique. We further elaborate on the connection between our results and the approach of Sims & Zha—which uses one policy shock, set to a new level at each date t—in Section 2.4.

Our results also resonate with recent attempts to bring insights from the "sufficient statistics" approach popular in public finance to macroeconomics (Chetty, 2009; Nakamura & Steinsson, 2018). For a large family of structural models and policy rule counterfactuals, policy shock impulse responses turn out to be precisely such sufficient statistics.

2.4 Illustration & relation to Sims & Zha (1995)

This section provides a visual illustration of our theoretical identification results and their relationship to the canonical approach of Sims & Zha (1995). As our laboratory we use the structural HANK model of Wolf (2021), with details of the model parameterization relegated to Appendix A.1. In this environment we compute policy counterfactuals in multiple ways: by using the structural equations of the model to simply solve the model with a counterfactual policy rule; by using the approach of Sims & Zha; and by using our identification results.

We begin by solving the model with a baseline policy rule of

$$i_t = \phi_\pi \pi_t + \sum_{\ell=0}^{\infty} \nu_{\ell,t-\ell} \tag{28}$$

and where $\phi_\pi = 1.5$. In particular, we solve for a) the impulse responses $\{\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}), \boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\}$ to a contractionary cost-push shock ε_t under (28) and b) the causal effects of contemporaneous and news policy shocks $\boldsymbol{\nu}$ to (28), $\{\Theta_{x,\nu,\mathcal{A}}, \Theta_{z,\nu,\mathcal{A}}\}$. We emphasize that those causal effects would be estimable for an econometrician living in our model and with access to valid instruments for the cost-push shock ε_t as well as the policy shocks $\{\nu_{0,t}, \nu_{1,t}, \dots\}$.

We now entertain the following counterfactual policy rule:

$$i_t = \phi_i i_{t-1} + (1-\phi_i)(\phi_\pi \pi_t + \phi_y y_t) \tag{29}$$

for $\phi_i = 0.9$, $\phi_\pi = 2$, $\phi_y = 0.5$. The grey and orange lines in all three panels of Figure 1 show the true model-implied impulse responses of output and inflation to a cost-push shock ε_t under the baseline rule (28) (grey) and the counterfactual rule (29) (orange), where both

of these lines are computed from the structural equations of the model. We now seek to recover the desired counterfactual (orange) *only* through knowledge of the dynamic causal effects of policy shocks, and *without* using the structural equations of the model.

The panels of Figure 1 show results for three possible strategies to predict the counterfactual propagation of the cost-push shock. The top panel begins with the empirical strategy of Sims & Zha (1995). Here the econometrician was only able to estimate the dynamic causal effects of the first entry of $\boldsymbol{\nu}$ (i.e., the contemporaneous shock $\nu_{0,t}$), and then uses a sequence of such shocks—one at each $t = 0, 1, 2, \ldots$—to enforce the counterfactual rule (29) ex post along the equilibrium transition path. The right panel shows the sequence of policy shocks that implements this strategy, and the blue dashed lines in the left and middle panels give the responses of output and inflation to the original cost-push shock *plus* the derived sequence of policy shocks. The main takeaway is that those blue dashed lines are not equal to the true counterfactual (orange). Intuitively, the issue is that the contemplated counterfactual rule is only imposed ex post, but not in ex ante expectation. Since expectations about the future affect the present, enforcing the rule through ex post surprises is not the same as switching and committing to a different rule from time $t = 0$ onwards.[16] Visually, the importance of ex post surprises is evident in the right panel: to map the baseline rule into the counterfactual rule, the econometrician requires a *sequence* of expansionary policy shocks $\nu_{0,t}$, with those shocks remaining large throughout the entire first year after the shock.

The middle and bottom panels now illustrate the core logic of our identification result—with multiple policy shocks, the econometrician can impose the counterfactual rule not just ex post, but also in expectation. As a warmup, the middle panel considers a case in which the econometrician is able to estimate the causal effects of the first $n_s = 2$ entries of $\boldsymbol{\nu}$ (i.e., a contemporaneous and a one-period forward guidance shock). Such access to multiple shocks suggests a natural generalization of Sims & Zha: use the available n_s policy shocks at each $t \geq 0$ to enforce the desired counterfactual rule not only ex post (as Sims & Zha do with one shock), but also in ex ante expectation for the next $n_s - 1$ periods.[17] Since the counterfactual policy rule is now imposed both ex post and in ex ante expectation for at least one period, the predicted counterfactuals (blue dashed) are closer to the truth (orange); correspondingly, the policy shock sequences in the right panel feature smaller ex post surprises dated $t = 1, 2, \ldots$. The bottom panel—which corresponds to our identification result—simply continues this

[16] It follows from this discussion that, if the private sector were not at all forward-looking, then one shock would already be enough for Lucas critique-robust counterfactuals.

[17] We present implementation details for this approach in Appendix A.6.

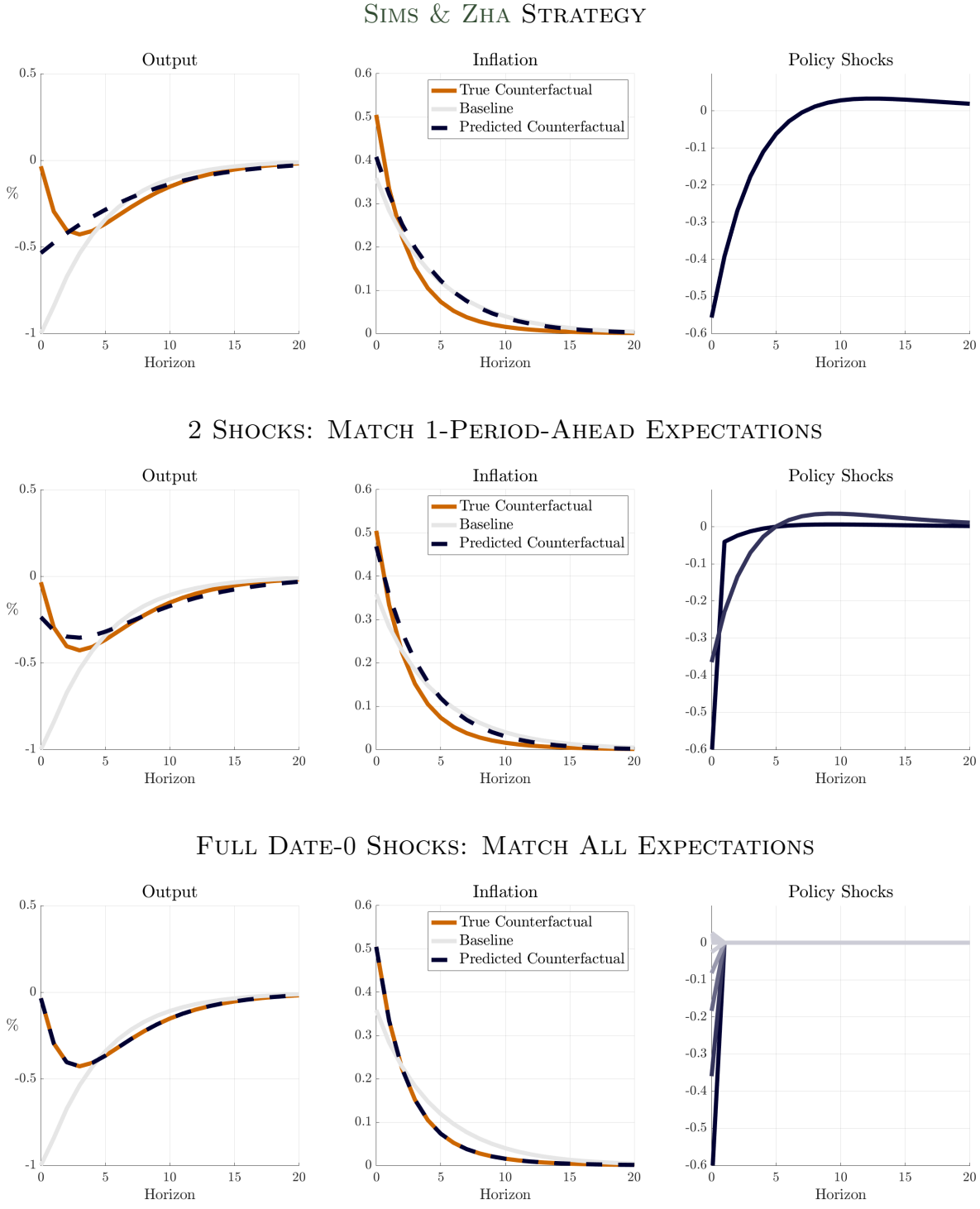

Figure 1: The grey and orange lines in the left and middle panels show output and inflation responses to the cost-push shock ε_t under the policy rules (28) and (29) in the HANK model. The dark blue dashed lines give output and inflation counterfactuals constructed through the policy shocks on the right, set to enforce the counterfactual rule ex post and in expectation for the next $n_s - 1$ periods, for $n_s = 1$ (top panel, = Sims & Zha), $n_s = 2$ (middle panel) and $n_s = \infty$ (bottom panel). Lighter shades of blue correspond to news about policy at longer horizons.

logic. With access to the causal effects of the full vector of policy shocks $\boldsymbol{\nu}$, the econometrician can rely purely on date-0 shocks (right panel) to enforce the counterfactual rule not just ex post but also in ex ante expectation. Under our assumptions, doing so suffices to circumvent the Lucas critique and recover the correct systematic policy rule counterfactual.

To summarize, the top and bottom right panels illustrate the core difference between the empirical method of Sims & Zha and our identification result. In the former, the researcher has access to a single policy shock, and uses a sequence of realizations of that shock to enforce the counterfactual rule. In our approach, the researcher has access to many shocks and only uses shocks at date-0 to enforce the counterfactual rule. Our identification result thus clearly has substantially higher informational requirements, but this increase in information brings with it the similarly substantial benefit of robustness to Lucas critique concerns.

2.5 Discussion

The central takeaway from the analysis in this section is that—under our maintained structural assumptions—policy rule counterfactuals can at least in principle be constructed purely through empirical measurement, in a way that is robust to Lucas critique concerns. In the remainder of the paper we discuss how to operationalize our insights. The main challenge is that the informational requirements underlying our identification results are quite high: the researcher needs evidence on the dynamic causal effects of the full menu of contemporaneous and news policy shocks *at all possible horizons*. Section 3 presents an empirical strategy for the relevant case of researchers with access only to a few distinct identified policy shocks. We then in Section 4 demonstrate the applicability of the method by constructing several systematic monetary policy rule counterfactuals.

3 Empirical method

This section presents our empirical method for constructing policy counterfactuals with evidence on multiple, but a limited number of, distinct policy shocks. Section 3.1 illustrates the basic logic of our method with a simple example based on the famous oil shock application of Bernanke et al. (1997). Section 3.2 then introduces the general methodology.

Throughout, the discussion in this section will leverage the following connection between our theoretical identification results in Section 2.3 and empirical evidence on policy shock propagation. Our theoretical identification analysis was phrased in terms of policy shocks $\boldsymbol{\nu}$ that perturb the prevailing policy rule $\{\mathcal{A}_x, \mathcal{A}_z\}$ horizon by horizon. Rather than expressing

everything in terms of shocks $\boldsymbol{\nu}$, we could however equivalently phrase the informational requirements underlying our identification results in terms of *policy instrument paths*: to implement our results, the econometrician needs to know the dynamic causal effects associated with all possible time paths of the policy instrument $\boldsymbol{z}$.[18] Empirical work that studies a given policy shock simply gives us the dynamic causal effects associated with a *particular* path of the policy instrument. Our empirical method takes this information as given and uses it to construct the desired policy counterfactual.

3.1 Illustrative example

Like Bernanke et al. (1997), we consider an econometrician that wishes to predict the (counterfactual) propagation of oil price shocks in the absence of a monetary policy reaction—the canonical "zeroing-out" policy counterfactual.[19]

Figure 2 provides a stylized representation of how the econometrician could use our identification result to construct the desired policy counterfactual. She begins by estimating the effects of an oil price shock under the prevailing monetary reaction function, exactly as in Bernanke et al. (1997). In the stylized example here, the oil shock leads to an increase in prices (top left panel); the monetary authority furthermore leans against this inflationary pressure through an increase in nominal interest rates (bottom left panel). By our identification result, she next needs to estimate the effects of a monetary policy shock—or a linear combination of such policy shocks—that moves nominal interest rates from date-0 onwards exactly like the observed endogenous interest rate response to the oil shock. The two middle panels show two possible scenarios. In the left one, the econometrician was able to identify a single monetary policy shock that induces the exact same path of nominal interest rates as the oil shock. In the right one, she estimated two separate policy shocks (one solid, one dashed), with the sum of the two replicating the rate path after the oil shock. In both cases,

[18] Formally, what we are discussing here is nothing but a change of basis: we solve for the policy rule counterfactual not in terms of shocks to some (arbitrary) baseline rule $\{\mathcal{A}_x, \mathcal{A}_z\}$, but directly in terms of policy instrument paths. This switch of basis is without loss of generality as long as the policymaker can implement any possible path of the policy instrument (i.e., the map $\Theta_{z,\nu,\mathcal{A}}$ is invertible). While the "rule shock" $\boldsymbol{\nu}$ perspective is much more natural in theory, the "policy instrument path" $\boldsymbol{z}$ perspective allows a more straightforward connection with data.

[19] In notation of Section 2, such "zeroing-out" corresponds to a counterfactual policy rule that sets $\boldsymbol{z} = \mathbf{0}$. It is of course well-known that rules of this sort—for example a nominal interest rate peg—often lead to equilibrium indeterminacy, violating Assumption 2 (Sargent & Wallace, 1981). As discussed in Footnote 14, the counterfactuals presented here should thus be interpreted as corresponding to the fundamental MSV equilibrium associated with this policy rule.

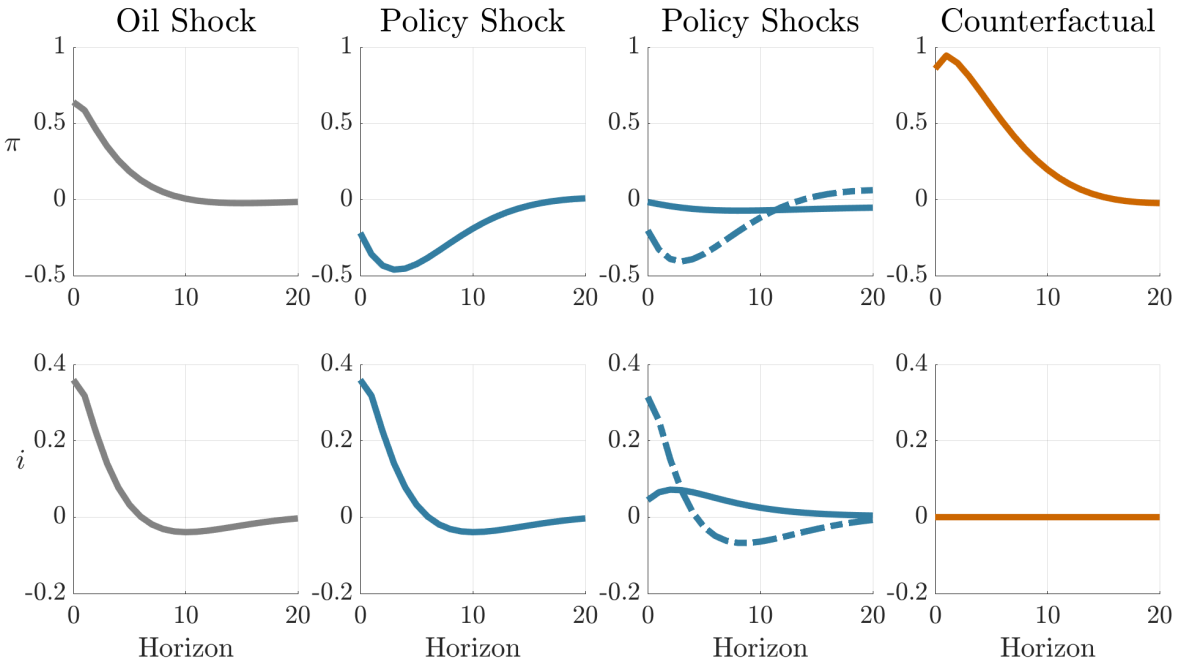

Figure 2: Inflation (π) and interest rate (i) impulse responses to: oil shock under the baseline rule (left panel, grey); monetary policy shocks to the baseline rule (two middle panels, blue and blue dashed); and oil shock under the counterfactual rule (right panel, orange). All impulse responses are purely illustrative; they do not come from any empirical exercise or structural model.

the identified policy shocks decrease inflation (top panels). Given either of these estimates, the econometrician can apply our identification result: she simply needs to subtract the impulse responses shown in the second or third column from those in the first column. The results are shown in the fourth column: interest rates are now by construction unresponsive, and inflation increases by more than under the baseline policy response. It follows from Proposition 1 that any structural model consistent with (i) our general model framework (6) - (7), (ii) the original propagation of the oil shock (first column) and (iii) either one of the two middle columns on monetary policy shock propagation will necessarily agree with this "zeroing-out" counterfactual (orange) displayed in the right panel.

The illustrative example in Figure 2 is stylized in two ways. First, using either of the estimated monetary policy shocks, the econometrician was able to *perfectly* enforce the desired policy counterfactual using only date-0 shocks. In actual applications this will not be possible in general. Second, the counterfactual rule that we considered was particularly simple, taking the form of an exogenous interest rate path rather than a more complicated

relationship between endogenous equilibrium outcomes (like, e.g., a Taylor rule). Our empirical method, presented in the next section, is the natural generalization of the stylized example: the researcher considers an arbitrary counterfactual rule of our general form (8), and enforces it *as well as possible* using the available policy shock evidence.

3.2 Counterfactuals with a limited number of policy shocks

We consider a researcher that has access to estimates of n_s distinct policy shocks associated with n_s distinct response paths of the policy instrument z.[20] We denote the dynamic causal effects of these shocks $\{\Omega_{x,\mathcal{A}}, \Omega_{z,\mathcal{A}}\}$, where each of the n_s columns of the Ω's gives the impulse response to a distinct identified policy shock. Given these lower-dimensional causal effect maps, and given a non-policy shock $\boldsymbol{\varepsilon}$ and a counterfactual rule $\{\tilde{\mathcal{A}}_x, \tilde{\mathcal{A}}_z\}$, the proof strategy of Proposition 1 will fail in general. We would now need to set

$$\tilde{\mathcal{A}}_x(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{x,\mathcal{A}} \times \boldsymbol{s}) + \tilde{\mathcal{A}}_z(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{z,\mathcal{A}} \times \boldsymbol{s}) = \mathbf{0} \tag{30}$$

where $\boldsymbol{s} \in \mathbb{R}^{n_s}$ denotes weights assigned to the n_s empirically identified policy shocks at date 0. The problem is that this system of T equations (where T is the large maximal transition horizon) in n_s unknowns will generically not have a solution. So how can researchers proceed?

Lucas critique-robust method. Our main proposal is to simply select the weights $\boldsymbol{s}$ on the n_s date-0 shocks to enforce the desired counterfactual rule *as well as possible*. In practice, this means solving a straightforward regression problem:

$$\min_{\boldsymbol{s}} \quad ||\tilde{\mathcal{A}}_x(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{x,\mathcal{A}} \times \boldsymbol{s}) + \tilde{\mathcal{A}}_z(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \Omega_{z,\mathcal{A}} \times \boldsymbol{s})||. \tag{31}$$

The output of the simple problem (31) is the best approximation to the desired policy counterfactual within the space of empirically identified policy shock paths. By our identification results in Section 2 and because all shocks are dated $t = 0$ (i.e., no ex post surprises), this approach is robust to the Lucas critique. In the illustrative example of Figure 2, the available evidence on policy shocks in the middle panels was sufficient to set the argument of (31) exactly to zero. In actual applications, on the other hand, we will not perfectly enforce

[20] In saying that a researcher has access to policy shocks that induce different instrument paths, we are implicitly assuming that these differences in instrument paths reflect different identification strategies capturing different linear combinations of the shocks $\boldsymbol{\nu}$, rather than statistical noise or violations of the identifying assumptions. We justify this interpretation in our empirical application in Section 4.

the desired policy counterfactual; rather, we will approximate it as closely as possible. The richer the menu of policy shocks we have access to, the better the approximation will become, eventually converging to the truth (as $n_s \to \infty$). The important limitation of our approach is thus that, for small n_s, it will not always be possible to construct an accurate approximation of the desired counterfactual rule—sometimes we will be able to set the implementation error in (31) close to zero, other times it will be large. The practical usefulness of our proposed method is thus an inherently application-dependent question.

By Proposition 2, our identification results also allow researchers to learn about *optimal* counterfactual policy rules, given some exogenously specified loss function. Appendix B.2 shows how to apply our Lucas critique-robust method to such questions of optimal policy design. Very briefly, the idea is to use date-0 policy shocks to reduce the policymaker loss as much as possible. Our approach thus minimizes the loss function by perturbing the baseline policy response in directions spanned by the set of empirically identified policy shocks.[21]

Alternative: a multi-shock refinement of Sims & Zha (1995). In keeping with this paper's overarching focus on robustness to Lucas critique concerns, we will in our applications in Section 4 present results only for our baseline method. However, we note that our results also suggest a refinement of Sims & Zha (1995)—a refinement that relies on stronger assumptions than our baseline method, but weaker assumptions than the original one-shock Sims & Zha approach. Given the popularity of the Sims & Zha strategy we briefly discuss this refinement here, with implementation details provided in Appendix B.1.

Recall that the Sims & Zha approach enforces the desired counterfactual policy rule by subjecting the economy to a sequence of policy shocks at $t = 0, 1, \ldots$. As we discussed in Section 2.4, it is precisely the ex post surprises at $t \geq 1$ that cause Lucas critique concerns: the counterfactual policy rule holds at each t, but is not expected to hold from $t+1$ onwards. The idea of our refinement is that a researcher with access to *multiple* empirically identified policy shocks can do more than just enforce the desired counterfactual rule ex post—she can use the additional degrees of freedom to also at least partially enforce the counterfactual rule in expectation, exactly as we did for illustration purposes in the middle panel of Figure 1. While our main methodology relies *only* on date-0 shocks, this refinement of Sims & Zha

[21] This part of our method is related to work by Barnichon & Mesters (2021). Those authors argue that, under quite general conditions, evidence on policy shock impulse responses can be used to test the optimality of a policy decision. Our method makes stronger assumptions—notably the separation of the policy and non-policy blocks in (6) - (7)—allowing us to explicitly *characterize* optimal policy (and optimal policy rules), as in Proposition 2.

instead still features ex post surprises, though strictly smaller than under the original single-shock Sims & Zha approach. If these ex post surprises can be made small enough, then the researcher may actually feel comfortable in ignoring any possible expectational effects related to the anticipation of such shocks.

4 Application to monetary policy counterfactuals

This section applies our empirical method to construct monetary policy rule counterfactuals. We proceed in two steps. First, in Section 4.1, we provide a brief review of existing evidence on monetary policy shock transmission—the key input to our empirical method. Second, in Section 4.2, we apply our method to study the propagation of investment-specific technology shocks under various counterfactual monetary rules.

4.1 A review of monetary policy shock evidence

In order to implement our empirical method, we require evidence on multiple distinct monetary policy shocks that induce different time paths for nominal interest rates. The empirical literature has devised many different strategies to isolate quasi-random variation in the conduct of monetary policy (see Ramey, 2016, as well as the discussion below). Since monetary authorities control current and future expected interest rates, monetary policy is inherently multi-dimensional, and so it is not surprising that different policy shocks are likely to capture different dimensions of policy: some experiments will capture transitory impulses, while others reflect more persistent deviations from the policy rule.[22] The empirical evidence that we leverage is consistent with this observation.

Our applications in Section 4.2 will use the two arguably most canonical available monetary policy shock series: those of Romer & Romer (2004) and Gertler & Karadi (2015). Importantly, those two shock series are likely to be informative about very different monetary experiments. While the Romer & Romer shock is rather short-lived (i.e., mostly reflecting contemporaneous shocks $\nu_{0,t}$), the Gertler & Karadi shock is well-known to move longer-term nominal interest rates and is thus more likely to have a larger forward guidance component (i.e., in greater part reflecting $\nu_{\ell,t}$ for $\ell > 0$). Our applications in the next section reveal

[22] A related argument was made by Sims (1998): there is no need for different identification strategies to yield correlated measures of policy shocks, simply because the identified shocks may capture different sources of variation in policy. We thank our discussant Valerie Ramey for pointing out that connection.

that even this relatively modest amount of evidence is in fact enough to tightly characterize several important monetary policy rule counterfactuals.

While we have chosen to focus on the most well-known and well-understood policy shock series for our main applications, we emphasize that similar arguments about interest rate time profiles apply just as well to several other popular monetary policy shock series. First, as we discuss in detail in Appendix C.4, the shock series of Miranda-Agrippino & Ricco (2021) and Aruoba & Drechsel (2022)—shock measures that seek to improve on the original series of Romer & Romer and Gertler & Karadi in various ways—induce similar dynamics, with one shock more transitory and the other more persistent. Second, some prior work has explicitly split monetary shock series by their effects on different points of the yield curve, exactly as required by our theory. Estimates of this type are available from Gürkaynak et al. (2005), Antolin-Diaz et al. (2021), and Inoue & Rossi (2021), and would offer natural alternatives as an input to our empirical method.[23]

4.2 Counterfactual policy rule exercises

We apply our empirical method to predict the effects of investment-specific technology shocks under various counterfactual monetary policy rules. In particular, our objects of interest are the counterfactual behavior of the output gap, inflation, and the short-term nominal rate. We choose to focus on investment-specific technology shocks since such shocks are widely argued to be one of the main drivers of aggregate business-cycle fluctuations, at least in the U.S. (e.g., see Justiniano et al., 2010; Ramey, 2016).

We proceed as follows: we estimate the inputs required by our methodology, apply the method and present the main results, and then discuss how to interpret those results in light of our theoretical identification results in Section 2. Appendix C provides the details of the empirical implementation.

Inputs. The first input to our analysis are the aggregate effects of the non-policy shock of interest $\boldsymbol{\varepsilon}$ under the prevailing baseline policy rule. To recover those effects we rely on the investment-specific technology news shock series identified by Ben Zeev & Khan (2015)—

[23]Finally, we note that this discussion also extends to fiscal shocks. For government spending, Ramey (2011) explicitly distinguishes between shocks reflecting gradual military build-ups and more transitory upticks in purchases. For taxes, Mertens & Ravn (2014) separate unanticipated (transitory) and anticipated (gradual) tax shocks. We leave applications of our methodology to fiscal policy counterfactuals using those shocks to future work.

a shock that induces an anticipated change in the relative price of investment goods. We estimate the propagation of this shock by ordering it first in a recursive Vector Autoregression (VAR) (as recommended in Plagborg-Møller & Wolf, 2021b).

The second input are the causal effects of a menu of different monetary policy shocks. For this we consider the shock series of Romer & Romer (2004) and Gertler & Karadi (2015), as already discussed in Section 4.1. To correctly account for joint uncertainty in the estimation of the two shocks, we study their propagation through a single VAR. For robustness, we also repeat all of our policy counterfactual applications with the shock series of Miranda-Agrippino & Ricco and Aruoba & Drechsel—two less well-known but arguably somewhat more robust shock series—and find similar results. All results for these alternative shock measures are reported in Appendix C.4.

Counterfactual policy results. We use our methodology to construct counterfactuals for several different alternative monetary policy rules: output gap targeting; a standard Taylor (1999) rule; a nominal rate peg; nominal GDP targeting; and the optimal policy rule corresponding to a loss function with equal weight on the output gap and a weighted average of current and lagged inflation (i.e., average inflation targeting).

First, Figure 3 shows our counterfactual results for output gap stabilization. The identified investment technology shock has both a cost-push as well as a negative demand component, consistent with theory (e.g., Justiniano et al., 2010). Under the baseline policy rule (dotted grey), interest rates are cut relatively aggressively, though by not enough to stabilize the output gap; furthermore inflation stays moderately above target. Under our approximation to output gap targeting, interest rates are cut much more aggressively, essentially stabilizing the output gap from around a couple of quarters after the shock, at the cost of persistently higher inflation. Given the well-documented lags in monetary policy transmission, it seems unlikely that *any* interest rate path could actually stabilize the output gap in the immediate aftermath of the shock; we thus believe that our empirical analysis yields an accurate approximation to what output gap targeting can achieve in practice.[24]

Second, Figure 4 shows the results for a Taylor-type rule with strong responses to inflation and the output gap as well as moderate nominal interest rate smoothing. Due to the observed increase in inflation, this policy rule actually dictates a much less aggressive rate cut, resulting in somewhat lower output and inflation at medium horizons. In the right panel, the distance

[24]In the notation of Section 2, this would mean that perfect output gap targeting—i.e., the rule $\boldsymbol{y} = 0$, with y denoting the output gap—is not implementable (i.e., Assumption 2 is violated).

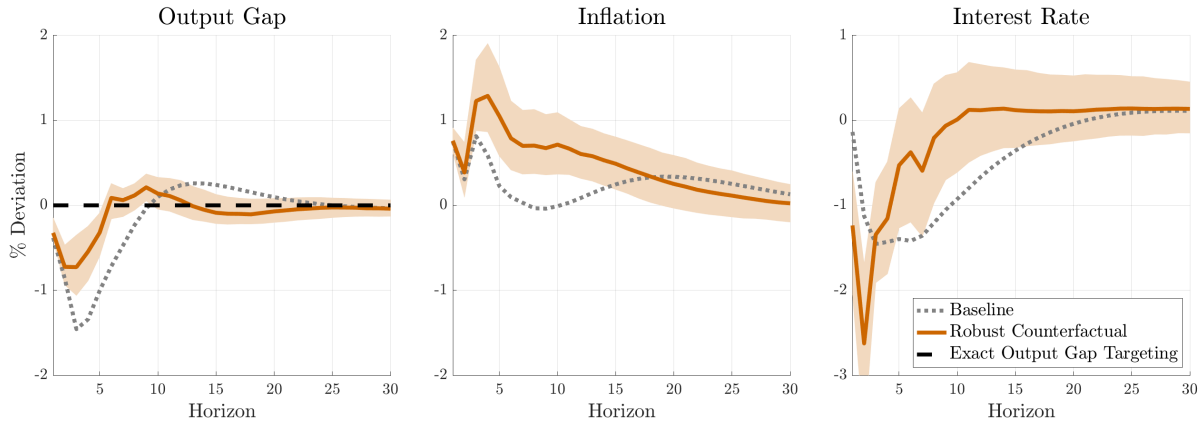

Figure 3: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to output gap targeting (orange), computed following (31). The shaded areas correspond to 16th and 84th percentile confidence bands. Perfect output gap targeting is displayed as the black dashed line.

between the black dashed and orange lines indicates whether or not our method is able to accurately implement the counterfactual rule. While the orange lines show our counterfactual path of interest rates, the black lines instead use the Taylor rule to map the output gap and inflation paths shown in the left and middle panels into paths of nominal interest rates. The distance between the two is thus simply the argument of (31)—i.e., the implementation error. We see that the counterfactual Taylor rule is imposed relatively well throughout, except at a couple of quarters after the initial shock (where interest rates are still cut by too much relative to the Taylor rule prescription).

Third, we proceed in the spirit of the recent change in the Federal Reserve's strategy and consider a policymaker with preferences over output and *average* inflation $\bar{\pi}_t$, where $\bar{\pi}_t = \sum_{\ell=0}^{K} \omega_\ell \pi_{t-\ell}$.[25] We then represent the loss function of a dual mandate policymaker with preferences over average inflation as

$$\mathcal{L} = \lambda_\pi \bar{\boldsymbol{\pi}}' W \bar{\boldsymbol{\pi}} + \lambda_y \boldsymbol{y}' W \boldsymbol{y}$$

[25]Here K denotes the maximal (lagged) horizon that enters the inflation averaging, and ω_ℓ denotes the weight on the ℓth lag, with $\sum_\ell \omega_\ell = 1$ and $\omega_\ell \geq 0 \; \forall \ell$. For our application we set $K = 20$ and $\omega_\ell \propto \exp(-0.1\ell)$. Suitably stacking the weights $\{\omega_\ell\}$, we can define a linear map $\bar{\Pi}$ such that $\bar{\boldsymbol{\pi}} = \bar{\Pi} \times \boldsymbol{\pi}$.

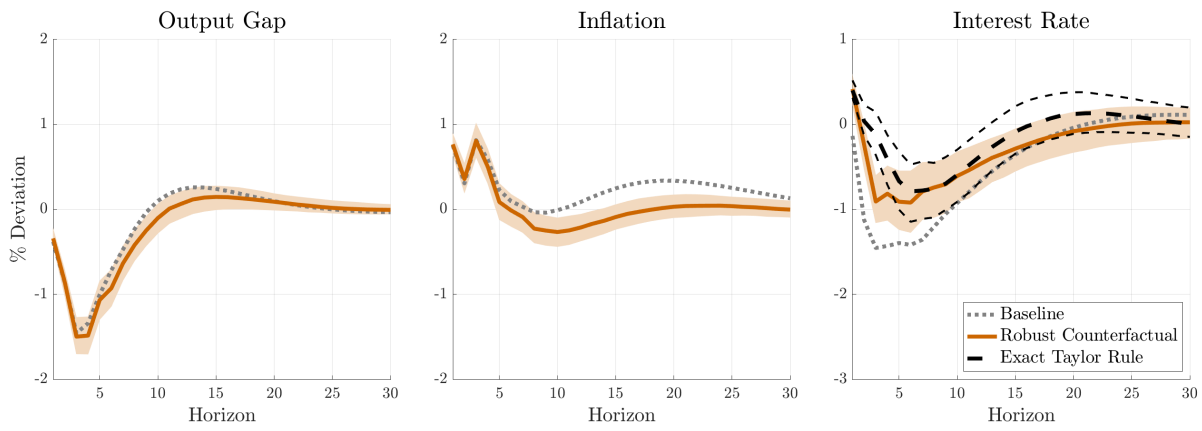

Figure 4: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to a simple Taylor-type rule $\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.5 \times (1.5\widehat{\pi}_t + \widehat{y}_t)$ (orange), computed following (31). The shaded areas correspond to 16th and 84th percentile confidence bands. The distance between black dashed and orange lines in the right panel is the implementation error (i.e., the argument of (31)).

with $\lambda_\pi = \lambda_y = 1$, $W = \text{diag}(1, \beta, \beta^2, \cdots)$ and $\beta = 1/1.01$. Results for our optimal policy counterfactual are displayed in Figure 5. The key takeaway here is that this optimal policy counterfactual differs very little from actually observed outcomes. In other words, there is little room to improve upon the observed allocation by changing policy within the space of policy instrument paths spanned by our two identified policy shocks.

Appendices C.3 and C.4 present several further applications. First, we consider the two remaining policy counterfactuals: nominal GDP targeting and a nominal interest rate peg. We find that nominal GDP targeting can be implemented very accurately; interestingly, this counterfactual looks quite similar to our estimated outcomes under the baseline rule, with interest rates cut only slightly less aggressively. Matters look different for a nominal interest rate peg, however. Here, nominal interest rates in our best Lucas critique-robust counterfactual still fall by quite a bit too much, in particular at short horizons. Our empirical method thus in this case does not allow an accurate characterization of the desired counterfactual. Second, we for all five counterfactual rules present results for the multi-shock Sims & Zha refinement discussed in Section 3.2. For four of our five counterfactuals, allowing for ex post shocks to further improve the rule fit does not materially alter our conclusions. The reason is simple: the contemplated counterfactual policy rules are already implemented well using

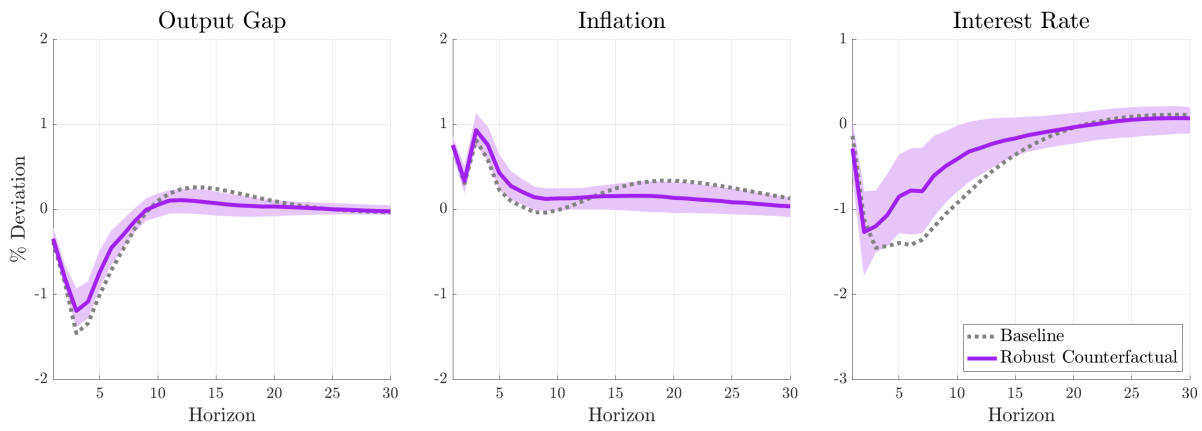

Figure 5: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to an optimal average inflation targeting monetary policy rule (purple), computed as discussed in Appendix B.2. The shaded areas correspond to 16th and 84th percentile confidence bands.

date-0 shocks only, so there is little need to additionally rely on ex post shocks. For the nominal interest rate peg, on the other hand, the date-0 shocks are not sufficient, yet moderately sized ex post surprises allow for an almost perfect stabilization of interest rates. Output in this counterfactual contracts by more, and inflation is materially lower at medium horizons. Third, we repeat our analysis with the alternative shock series of Miranda-Agrippino & Ricco and Aruoba & Drechsel. Those two shocks give similar impulse responses to our baseline shock measures, and so our systematic policy rule counterfactuals are not affected much.

Discussion. The results from our applications reveal that existing empirical evidence on policy shocks is already sufficient to tightly restrict policy rule counterfactuals for several prominent alternative monetary policy strategies. At the same time, we emphasize that our empirical method is clearly not *always* applicable: for some non-policy shocks and some counterfactual rules, it will not be possible to enforce the counterfactual rule accurately. In particular, the counterfactuals that we constructed for the investment shock application were relatively accurate precisely because the investment shock is rather transitory, thus only requiring knowledge of the effects of similarly transitory interest rate changes, along the lines of those implied by the Romer & Romer and Gertler & Karadi shocks (see Appendix C.2 for the exact paths). More persistent non-policy shocks $\boldsymbol{\varepsilon}$ necessarily induce more persistent

policy instrument movements and thus would correspondingly require empirical evidence on highly persistent policy shocks (e.g., far-ahead forward guidance).

5 Conclusions

The standard approach to counterfactual analysis for changes in systematic policy rules relies on fully-specified structural general equilibrium models. Our identification results instead point in a different direction: researchers can estimate the causal effects of distinct policy shocks and combine them to form policy counterfactuals. Importantly, these counterfactuals are valid in a large class of models that encompasses the majority of structural business-cycle models that are currently used for policy analysis.

An important challenge in implementing this strategy is that its informational requirements are high. We showed how to proceed in the empirically relevant case of evidence on a small number of policy shocks. We illustrated through several examples that empirical evidence is already sufficient to tightly characterize a variety of interesting monetary policy rule change counterfactuals, reducing the need for explicit structural modeling. More generally, a key message of this paper is to emphasize the value of empirical strategies that recover the dynamic causal effects associated with different *time paths* of policy instruments. Every additional piece of empirical evidence on a different policy instrument path will expand the space of counterfactual policy rules that can be analyzed with our method.

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Online Appendix for: What Can Time-Series Regressions Tell Us About Policy Counterfactuals?

This online appendix contains supplemental material for the article "What Can Time-Series Regressions Tell Us About Policy Counterfactuals?". We provide (i) supplementary results complementing our theoretical identification analysis in Section 2, (ii) implementation details for our empirical methodology in Section 3, and (iii) several supplementary findings and alternative experiments complementing our applications in Section 4.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "A."—"C." refer to the main article.

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A Supplementary theoretical results

This appendix provides several results complementing our theoretical identification analysis of Section 2. Appendix A.1 discusses examples of structural models that are nested by our results, Appendix A.2 gives an example of a model that is not, Appendix A.3 extends our optimal policy arguments to more general loss functions, Appendix A.4 provides the details for unconditional second-moment counterfactuals, Appendix A.5 studies optimal policy in our illustrative HANK model, Appendix A.6 shows how we construct counterfactuals with a limited number of policy shocks (as displayed in Figure 1), and finally Appendix A.7 provides a global identification analysis with even higher informational requirements.

A.1 Examples of nested models

We provide further details on three sets of models: the three-equation New Keynesian model of Section 2.1, a general class of behavioral models, and the HANK model of Section 2.4.

Three-equation NK model. We here state the three-equation model of Section 2.1 in the form of our general matrix system (6) - (7). We begin with the non-policy block. The Phillips curve can be written as

$$\begin{pmatrix} 1 & -\beta & 0 & \dots \\ 0 & 1 & -\beta & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \boldsymbol{\pi} - \kappa \boldsymbol{y} - \boldsymbol{\varepsilon}^s = 0,$$

while the Euler equation can be written as

$$-\sigma \begin{pmatrix} 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \boldsymbol{\pi} + \begin{pmatrix} 1 & -1 & 0 & \dots \\ 0 & 1 & -1 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \boldsymbol{y} + \sigma \boldsymbol{i} = 0.$$

Letting $\boldsymbol{x} \equiv (\boldsymbol{\pi}', \boldsymbol{y}')'$, we can stack these linear maps into the form (6). Finally the policy rule can be written as

$$\phi_\pi \boldsymbol{\pi} - \boldsymbol{i} + \boldsymbol{\nu} = 0,$$

which directly fits into the form of (7) with $\boldsymbol{z} = \boldsymbol{i}$.

Behavioral model. Our general framework (6) - (7) is rich enough to nest popular behavioral models such as the cognitive discounting framework of Gabaix (2020) or the sticky information set-up of Mankiw & Reis (2002). We here provide a sketch of the argument for a particular example—the consumption-savings decision of behavioral consumers. Our discussion leverages sequence-space arguments as in Auclert et al. (2021).

Let the linear map $\mathcal{E}$ summarize the informational structure of the consumption-savings problem, with entry (t, s) giving the expectations of consumers at time t about shocks at time s. Here an entry of 1 corresponds to full information and rational expectations, while entries between 0 and 1 can capture behavioral discounting or incomplete information. For example, cognitive discounting at rate θ would correspond to

$$\mathcal{E} = \begin{pmatrix} 1 & \theta & \theta^2 & \dots \\ 1 & 1 & \theta & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

while sticky information with a fraction $1 - \theta$ receiving the latest information could be summarized as

$$\mathcal{E} = \begin{pmatrix} 1 & 1-\theta & 1-\theta & \dots \\ 1 & 1 & 1-\theta^2 & \dots \\ 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Let p denote an input to the household consumption-savings problem (e.g., income or interest rates). In sequence space, we can use the matrix $\mathcal{E}$ to map derivatives of the aggregate consumption function with respect to p, denoted $\mathcal{C}_p$, into their behavioral analogues $\tilde{\mathcal{C}}_p$ via

$$\tilde{\mathcal{C}}_p(t, s) = \sum_{q=1}^{\min(t,s)} [\mathcal{E}(q, s) - \mathcal{E}(q-1, s)]\mathcal{C}_p(t - q + 1, s - q + 1).$$

Typical behavioral frictions thus merely affect the matrices that enter our general non-policy block (6), but do not affect the separation of policy- and non-policy blocks at the heart of our identification result.

QUANTITATIVE HANK MODEL. The HANK model used for our quantitative illustration in Section 2.4 is exactly the same as in Wolf (2021) (including the parameterization, except of course for the monetary policy rule). The non-policy shock $\boldsymbol{\varepsilon}$ is an AR(1) innovation to the model's Phillips curve with persistence 0.8.

A.2 Filtering problems

To illustrate how an asymmetry in information between the private sector and the policy authority can break our separation of the policy and non-policy blocks in (6) - (7) even for a linear model, we consider a standard Lucas (1972) island model with a slightly generalized policy rule. The policy authority sets nominal demand x_t according to the rule

$$x_t = \phi_y y_t + x_{t-1} + \varepsilon_t^m$$

where y_t denotes real aggregate output and ε_t^m is a policy shock with volatility σ_m. The private sector of the economy as usual yields an aggregate supply curve of the form

$$y_t = \theta(p_t - \mathbb{E}_{t-1} p_t)$$

where the response coefficient θ follows from a filtering problem and is given as

$$\theta = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_p^2}$$

with σ_z denoting the (exogenous) volatility of idiosyncratic demand shocks and σ_p denoting the (endogenous) volatility of the surprise component of prices, $p_t - \mathbb{E}_{t-1} p_t$. A straightforward guess-and-verify solution of the model gives

$$p_t = \frac{1}{1+\theta} x_t + \frac{\theta}{1+\theta} x_{t-1}$$

and so

$$\sigma_p^2 = \left(\frac{1}{1+\theta}\right)^2 \mathrm{Var}(\phi_y y_t + \varepsilon_t^m).$$

But since

$$y_t = \frac{1}{1 - \frac{\theta}{1+\theta}\phi_y} \frac{\theta}{1+\theta} \varepsilon_t^m$$

it follows that θ depends on the policy rule coefficient ϕ_y, breaking our separation assumption.

A.3 More general loss functions

Proposition 2 can be generalized to allow for a non-separable quadratic loss function. Suppose the policymaker's loss function takes the form

$$\mathcal{L} = \boldsymbol{x}' Q \boldsymbol{x} \tag{A.1}$$

where Q is a weighting matrix. Following the same steps as the proof of Proposition 2, we can formulate the policy problem as minimizing the loss function (A.1) subject to (25). The first-order conditions of this problem are

$$\Theta'_{\nu,x,A}(Q + Q')\boldsymbol{x} = 0$$

so we can recover the optimal policy rule as

$$\begin{aligned} \mathcal{A}^*_x &= \Theta'_{\nu,x,A}(Q + Q') \\ \mathcal{A}^*_z &= \mathbf{0} \end{aligned}$$

Even outside of the quadratic case, the causal effects of policy shocks on $\boldsymbol{x}$ are still enough to formulate a set of necessary conditions for optimal policy, but in this general case the resulting optimal policy rule will not fit into the linear form (7).

A.4 Counterfactual second-moment properties

Our analysis is largely focussed on constructing counterfactuals conditional on particular non-policy shock paths $\boldsymbol{\varepsilon}$. This is in keeping with much of the empirical policy counterfactual literature that followed the lead of Sims & Zha (1995) (e.g., Bernanke et al., 1997; Eberly et al., 2020; Antolin-Diaz et al., 2021). However, under some additional assumptions, our results can also be used to construct *unconditional* counterfactual second-moment properties—that is, predict how variances and covariances of macroeconomic aggregates would change under a counterfactual rule. This section provides the detailed argument.

Setting. We consider a researcher that observes and is interested in the counterfactual properties of some vector of macroeconomic aggregates $y = (x, z)$—the endogenous outcomes and policy instruments of our main analysis. We assume that, under the prevailing baseline policy rule, this vector of macroeconomic aggregates follows a standard structural vector

moving average representation:

$$y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} = \Theta(L)\varepsilon_t \tag{A.2}$$

where $\varepsilon_t \sim N(0, I)$.[26] We would like to predict the second-moment properties of y_t under some counterfactual policy rule (8).

If the researcher can estimate the causal effects of all shocks ε_t on the outcomes y_t, then the identification argument is trivial: she simply applies Proposition 1 for each individual shock, stacks the resulting impulse responses into a new vector moving average representation $\tilde{\Theta}(L)$, and from here computes the counterfactual second-moment properties. This approach may however not be feasible, as it requires the researcher to be able to correctly disentangle all of the structural shocks driving the macro-economy.

Procedure. Our proposed procedure has three steps. First, the researcher estimates the Wold representation of the observables y_t. Second, using Proposition 1, she maps the impulse responses to the Wold errors into new impulse responses corresponding to the counterfactual policy rule. Third, she stacks those new impulse responses to arrive at a new vector moving average representation, and from this representation constructs a new set of second-moment properties. Our identification result states that, if the vector moving representation (A.2) under the baseline rule is invertible, then this procedure correctly recovers the desired counterfactual second moments.

Identification result. Let $\tilde{\Theta}_\ell$ denote the lag-ℓ impulse responses of the observables y_t to the shocks ε_t under the counterfactual policy rule. The process for y_t under the counterfactual policy rule thus becomes

$$y_t = \sum_{\ell=0}^{\infty} \tilde{\Theta}_\ell \varepsilon_{t-\ell} = \tilde{\Theta}(L)\varepsilon_t$$

and so the second moments of the true counterfactual process are given by

$$\Gamma_y(\ell) = \sum_{m=0}^{\infty} \tilde{\Theta}_m \tilde{\Theta}'_{m+\ell}. \tag{A.3}$$

[26] Given our focus on second moments, the normality restriction is purely for notational convenience (see e.g., Plagborg-Møller & Wolf, 2021b).

Now consider instead the output of our proposed procedure. Let u_t denote the Wold errors under the observed policy rule, and let ε_t^* denote any unit-variance orthogonalization of these Wold errors (e.g., $\varepsilon_t^* = \text{chol}(\text{Var}(u_t))^{-1} \times u_t$). Then y_t under the observed policy rule satisfies

$$y_t = \Psi(L)\varepsilon_t^* = \sum_{\ell=0}^{\infty} \Psi_\ell \varepsilon_{t-\ell}^*$$

where $\varepsilon_t^* \sim N(0, I)$. Under invertibility—i.e., $\Theta(L)$ has a one-sided inverse—we in fact know that $\varepsilon_t^* = P\varepsilon_t$, $\Psi(L) = \Theta(L)P'$, $PP' = P'P = I$. The second step of our procedure gives the counterfactual vector moving average representation

$$y_t = \tilde{\Psi}(L)\varepsilon_t^*$$

where $\tilde{\Psi}(L)$ gives the dynamic causal effects of $\varepsilon_t^* = P\varepsilon_t$ on y_t under the counterfactual rule. But since the causal effects of ε_t under the baseline rule are given as $\tilde{\Theta}(L)$, it follows that we must also have

$$\tilde{\Psi}(L) = \tilde{\Theta}(L)P'.$$

But then the implied second-moment properties of y_t are given as

$$\Gamma_y(\ell) = \sum_{m=0}^{\infty} \tilde{\Psi}_m \tilde{\Psi}'_{m+\ell} = \sum_{m=0}^{\infty} \tilde{\Theta}_m P'P \tilde{\Theta}'_{m+\ell} = \sum_{m=0}^{\infty} \tilde{\Theta}_m \tilde{\Theta}'_{m+\ell} \tag{A.4}$$

which is exactly equal to (A.3), completing the argument.

Finally, we emphasize that this identification result inherently rests on the assumption of invertibility. Under invertibility, there is a static one-to-one mapping between true shocks ε_t and Wold errors ε_t^*; thus, if we can predict the propagation of the Wold errors under the counterfactual rule, then we also match the propagation of the true shocks, and so we correctly recover second-moment properties. Under non-invertibility, however, there is no analogous one-to-one mapping, and so it is not guaranteed that second moments will be matched by our procedure.

A.5 Optimal policy counterfactual in HANK

Section 2.4 used a quantitative HANK model to illustrate the logic of Proposition 1—the general counterfactual rule identification result. We here do the same for the analogous optimal policy identification result in Proposition 2.

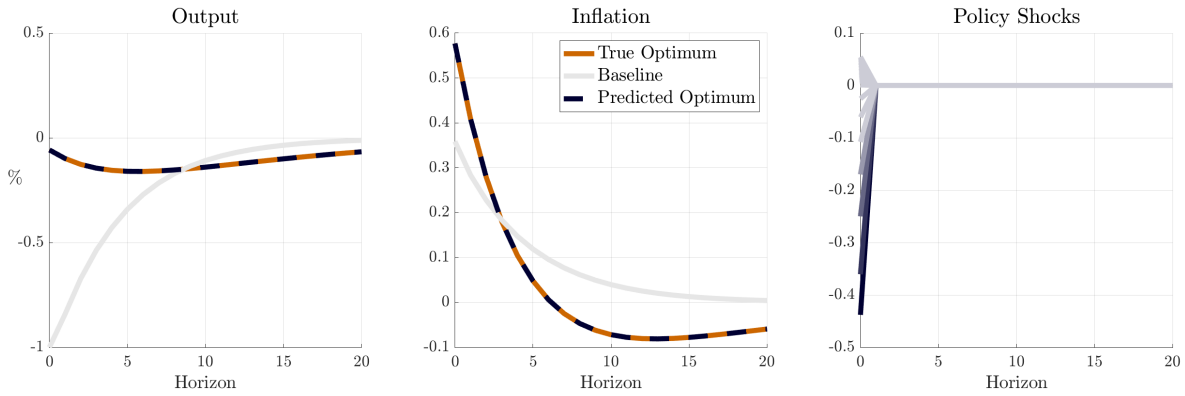

Figure A.1: The grey and orange lines in the left and middle panels show output and inflation responses to the cost-push shock ε_t for the HANK model with policy rule (28) and the optimal rule for the loss function (A.5). The dark blue lines give output and inflation counterfactuals constructed through the policy shocks on the right, set in line with Proposition 2. Lighter shades of blue indicate farther-out policy news shocks.

We consider a policymaker with a standard dual mandate loss function

$$\mathcal{L} = \lambda_\pi \boldsymbol{\pi}'\boldsymbol{\pi} + \lambda_y \boldsymbol{y}'\boldsymbol{y} \tag{A.5}$$

with $\lambda_\pi = \lambda_y = 1$. As in Section 2.4 we start by solving for the optimal policy using conventional methods: we derive the policy rule corresponding to the first-order conditions (18) - (20), solve the model given that policy rule, and report the result as the orange lines in the left and middle panels of Figure A.1. We see that, at the optimum, the cost-push shock moves inflation by much more than output, consistent with the assumed policy weights and the relatively flat Phillips curve. Compared to this optimal policy, the simple baseline rule of the form (28) tightens too much.

We then instead use Proposition 2 to equivalently recover the optimal policy rule and the corresponding impulse responses. We begin with the optimal rule itself. By (26), the optimal rule is given as

$$\lambda_\pi \Theta'_{\pi,\nu,\mathcal{A}} \boldsymbol{\pi} + \lambda_y \Theta'_{y,\nu,\mathcal{A}} \boldsymbol{y} = \mathbf{0}.$$

A researcher with knowledge of the effects of monetary policy shocks on inflation and output, $\{\Theta_{\pi,\nu,\mathcal{A}}, \Theta_{y,\nu,\mathcal{A}}\}$, is able to construct this optimal policy rule. We can then create a counterfactual response to the cost-push shock using (11)-(13), again requiring only knowledge of

the causal effects of policy shocks as well as the impulse responses to the cost-push shock under the baseline rule. As expected, the resulting impulse responses—the dark blue lines—are identical to those obtained by explicitly solving the optimal policy problem. Finally, the right panel of Figure A.1 shows the optimal policy as a deviation $\tilde{\boldsymbol{\nu}}$ from the prevailing rule. The optimal rule accommodates the inflationary cost-push shock more than the baseline rule (28), so the required policy "shock" is persistently negative (i.e., expansionary). Consistent with our discussion in Figure 1, we choose to display those shocks $\tilde{\boldsymbol{\nu}}$ in a way that emphasizes that the optimum is achieved through a sequence of *date-0* policy shocks.

A.6 Counterfactuals with a limited number of shocks

In Figure 1 we constructed counterfactuals using a limited number n_s of policy shocks. We here provide the computational details for this construction. We discuss the general case of a researcher with access to n_s shocks (which converges to our identification result for $n_s \to \infty$), with the original proposal of Sims & Zha (1995) nested as the $n_s = 1$ special case.

The approach of Sims & Zha leverages the idea that evidence on one policy shock—i.e., any single fixed path $\boldsymbol{\nu}$—is sufficient to enforce any given counterfactual *ex post*. With n_s distinct shocks, the counterfactual rule can be implemented *ex post* as well as in *ex ante* expectation for the next $n_s - 1$ time periods. To compute the counterfactuals corresponding to this multi-shock case we proceed as follows. First, at $t = 0$, we solve for the n_s-dimensional vector of policy shocks $\boldsymbol{\nu}^0_{1:n_s} \equiv (\nu^0_0, \ldots, \nu^0_{n_s-1})'$ such that, in response to $\boldsymbol{\varepsilon}$ and $\boldsymbol{\nu}^0_{1:n_s}$ the counterfactual rule holds at $t = 0$ and is expected to hold for $t = 1, \ldots, n_s - 1$. Output and inflation at $t = 0$ are simply given as the thus-derived impulse responses to $\boldsymbol{\varepsilon}$ and $\boldsymbol{\nu}^0_{1:n_s}$. Second, at $t = 1$, we solve for the n_s-dimensional vector of shocks $\boldsymbol{\nu}^1_{1:n_s} \equiv (\nu^1_0, \ldots, \nu^1_{n_s-1})'$ such that, in response to the time-0 shocks $\{\boldsymbol{\varepsilon}, \boldsymbol{\nu}^0_{1:n_s}\}$ and the time-1 shocks $\boldsymbol{\nu}^1_{1:n_s}$, the counterfactual policy rule holds at $t = 1$ and in expectation for $t = 2, \ldots, n_s$. These impulse responses then give us output and inflation at $t = 1$. Continuing iteratively, we obtain the entire output and inflation impulse responses, as plotted in the left and middle panels of Figure 1. The corresponding shock paths are shown in the right panel.

A.7 Global identification argument

We here extend our identification results to a general non-linear model with aggregate risk.

SETTING. We consider an economy that runs for T periods overall. As in our main analysis, the economy consists of a private block and a policy block. Differently from our main analysis, there is no exogenous non-policy shock sequence $\boldsymbol{\varepsilon}$; rather, there is a stochastic event ω_t each period, with stochastic events drawn from a finite (n_ω-dimensional) set. Let $x_t(\omega^t)$ be the value of the endogenous variables after history $\omega^t \equiv \{\omega_0, \omega_1, \cdots, \omega_t\}$ and let $z_t(\omega^t)$ be the realization of the policy instruments after history ω^t. Let $\boldsymbol{x}$ and $\boldsymbol{z}$ be the full contingent plans for for all $t \in \{0, 1, \cdots, T\}$ and all histories. $\boldsymbol{x}$ and $\boldsymbol{z}$ are vectors in $\mathbb{R}^{n_x \times N}$ and $\mathbb{R}^{n_z \times N}$ respectively, where $N = n_\omega + n_\omega^2 + \cdots + n_\omega^{T+1}$.

We can write the private-sector block of the model as the non-linear equation

$$\mathcal{H}(\boldsymbol{x}, \boldsymbol{z}) = \mathbf{0}. \tag{A.6}$$

Similarly, we can write the policy block corresponding to a baseline policy rule as

$$\mathcal{A}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{\nu} = \mathbf{0} \tag{A.7}$$

where the vector of policy shocks $\boldsymbol{\nu}$ is now $n_z \times N$ dimensional. We assume that, for any $\boldsymbol{\nu} \in \mathbb{R}^{n_z \times N}$, the system (A.6) - (A.7) has a unique solution. We write this solution as

$$\boldsymbol{x} = x(\boldsymbol{\nu}), \quad \boldsymbol{z} = z(\boldsymbol{\nu}).$$

We want to construct counterfactuals under the alternative policy rule

$$\tilde{\mathcal{A}}(\boldsymbol{x}, \boldsymbol{z}) = \mathbf{0} \tag{A.8}$$

replacing (A.7). We again assume that the system (A.6) and (A.8) has a unique solution, now written as $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}})$. If we are interested in the counterfactual following a particular path of exogenous events, then we are interested in selections from these vectors.

Proposition A.1. *For any alternative policy rule $\tilde{\mathcal{A}}$ we can construct the desired counterfactuals as*

$$x(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{x}}, \quad z(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{z}} \tag{A.9}$$

where $\tilde{\boldsymbol{\nu}}$ solves

$$\tilde{\mathcal{A}}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) = \mathbf{0}. \tag{A.10}$$

The solution $\tilde{\boldsymbol{\nu}}$ to this system exists and any such solution generates the unique counterfactual $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}})$.

Proof. We construct the solution $\tilde{\boldsymbol{\nu}}$ as

$$\tilde{\boldsymbol{\nu}} \equiv \tilde{\mathcal{A}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}}) - \mathcal{A}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}}).$$

By the definition of the functions of $x(\bullet)$ and $z(\bullet)$, we know that

$$\begin{aligned} \mathcal{H}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) &= \mathbf{0} && \text{(A.11)} \\ \mathcal{A}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) + \tilde{\mathcal{A}}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}}) - \mathcal{A}(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}}) &= \mathbf{0} && \text{(A.12)} \end{aligned}$$

Similarly, by the definition of the functions $\tilde{x}(\bullet)$ and $\tilde{z}(\bullet)$, we also know that

$$\begin{aligned} \mathcal{H}(\tilde{x}(\mathbf{0}), \tilde{z}(\mathbf{0})) &= \mathbf{0} && \text{(A.13)} \\ \tilde{\mathcal{A}}(\tilde{x}(\mathbf{0}), \tilde{z}(\mathbf{0})) &= \mathbf{0} && \text{(A.14)} \end{aligned}$$

It follows that $\{x(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{x}}, z(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{z}}\}$ is a solution of the system (A.11) - (A.12). By assumption this system has a unique solution, so it must be that $\tilde{\boldsymbol{\nu}}$ satisfies $\{x(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{x}}, z(\tilde{\boldsymbol{\nu}}) = \tilde{\boldsymbol{z}}\}$.

We now show that any solution to (A.10) must generate $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{z}})$. Proceeding by contradiction, consider any other $\tilde{\boldsymbol{\nu}}$ that solves (A.10) and suppose that either $x(\tilde{\boldsymbol{\nu}}) \neq \tilde{\boldsymbol{x}}$ and/or $z(\tilde{\boldsymbol{\nu}}) \neq \tilde{\boldsymbol{z}}$. By definition of the functions $x(\bullet)$ and $z(\bullet)$ together with the property (A.10) we know that

$$\begin{aligned} \mathcal{H}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) &= \mathbf{0} \\ \tilde{\mathcal{A}}(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}})) &= \mathbf{0} \end{aligned}$$

and so $(x(\tilde{\boldsymbol{\nu}}), z(\tilde{\boldsymbol{\nu}}))$ is a solution of (A.6) and (A.8) that is distinct from $(\tilde{x}, \tilde{z})$. But by assumption only one such solution exists, so we have a contradiction.

□

Informational requirements. To construct the desired policy counterfactual for all possible alternative policy rules, we in general need to be able to evaluate the functions $x(\bullet)$ and $z(\bullet)$ for every possible $\boldsymbol{\nu} \in \mathbb{R}^{n_z \times N}$. That is, we need to know the effects of policy shocks of all possible sizes at all possible dates and all possible histories.

To understand how our baseline analysis relaxes these informational requirements, it is useful to proceed in two steps: first removing uncertainty (but keeping non-linearity), and then moving to a linear system.

1. **Non-linear perfect foresight.** For a non-linear perfect foresight economy, we replace our general $(n_x + n_z) \times N$-dimensional system with an $(n_x + n_z) \times T$-dimensional one:

$$\begin{aligned} \mathcal{H}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\varepsilon}) &= \mathbf{0} \\ \mathcal{A}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{\nu} &= \mathbf{0} \end{aligned}$$

   Because of the lack of uncertainty, other possible realizations of the exogenous events do not matter—only the particular time path, now denoted $\boldsymbol{\varepsilon}$, is relevant. Proceeding exactly in line with the analysis above, we can conclude that now we need the causal effects of all possible policy shocks $\boldsymbol{\nu} \in \mathbb{R}^{n_z \times T}$ *at the equilibrium path induced by* $\boldsymbol{\varepsilon}$. Thus, since we only care about the actual realized history of the exogenous inputs, the dimensionality of the informational requirements has been reduced substantially.

2. **Linear perfect foresight/first-order perturbation.** Linearity further reduces our informational requirements in two respects. First, because of linearity, to know the effects of every possible $\boldsymbol{\nu} \in \mathbb{R}^{n_z \times T}$, it suffices to know the effects of $n_z \times T$ distinct paths $\boldsymbol{\nu}$ that together span $\mathbb{R}^{n_z \times T}$. Second, estimates given any possible exogenous state path of the economy suffice, simply because the effects of policy and non-policy shocks are additively separable. We have thus reduced the problem to the (still formidable) one of finding the effects of $n_z \times T$ distinct policy shock paths.

B Details for empirical method

This appendix provides further details for our empirical methodology. Appendix B.1 begins with the Sims & Zha refinement, and Appendix B.2 presents econometric implementation details for both our main method and the refinement.

B.1 Multi-shock refinement of Sims & Zha

An intuitive description of our refinement of the method of Sims & Zha (1995) was provided in Section 3.2. We here present the mathematical details.

Our proposed extension of the Sims & Zha method trades off rule accuracy versus ex post surprises in the form of a simple ridge regression, generalizing our baseline method (31). To formally state this approach we require some additional notation. We let $\{\Omega^{(h)}_{x,\mathcal{A}}, \Omega^{(h)}_{z,\mathcal{A}}\}$ denote impulse responses to policy shocks that materialize at horizon h; that is, for $h = 0$ those impulse responses are simply given as $\{\Omega_{x,\mathcal{A}}, \Omega_{z,\mathcal{A}}\}$, while for $h > 0$ impulse responses at the first $h-1$ horizons are exactly zero, and impulse responses from horizon h onwards are equal to $\{\Omega_{x,\mathcal{A}}, \Omega_{z,\mathcal{A}}\}$. Now let $\boldsymbol{s}^h \in \mathbb{R}^{n_s}$ denote the weights assigned to the n_s shocks at horizon h. Our refinement of Sims & Zha then solves the following ridge regression problem:

$$\min_{\{\boldsymbol{s}^h\}_{h=0}^{H}} ||\tilde{\mathcal{A}}_x(\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \sum_{h=0}^{H} \Omega^{(h)}_{x,\mathcal{A}} \times \boldsymbol{s}^h) + \tilde{\mathcal{A}}_z(\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \sum_{h=0}^{H} \Omega^{(h)}_{z,\mathcal{A}} \times \boldsymbol{s}^h)|| + \psi \sum_{h=1}^{H} ||\boldsymbol{s}^h||, \qquad \text{(B.1)}$$

where the tuning parameter ψ penalizes ex post policy surprises, and $H \gg 0$ is the maximal shock horizon. For $\psi = \infty$ this method simply reduces to our baseline method, with only the date-0 shocks $\boldsymbol{s}^0$ allowed to be different from zero. For $\psi = 0$ (and large H) the counterfactual rule is instead imposed perfectly ex post as in the original proposal of Sims & Zha, with $n_s = 1$ corresponding exactly to their procedure. For intermediate ψ, the researcher is willing to trade off ex post surprises $\boldsymbol{s}^h$ for $h \geq 1$ in return for higher accuracy in implementing the desired counterfactual policy rule.[27] If those ex post surprises are small enough, then researchers may be willing to accept the expectational errors they entail in return for more accurately imposing the counterfactual rule ex post.

[27] Rather than smoothly penalizing ex post surprises as in (B.1), researchers may instead consider using n_s shocks to enforce a given counterfactual rule ex post and in expectation for the next $n_s - 1$ periods, as we did in Figure 1. Unfortunately we have found this method often yields explosive dynamics in actual data—a problem that actually also arises with the original approach of Sims & Zha (1995).

B.2 Econometric implementation

We here discuss the practical implementation of our baseline Lucas critique-robust empirical method as well as the refinement of the Sims & Zha method. Since our robust procedure is a general case of the general ridge regression problem (B.1) for $\psi = \infty$, we here simply present implementation details for the ridge regression version.

To express the solution to our basic ridge regression problem (B.1), we stack the policy shocks in the vector $\boldsymbol{s}^H$ and the corresponding causal effects in the matrix $\Omega^H_{x,\mathcal{A}}$. We furthermore let P denote a matrix that is equal to an $(n_s \cdot H) \times (n_s \cdot H)$-dimensional identity matrix except for the first n_s diagonal entries, which are equal to zero. The ridge regression solution is then given as

$$\boldsymbol{s}^H = -\left[\left(\tilde{\mathcal{A}}_x\Omega^H_{x,\mathcal{A}} + \tilde{\mathcal{A}}_z\Omega^H_{z,\mathcal{A}}\right)'\left(\tilde{\mathcal{A}}_x\Omega^H_{x,\mathcal{A}} + \tilde{\mathcal{A}}_z\Omega^H_{z,\mathcal{A}}\right) + \psi P'P\right]^{-1} \times \left[\left(\tilde{\mathcal{A}}_x\Omega^H_{x,\mathcal{A}} + \tilde{\mathcal{A}}_z\Omega^H_{z,\mathcal{A}}\right)'\left(\tilde{\mathcal{A}}_x\boldsymbol{x}_{\mathcal{A}}(\boldsymbol{\varepsilon}) + \tilde{\mathcal{A}}_z\boldsymbol{z}_{\mathcal{A}}(\boldsymbol{\varepsilon})\right)\right].$$

For our optimal policy counterfactual, we analogously consider the following regularized optimal policy problem:

$$\min_{\boldsymbol{s}^H} \sum_{i=1}^{n_x} \lambda_i \boldsymbol{x}_i' W \boldsymbol{x}_i + \psi ||P\boldsymbol{s}^H|| \tag{B.2}$$

such that

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\varepsilon}) + \Omega^H_{x,\mathcal{A}}\boldsymbol{s}^H$$

This gives the optimality conditions:

$$\begin{aligned} (W \otimes \Lambda)\boldsymbol{x} + \boldsymbol{\varphi}_x &= \boldsymbol{0} \\ -\psi P\boldsymbol{s}^H + (\Omega^H_{x,\mathcal{A}})'\boldsymbol{\varphi}_x &= \boldsymbol{0}, \end{aligned}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots)$. Solving this system (together with the constraint of the problem) gives our optimal policy counterfactual. In particular, for $\psi = \infty$, we find the optimal counterfactual within the space of identified time-0 policy shock causal effects, without any ex post surprises.

C Supplementary details for monetary applications

This appendix provides further results supplementing the discussion in Section 4 on systematic monetary policy rule counterfactuals. Appendices C.1 and C.2 begin by describing the data and our baseline policy shock dynamic causal effect estimates. Results for the counterfactuals omitted in the main text are presented in Appendix C.3, and we investigate the robustness of our results to the use of other shock measures in Appendix C.4.

C.1 Data

Our analysis of investment-specific technology shocks follows Ben Zeev & Khan (2015), while our monetary policy shock identification closely mimics that of (i) Romer & Romer (2004) and (ii) Gertler & Karadi (2015).

Outcomes. We are interested in impulse responses of three outcome variables: the output gap, inflation, and the policy rate. For the output gap, we use the series `ygap_hp` of Barnichon & Mesters (2020).[28] For inflation, we compute annual changes in the GDP deflator (using the series `pgdp` from the replication files of Ramey (2016)). Finally, we consider the federal funds rate as our measure of the policy rate, obtained from the St. Louis Federal Reserve FRED database. In keeping with much prior work, we also additionally control for commodity prices, with our measure obtained from the replication files of Ramey (2016) (`lpcom`). All series are quarterly.

Shocks & identification. We take the investment-specific technology shock series from Ben Zeev & Khan (2015) (`bzk_ist_news` in the replication files of Ramey (2016)), the Romer & Romer (2004) shock series from the replication and extension of Wieland & Yang (2020) (`rr_3`), and the high-frequency monetary policy surprise series from Gertler & Karadi (2015) (`mp1_tc` in the replication files of Ramey (2016)).[29] When applicable, the shock series are aggregated to quarterly frequency through simple averaging.

In Appendix C.4 we examine the robustness of our conclusions to other policy shock series—those of Aruoba & Drechsel (2022) and Miranda-Agrippino & Ricco (2021). For the former, we obtain the shock series directly from their replication files (`shock`). For the latter,

[28] All results are essentially unchanged if we use a measure of log real GDP instead (`rgdp` scaled by `pop`, taken from the replication files of Ramey (2016)).

[29] Results are very similar if we use the alternative surprise series `ff4_tc` instead.

we use the publicly available replication files to construct the SVAR-IV shock series for the full sample (from 1979:M1 onwards), with the shocks constructed at the posterior mode of the estimated reduced-form VAR (the specification for their Figure 3).

C.2 Shock & policy dynamic causal effects

For maximal consistency, we try to estimate all impulse responses within a common empirical specification. For the investment-specific technology shocks, we order the shock measure first in a recursive VAR containing our outcomes of interest (following Plagborg-Møller & Wolf, 2021b), estimated on a sample from 1969:Q1–2007:Q4. For our two monetary policy shocks, we estimate a single VAR in the two shock series, our three outcomes of interest, as well as commodity prices, also estimated from 1969:Q1–2007:Q4.[30] For identification, we order the Gertler & Karadi shock first (again consistent with the results in Plagborg-Møller & Wolf (2021b)) and the Romer & Romer shock second-to-last, before the federal funds rate (the additional "exogeneity insurance" as in Romer & Romer, 2004).

We use three lags in the technology shock specification, and four lags in the joint monetary policy VAR. We furthermore estimate all VARs with a constant as well as deterministic linear and quadratic trends. For the baseline investment-specific technology shock we fix the OLS point estimates. We then construct policy counterfactuals using our identified monetary policy shocks, taking into account their estimation uncertainty. Since the transmission of both shocks is estimated within a single VAR, we can draw from the posterior and compute the counterfactuals for each draw, thus taking into account joint estimation uncertainty.

Results. The OLS point estimates for the technology shocks of Ben Zeev & Khan (2015) are reported as the grey lines in Figure 3. For monetary policy, the estimated causal effects for our two outcomes of interest as well as the policy instrument are displayed in Figure C.1. The results are in line with prior work: both policy shocks induce the expected signs of the output gap and inflation responses, though the response shapes are quite distinct, consistent with the differences in the induced interest rate paths. We also note that the magnitudes of the estimated responses are at the lower end of empirical estimates (c.f. Table 2 and Figures 1-2 in Ramey, 2016).

[30] The Gertler & Karadi shock series is only available from 1988 onwards. We thus follow prior work in the macro IV literature (e.g., Känzig, 2021) and set the missing values to zero.

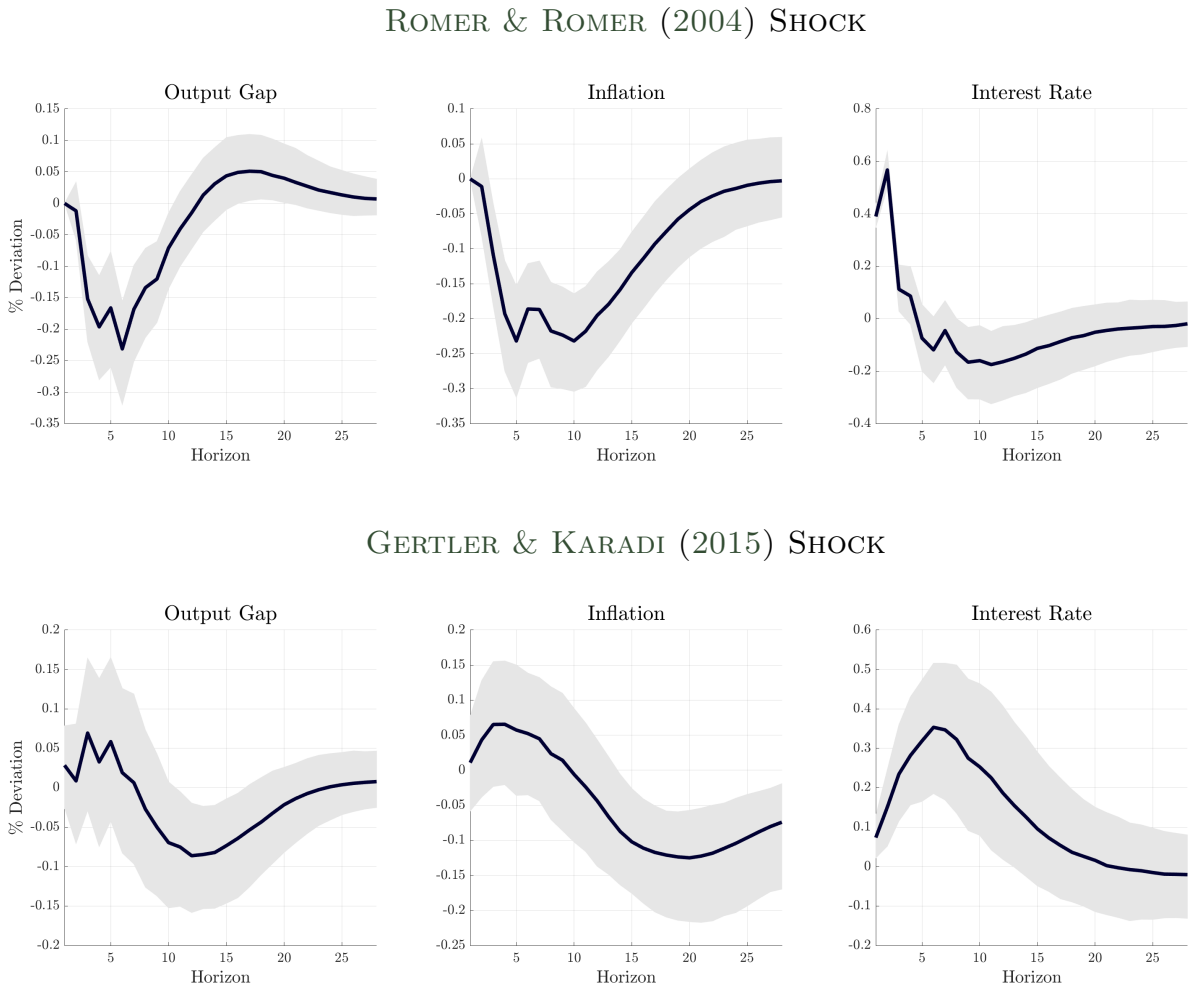

Figure C.1: Impulse responses after the Romer & Romer shock (top panel) and the Gertler & Karadi shock (bottom panel). The grey areas correspond to 16th and 84th percentile confidence bands, constructed using 10,000 draws from the posterior distribution of the reduced-form VAR parameters.

C.3 Results for omitted monetary policy counterfactuals

In Section 4 we presented detailed results for only three of our counterfactuals—output gap targeting, the Taylor rule, and optimal average inflation targeting policy—and only for our baseline method, not the Sims & Zha (1995) refinement. We here provide the remaining results. Throughout this section, our measure of rule accuracy is the horizon-by-horizon error in enforcing the desired counterfactual rule (i.e., the argument of (31) or (B.1)). For our Sims & Zha refinement we set $\psi = 1$, corresponding to an equal penalty on rule inaccuracy and ex post policy shock surprises.

Policy Counterfactual via Sims & Zha refinement, Output Gap Targeting

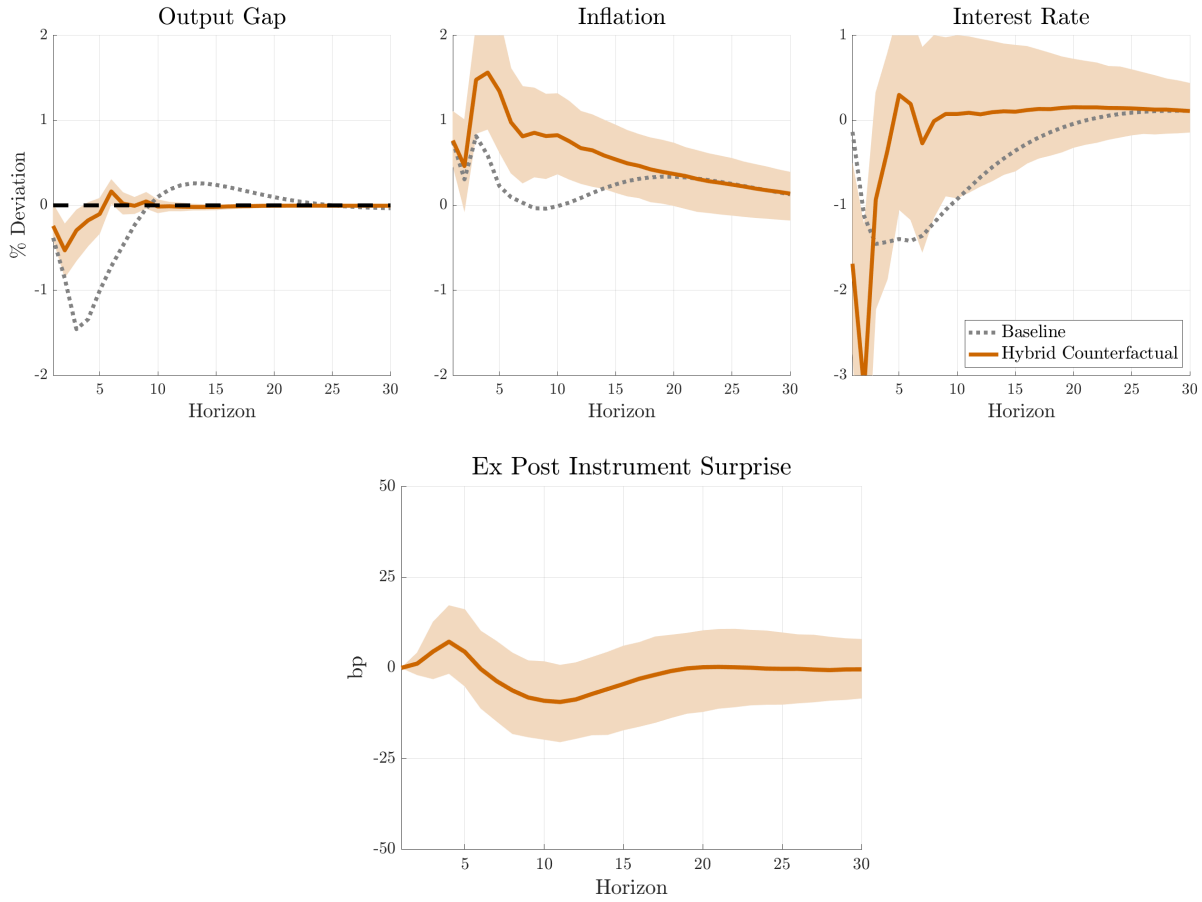

Figure C.2: Top panel: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to output gap targeting with ex post surprises (orange), computed following (B.1) for $\psi = 1$. Bottom panel: ex post nominal interest rate surprise at time t. The shaded areas correspond to 16th and 84th percentile confidence bands.

Output gap targeting. We begin in Figure C.2 with the output gap targeting counterfactual. Since we already discussed results from our baseline empirical method in the main text, we here only show results for the Sims & Zha refinement. We can conclude that allowing for some ex post shocks essentially does not change the picture: ex post shocks do not help with output gap stabilization right at the beginning, but after a couple of quarters the output gap is almost perfectly stabilized anyway using date-0 shocks.

Taylor rule. Results for the Taylor rule counterfactual computed using the Sims & Zha refinement are reported in Figure C.3. The orange lines and black dashed lines in the top

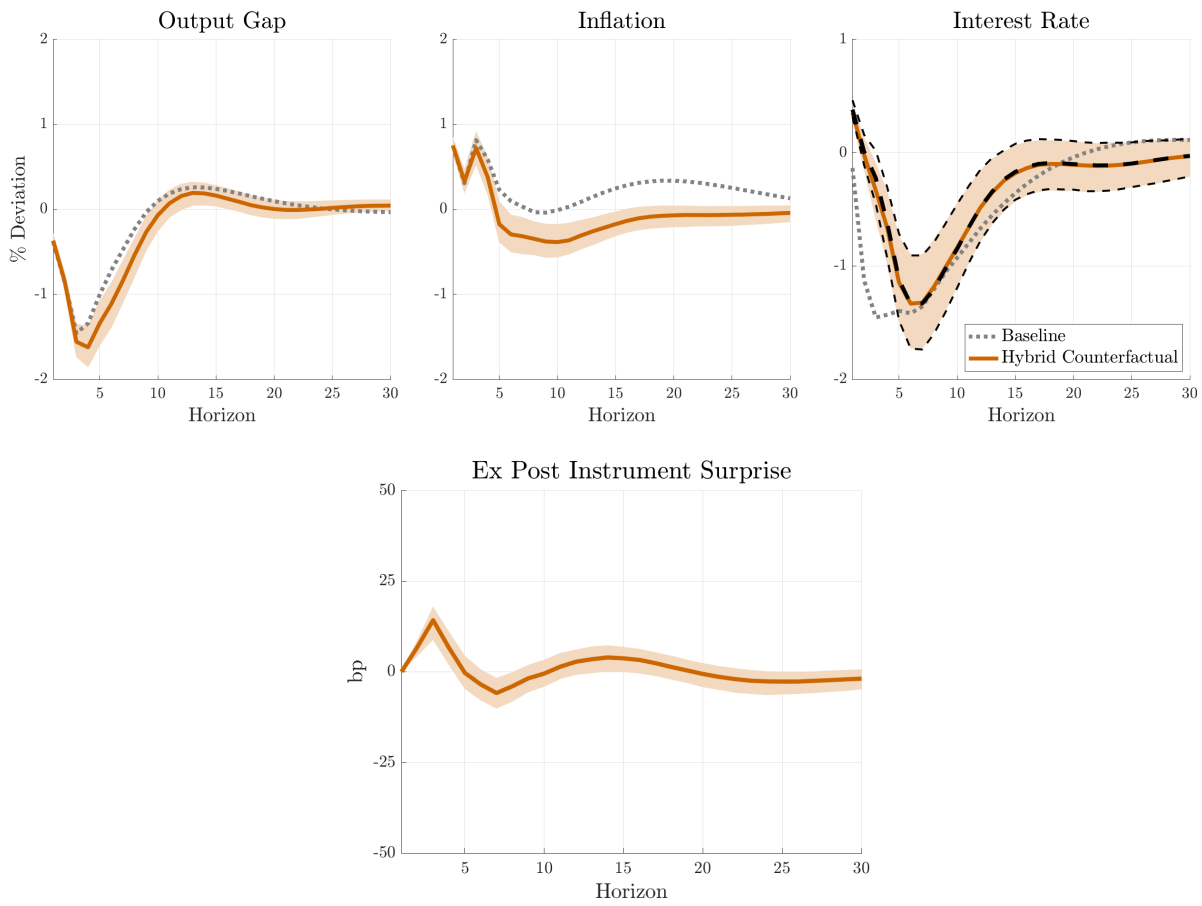

Figure C.3: Top panel: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to a Taylor-type rule $\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.5 \times (1.5\widehat{\pi}_t + \widehat{y}_t)$ with ex post surprises (orange), computed following (B.1) for $\psi = 1$. The distance between black dashed and orange lines in the right panel is the implementation error (i.e., the first part of the argument of (B.1)). Bottom panel: ex post nominal interest rate surprise at time t. The shaded areas correspond to 16th and 84th percentile confidence bands.

right panel reveal that the counterfactual policy rule is now implemented almost perfectly throughout; the bottom panel shows that this requires some moderate ex post instrument surprises a couple of quarters after the initial shock. Compared to Figure 4, these ex post surprises only have a moderate effect on the implied output gap and inflation dynamics (which are consistently below the baseline outcome for both methods). The nominal interest rate paths that generate these outcomes somewhat differ in their timing, with the Sims & Zha refinement suggesting a more gradual response.

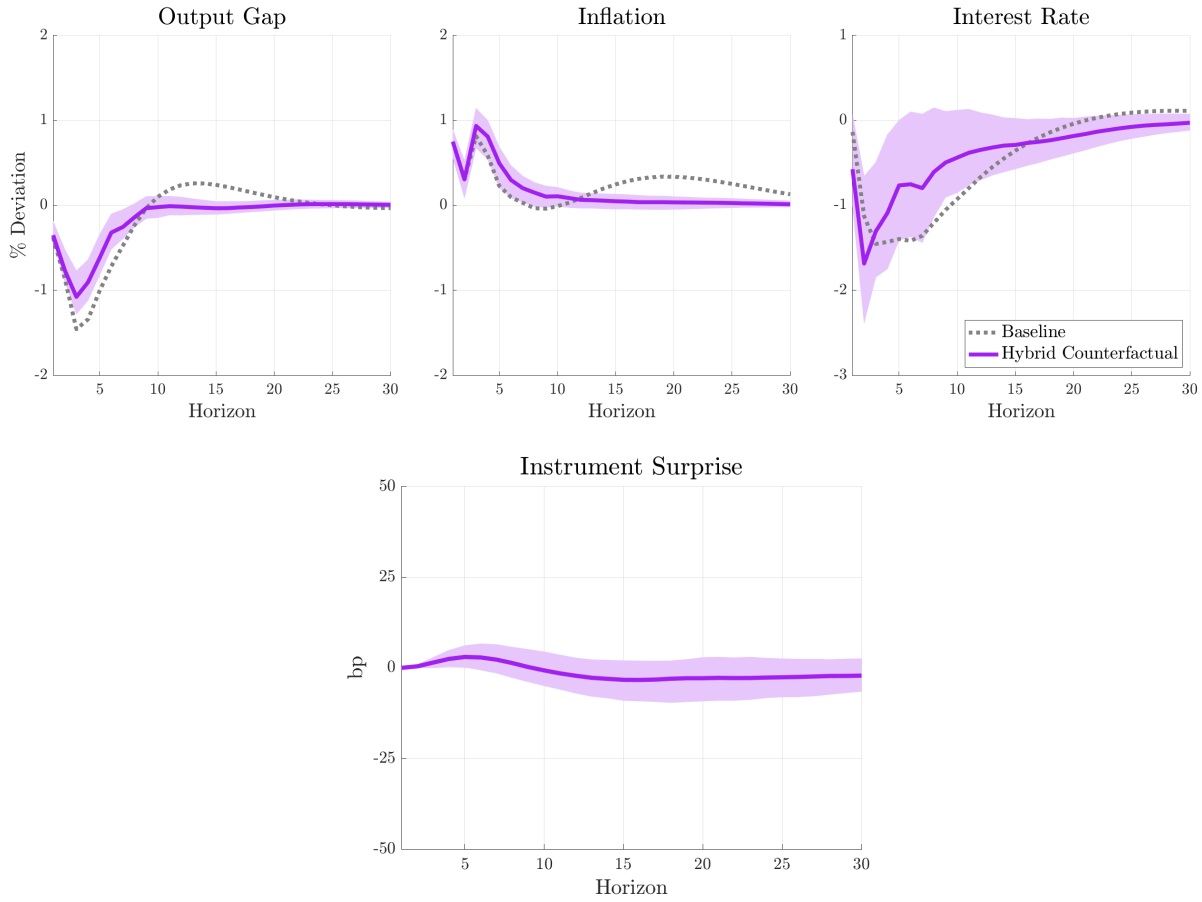

Figure C.4: Top panel: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to an optimal average inflation targeting monetary policy rule with ex post surprises (purple), computed by solving the problem (B.2) for $\psi = 1$. Bottom panel: ex post nominal interest rate surprise at time t. The shaded areas correspond to 16th and 84th percentile confidence bands.

Optimal average inflation targeting policy. Figure C.4 revisits our optimal average inflation targeting policy counterfactual. As discussed in Section 4, for our baseline method, the optimal policy counterfactual differs very little from actually observed outcomes. The figure reveals that furthermore allowing for ex post surprises does not materially change this headline conclusion.

Nominal interest rate peg. Results for the nominal interest rate peg are presented in Figure C.5. The figure shows results both for our baseline method (orange) as well as the

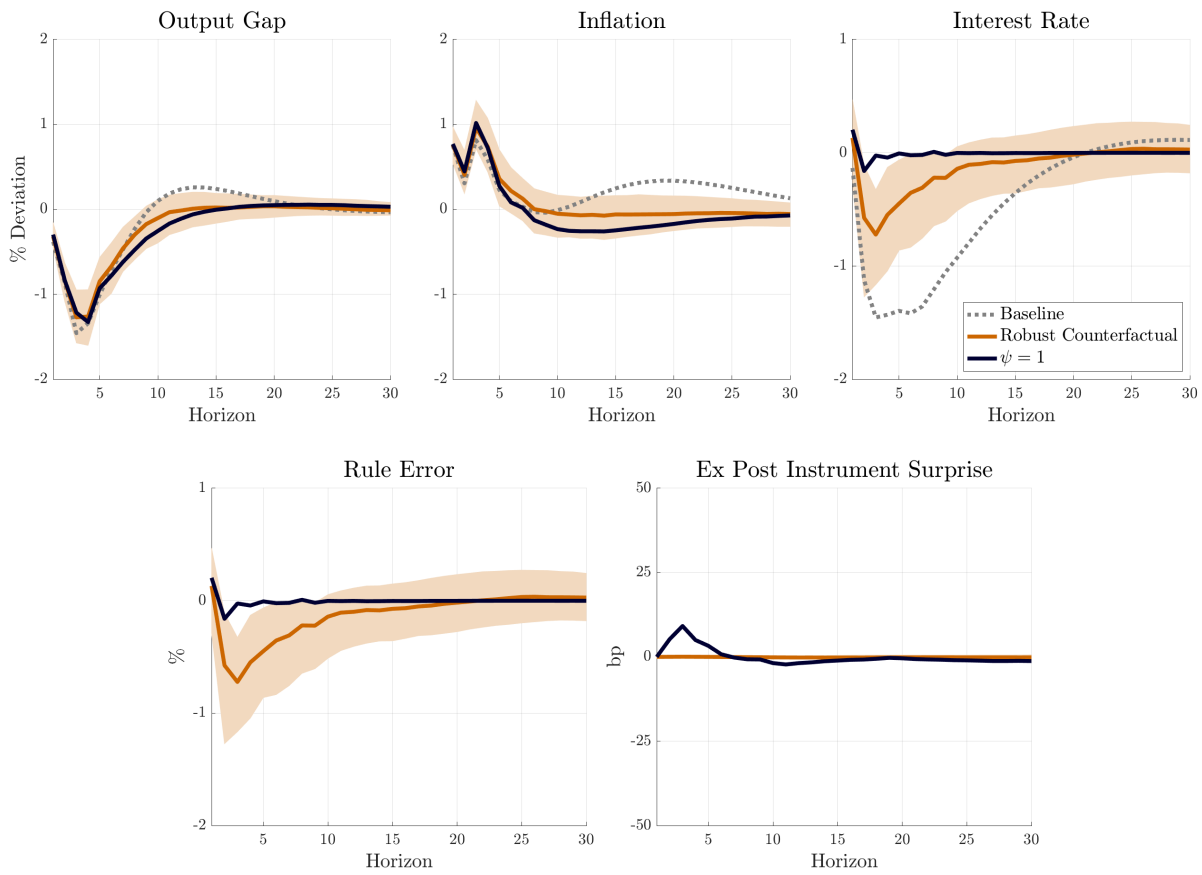

Figure C.5: Top panel: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to a nominal interest rate peg (orange and black), computed following (31) and (B.1) for $\psi = 1$. Bottom panel: implementation error for the counterfactual rule and ex post nominal interest rate surprise at time t. The shaded areas correspond to 16th and 84th percentile confidence bands.

Sims & Zha refinement (black). The implementation accuracy—the argument of (31)—is presented in the bottom left panel. We see that, for our baseline method, the counterfactual rule is implemented well from a couple of quarters out onwards, but rates are still cut by quite a bit too much immediately after the shock. Alternatively, at the cost of repeated (relatively small) interest rate surprises within the first year after the shock, the interest rate is fixed almost perfectly. Since interest rates are now not cut (as much), the output gap and inflation remain low for a longer period of time.

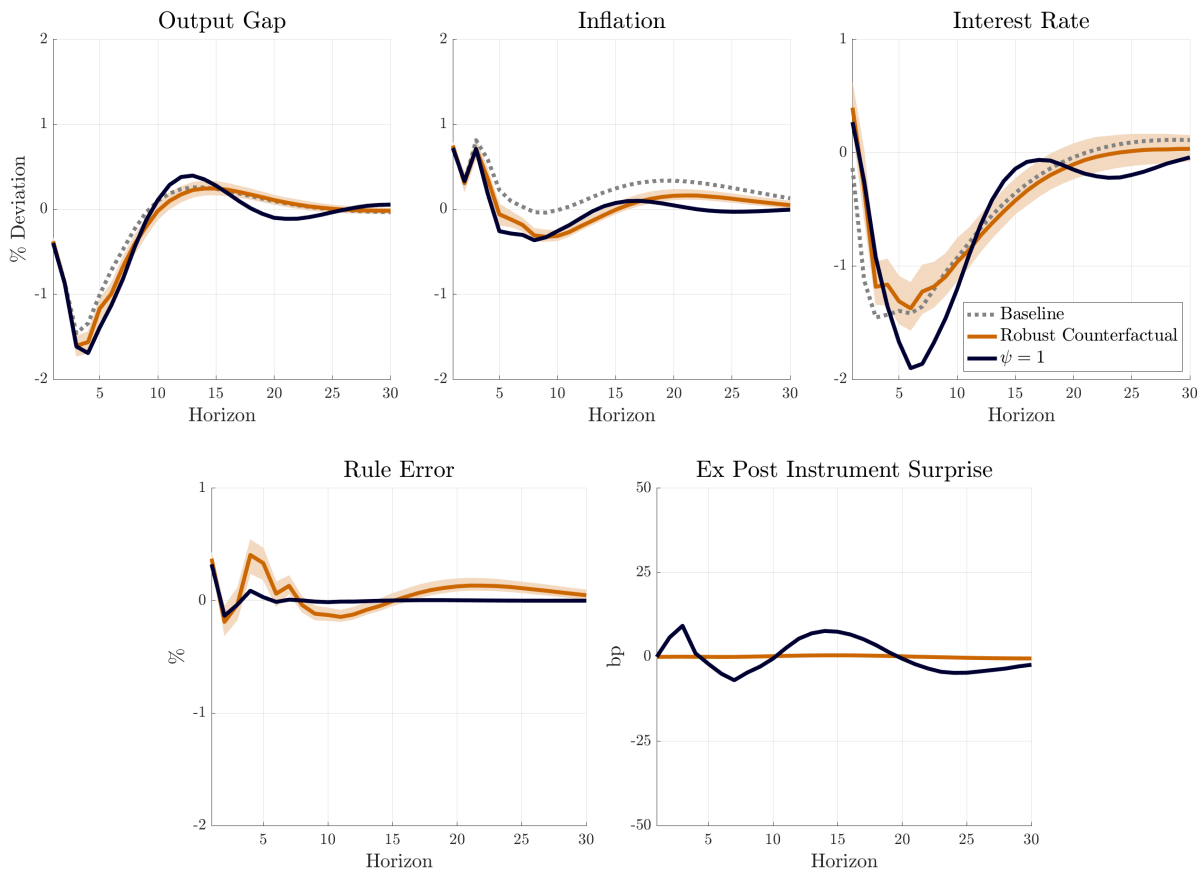

Figure C.6: Top panel: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to a nominal GDP targeting (orange and black), computed following (31) and (B.1) for $\psi = 1$. Bottom panel: implementation error for the counterfactual rule and ex post nominal interest rate surprise at time t. The shaded areas correspond to 16th and 84th percentile confidence bands.

Nominal GDP targeting. Results for nominal GDP targeting are presented in Figure C.6. The counterfactual policy is implicitly defined by the targeting rule

$$\widehat{\pi}_t + (\widehat{y}_t - \widehat{y}_{t-1}) = 0, \quad \forall t = 0, 1, \ldots$$

We find that implementation errors are quite small throughout. Interestingly, the policy instrument path is quite close to the estimated baseline (dotted grey), indicating that nominal GDP is stabilized quite well already under the prevailing rule.

C.4 Counterfactuals with alternative shock measures

Some recent work has questioned the validity of the canonical policy shocks of Romer & Romer and Gertler & Karadi (e.g., see Ramey, 2016; Nakamura & Steinsson, 2018, and the references therein). To examine the robustness of our conclusions to the use of alternative measures of monetary policy shocks, we now use the policy shock series of Miranda-Agrippino & Ricco (2021) and Aruoba & Drechsel (2022). These shock series are constructed using methods similar to those of Gertler & Karadi and Romer & Romer, but use a richer set of controls for the state of the economy as perceived by the Federal Reserve.

We study the propagation of these shocks in a single integrated VAR, exactly as in our baseline analysis. We find that the two shocks differ in the implied interest rate movements, with the shock of Miranda-Agrippino & Ricco (2021) mirroring the transitory rate movement of Romer & Romer (2004), and the shock of Aruoba & Drechsel (2022) similar to the gradual interest rate movement of Gertler & Karadi (2015). We then leverage these shock estimates to construct monetary policy rule counterfactuals, proceeding exactly as in Section 4. Results for our two main systematic policy rule counterfactuals—output gap targeting and the Taylor rule—are displayed in Figure C.7. The main takeaway is that the systematic monetary policy rule counterfactuals are very similar to our headline results. The underlying reason is simply that the impulse responses to the Miranda-Agrippino & Ricco and Aruoba & Drechsel shocks are quite similar to those displayed in Figure C.1 for Romer & Romer and Gertler & Karadi. The perhaps most notable difference is that the shocks of Miranda-Agrippino & Ricco and Aruoba & Drechsel have somewhat *larger* effects on output and inflation (for a given peak interest rate response), so the interest rate cut for the output gap targeting counterfactual is somewhat less steep, and the inflation spike is somewhat more pronounced.

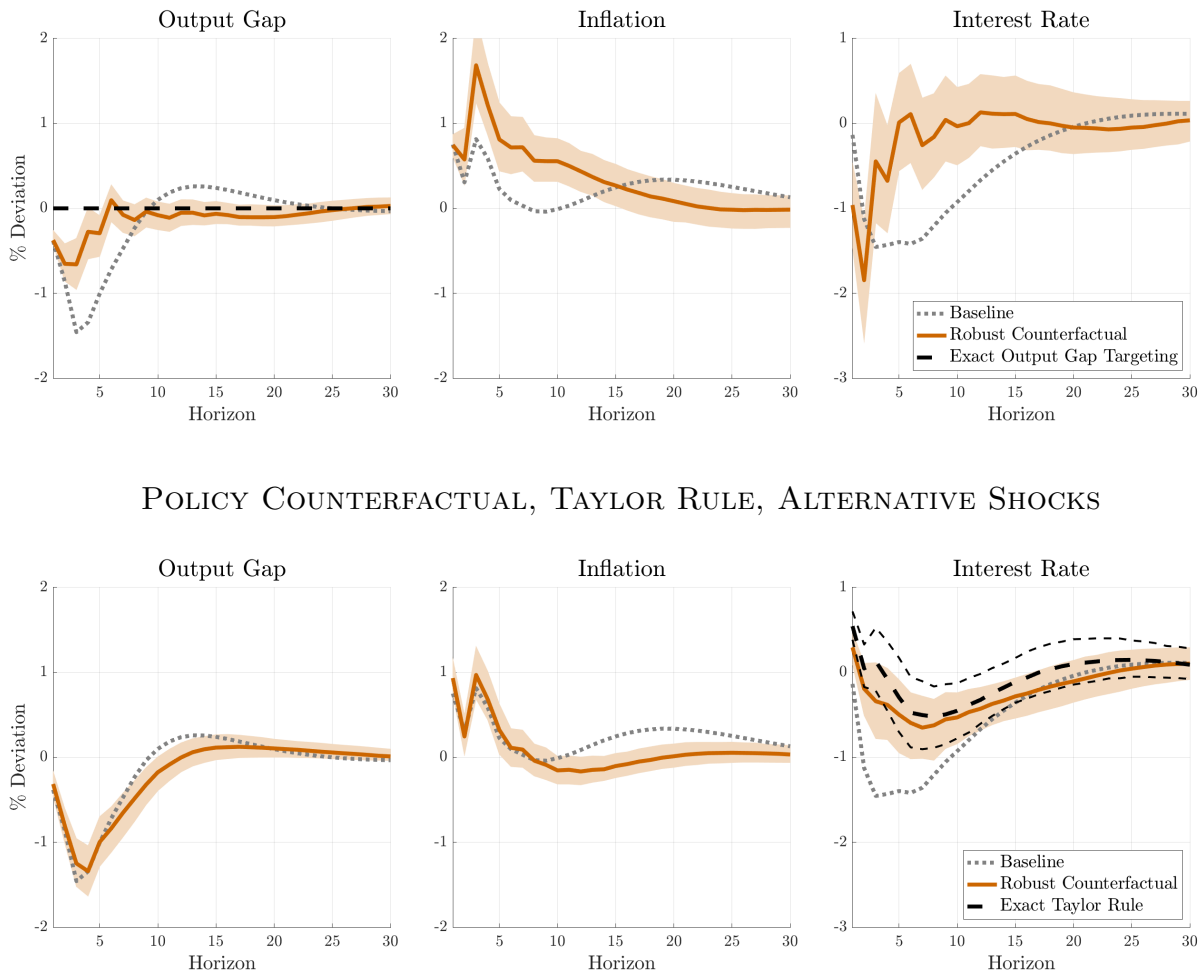

Figure C.7: Output gap, inflation and interest rate impulse responses to a contractionary investment-specific technology shock under the prevailing baseline rule (dotted grey) and the best feasible approximation to output gap targeting (orange, top panel) and a simple Taylor-type rule $\widehat{i}_t = 0.5\widehat{i}_{t-1} + 0.5 \times (1.5\widehat{\pi}_t + \widehat{y}_t)$ (orange, bottom panel) computed following (31) and using the monetary shocks of Miranda-Agrippino & Ricco (2021) and Aruoba & Drechsel (2022). The shaded areas correspond to 16th and 84th percentile confidence bands.