Leaning Against Housing Prices as Robustly Optimal Monetary Policy

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Housing Prices and Monetary Policy

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  Policy only needs to be sufficiently sensitive to inflation/inflation forecasts (Bernanke and Gertler (1999, 2001)).

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- Still prominent proponents today, e.g. Svensson (2017)

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- What are the consequences of alternative policies when housing prices not necessarily based on rational expectations?

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- **Natural question:** a role for asset prices in *fully optimal monetary policy* in the presence of **speculative mispricing of assets?**
- What are the consequences of alternative policies when housing prices not necessarily based on rational expectations?
- **Not easily determined:**

  Standard RE models assume away speculative mispricing

Housing Prices and Monetary Policy

- Rational bubble models allow addressing "mispricing"

  Struggle with EQ multiplicity: exogenously sunspot process

  Compare policies for specifically given sunspot:
  Bernanke and Gertler (1999, 2001).

- Which sunspot? Do sunspots vary with policy?

- RE theory silent.....

Robust Policy Analysis

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- CB chooses policy that is *least vulnerable* to deviations of PS expectations from model-consistency, as in models of "ambiguity aversion" or "robust control"

Robust Policy Analysis

Extend analysis of standard NK model in Adam & Woodford (2012):

1. Add housing sector to NK model:
   - house prices fluctuate: fundamentals + speculative mispricing
   - housing production: supply reacts to housing price
2. Allow for 'larger' belief distortions:
   affect inflation/output gap/housing prices to first order
3. New & simpler approach for computing the "upper bound" on what robustly optimal MP can achieve

Main Finding #1

- **Without robustness concerns/fear of speculative mispricing:**

  Optimal MP implemented by standard targeting rule:

$$\pi_t + \lambda_y (y_t^{gap} - y_{t-1}^{gap}) = 0 \quad \text{with } \lambda_y > 0$$

  - same rule as is optimal in a model w/o housing sector (!)

  - only difference: y_t^{gap} now also depends on housing shocks

  Important message: under RE no role for asset prices in MP

Main Finding #2

- **With robustness concerns/fear of speculative mispricing:**

  Generalized targeting rule:

$$\pi_t + \lambda_y (y_t^{gap} - y_{t-1}^{gap}) + \lambda_\pi (\pi_t - E_{t-1}\pi_t) + \lambda_q (\widehat{q}_t^u - E_{t-1}\widehat{q}_t^u) = 0$$

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- **Important message:** leaning against housing price *surprises* optimal

  - positive housing surprise $\Rightarrow$ tighter policy than under RE
  - symmetric response to negative surprises
  - no need to determine fund. housing price: familiar excuse for inaction

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Robust Policy: Defining the Problem

- Robustly optimal policy problem

$$\max_{c \in C} \min_{\zeta \in \mathcal{Z}} \quad U(y)$$

$$\text{s.t.} \quad y = O(c, \zeta) \text{ and } V(\zeta, y) \leq \overline{V}$$

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- $V(\zeta, y) \geq 0$: belief distortion measure
reflects notion of "near-rationality", $V(\zeta, y) = 0 \Leftrightarrow$RE
- $\overline{V} \geq 0$: measures "degree of robustness concerns"

  $\overline{V} = 0 \Leftrightarrow$ RE-optimal policy

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- **Unappealing features:**

1. Solution depends on assumed set of policy commitments C
   Asset prices only relevant because of the assumed set C?
2. A very hard problem: requires determining worst-case beliefs ζ^* for all $c \in C$

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- Then provide an **example** of a policy commitment that **achieves the upper bound** (as w.c. outcome): generalized targeting rule
- Next slides: present the upper bound problem...

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- **Equivalent Lagrangian formulation**

$$\max_{c \in C} \min_{\zeta \in \mathcal{Z}} U(O(c, \zeta)) + \theta V(\zeta, O(c, \zeta))$$

θ : (inverse) measure of the degree of robustness concerns

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- **Second upper bound:** let policymaker directly choose y

$$\begin{aligned} \min_{\zeta \in \mathcal{Z}} \max_{y \in Y} \quad & U(y) + \theta V(\zeta, y) \\ s.t. \quad : \quad & F(y, \zeta) = 0 \end{aligned}$$

Robust Policy: General Approach

- **Lagrangian formulation of second upper bound**

$$\min_{\zeta \in \mathcal{Z}} \max_{y \in Y} \min_{\varphi} U(y) + \theta V(\zeta, y) - \varphi F(y, \zeta)$$

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    - show that $\bar{\zeta}$ are the worst-case beliefs associated with $\bar{c}$
    - show that $\bar{y}$ is the unique outcome associated with $(\bar{c}, \bar{\zeta})$: outcome function then *must* satisfy $O(\bar{c}, \bar{\zeta}) = \bar{y}$

    => upper bound is attained by $\bar{c}$!

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  => Radon-Nikodym theorem: can represent distorted beliefs as

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- Size of belief distortions measured by relative entropy

  $$E_t m_{t+1} \log m_{t+1}$$

  and discounted relative entropy

  $$E_0 \left[\sum_{t=0}^{\infty} \beta^{t+1} m_{t+1} \log m_{t+1} \right]$$

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- New to our approach: we parameterize (!) belief distortions

$$\begin{aligned} \min_{m_{t+1}\geq 0} \quad & E_t\left[\theta m_{t+1}\log m_{t+1}\right] \\ s.t. \quad : \quad & E_t\left[m_{t+1}y_{t+1}\right] = y_t^e \\ & E_t\left[m_{t+1}\right] = 1 \end{aligned}$$

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ζ_t: Lagrange multipliers on $E_t[m_{t+1} y_{t+1}] = y_t^e$

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- Discounted relative entropy

$$\begin{aligned} E_0\left[\sum_{t=0}^{\infty} \beta^{t+1} m_{t+1} \log m_{t+1}\right] &= E_0\left[\sum_{t=0}^{\infty} \beta^{t+1} R(\zeta_t, y_{t+1})\right] \\ &\equiv V(\zeta, y) \end{aligned}$$

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- Lagrange multipliers ζ can be used to parameterize belief distortions.

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A Sticky Price Model with a Housing Sector

- Representative Household

$$U \equiv \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\tilde{u}(C_t; \xi_t) - \int_0^1 \tilde{v}(H_t(j); \xi_t) dj + \tilde{\omega}(D_t; \xi_t) \right],$$

- Flow budget constraint

$$\begin{aligned} & P_t C_t + B_t + (D_t + (1-\delta) D_{t-1})\, q_t P_t + k_t P_t \\ \leq & (1+s^d) \tilde{d}(k_t; \xi_t) q_t P_t + \int_0^1 w_t(j) P_t H_t(j) dj + B_{t-1}(1+i_{t-1}) \\ & + \Sigma_t + T_t, \end{aligned}$$

$s^d \lesseqgtr 0$: housing subsidy (or tax).

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- Isoelastic forms for $\tilde{u}(\cdot)$ and $\tilde{v}(\cdot)$ and in particular

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- 2 new housing related shocks (ξ_t^d, A_t^d) plus 1 new parameter (s^d)

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- Two new optimality conditions:

(1) Opt. housing investment

$$k_t = \left(\left(1 + s^d \right) A_t^d q_t \right)^{\frac{1}{1-\widetilde{\alpha}}}$$

(2) Asset pricing equation

$$q_t^u = \xi_t^d + \beta(1-d)\widehat{E}_t q_{t+1}^u$$

$q_t^u \equiv q_t \tilde{u}_C(C_t; \xi_t)$: housing price in marginal utility units

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s.t.:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + \kappa_q \left(\widehat{q}_t^u - \widehat{q}_t^{u*}\right) + u_t$$

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(+initial conditions.)

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- RE asset price eq. => constant & exogenous housing price gap:

$$\widehat{q}_t^u - \widehat{q}_t^{u*} = s^d$$

Optimal Policy under RE

- Dropping variables independent of policy, RE problem is

$$\min_{\{\pi_t, y_t^{gap}\}} E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{2} \left[\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap} \right)^2 \right]$$

s.t.:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y y_t^{gap} + s^d + u_t$$

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- **RE optimal policy does not need to refer to housing prices:**

$$\pi_t + \frac{\Lambda_y}{\Lambda_\pi \kappa_y} (y_t^{gap} - y_{t-1}^{gap}) = 0$$

Outline of the Presentation

1. Defining the robustly optimal policy problem
2. The near rational beliefs and distortion measure $V(\cdot, \cdot)$
3. Present nonlinear NK model with housing
4. LQ approx. to optimal policy problem under RE
5. **LQ approx. of upper-bound problem with robustness concerns**
6. Numerical illustration of result

Upper-Bound Problem w Robustness Concerns

- Restrict attention to (first-order) solutions of the form:

$$x_t = E_0[x_t] + \sigma \sum_{k}^{K} \sum_{j=0}^{t-1} x_{j,k} e_{k,t-j} + O(\sigma^2)$$

K: number of independent structural disturbances
$e_{k,t}$: the time t-innovation to k-th disturbance.
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- Restriction required for obtaining analytical solutions under robustly optimal policies

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- Present work allows for "larger" belief distortions (technically more demanding).

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- LQ approximation of upper-bound problem

$$\max_{\{\widehat{\zeta}_t \in R^2\}} \min_{\{\pi_t, y_t^{gap}, \widehat{q}_t^u\}} E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{2} \left(\Lambda_\pi \pi_t^2 + \Lambda_y \left(y_t^{gap}\right)^2 + \Lambda_q (\widehat{q}_t^u - \widehat{q}_t^{u*})^2 - \frac{\beta}{\theta} \widehat{\zeta}_t' V \widehat{\zeta}_t \right)$$

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- AP equation:

$$(\widehat{q}_t^u - \widehat{q}_t^{u*}) = \beta(1-\delta) \left(E_t [\widehat{q}_{t+1}^u - \widehat{q}_{t+1}^{u*}] + \frac{V_2}{\theta} \widehat{\zeta}_t \right) + (1 - \beta(1-\delta)) s^d$$

(+ initial conditions)

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- LQ approximation features positive semi-definite 2x2 matrix

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

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- Nonstandard feature: V is not a matrix of exogenous coefficients...
- Defining the conditional inflation and housing price surprises:

$$\begin{aligned} \widetilde{\pi}_{t+1} &\equiv \pi_{t+1} - E_t \pi_{t+1} \\ \widetilde{q}^u_{t+1} &\equiv \widehat{q}^u_{t+1} - E_t \widehat{q}^u_{t+1} \end{aligned}$$

  we have

$$V = E_t \begin{pmatrix} \left(\widetilde{\pi}_{t+1}\right)^2 & \widetilde{\pi}_{t+1} \widetilde{q}^u_{t+1} \\ \widetilde{\pi}_{t+1} \widetilde{q}^u_{t+1} & \left(\widetilde{q}^u_{t+1}\right)^2 \end{pmatrix}$$

Upper-Bound Problem w Robustness Concerns

- FOCs with respect to worst-case belief distortions

$$\widehat{\zeta}_t^* = \begin{pmatrix} \varphi_t^* \\ (1-\delta)\psi_t^* \end{pmatrix}$$

$$\begin{array}{lcl} \varphi_t^* & : & \text{Lagrange multiplier on NKPC in follower's solution} \\ \psi_t^* & : & \text{Lagrange multiplier on AP eqn in follower's solution} \end{array}$$

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- Belief distortions larger if constraints more binding!
- Worst-case distortions do not *directly* depend on V, but endogeneity of V relevant for follower's problem and thus for Lagrange multipliers
- Can substitute worst-case belief distortions $\widehat{\zeta}_t^*$ into upper-bound problem & derive follower's FOCs (under the restriction of conditionally linear policies)

Upper-Bound Problem w Robustness Concerns

- FOCs with respect to π_t :

$$\Lambda_\pi \pi_t - \varphi_t + \varphi_{t-1} + \theta^{-1} E_{\varphi\varphi} \left(\pi_t - E_{t-1} \pi_t \right) + \theta^{-1} (1-\delta) E_{\varphi\psi} \left(\widehat{q}_t^u - E_{t-1} \widehat{q}_t^u \right) = 0$$

where

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are endogenous coefficients (because V is endogenous in the follower's problem).

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Using this equation to substitute $\varphi_t + \varphi_{t-1}$ in the first FOC delivers the proposed target criterion!

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- Paper shows how SOC can be verified for upper-bound problem and optimality of target criterion

Upper-Bound Problem w Robustness Concerns

- The proposed target criterion:

$$\pi_t + \frac{\Lambda_y}{\Lambda_\pi \kappa_y}\left(y_t^{gap} - y_{t-1}^{gap}\right)$$

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- If SS housing supply too high and output subsidy too low

$$\varphi_t, \psi_t > 0 \text{ in steady state}$$

- If stochastic fluctuations not too large relative to SS distortions

$$E_{\varphi\psi} > 0$$

=> leaning against housing prices optimal!

Upper-Bound Problem w Robustness Concerns

- Economic intuition for "leaning-against" housing prices:

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- Output suboptimally low:
  upward distortions of expected inflation harmful
  NKPC => even lower output given current inflation
- Housing stock suboptimally high:
  upward distortion of housing price expectations harmful
  AP equation => higher current prices and even more housing supply

Upper-Bound Problem w Robustness Concerns

- If there are states of the world featuring
  a.) *positive* housing price and *positive* inflation surprises, or
  b) *negative* housing price and *negative* inflation surprises
  $=>$ belief distortions can achieve *simultaneous* upward distortions of inflation and housing price expectations:

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  => can generate **larger distortions for a given entropy bound**

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- How? Lean against housing price surprises

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6. **Numerical illustration of result**

Numerical Illustration

- Steady state distortions: housing subsidy of 15%, output subsidy $\underline{\tau}$ falls 15% below its efficient level
- Persistent mark-up shocks

$$u_t \equiv w + \widehat{u}_t$$

where

$$w \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\eta)} \log \frac{\eta}{\eta - 1} \frac{1-\underline{g}}{1-\underline{\tau}}$$

$$\widehat{u}_t = \rho_u \widehat{u}_{t-1} + e_t^u$$

- Persistent shocks to efficient housing prices

$$\widehat{q}_t^{u*} = \widehat{\bar{\xi}}_t^d - s^d \quad \text{, where } \widehat{\bar{\xi}}_t^d = \rho_\xi \widehat{\bar{\xi}}_{t-1}^d + e_t^\xi,$$

Numerical Illustration

Discount factor	β	0.99
Housing depreciation rate	δ	0.03/4
Phillips curve coeff. on output gap	κ_y	0.024
Phillips curve coeff. on house price gap	κ_q	-0.0023
Relative weight on output gap	$\frac{\Lambda_Y}{\Lambda_\pi}$	0.0031
Relative weight on housing gap	$\frac{\Lambda_q}{\Lambda_\pi}$	0.0014
Steady state housing subsidy	s^d	15%
Steady state mark-up gap	w	0.0057
Mark-up shock persistence	ρ_u	0.9907
Housing preference shock persistence	ρ_ξ	0.99
Std. dev. mark-up shock innovation	σ_{e^u}	0.0002
Std. dev. housing pref. shock innovation	σ_{e^ξ}	0.024
Robustness parameters	$\frac{\theta^{-1}}{\Lambda_\pi}$	50

Numerical Illustration: Steady State Effects

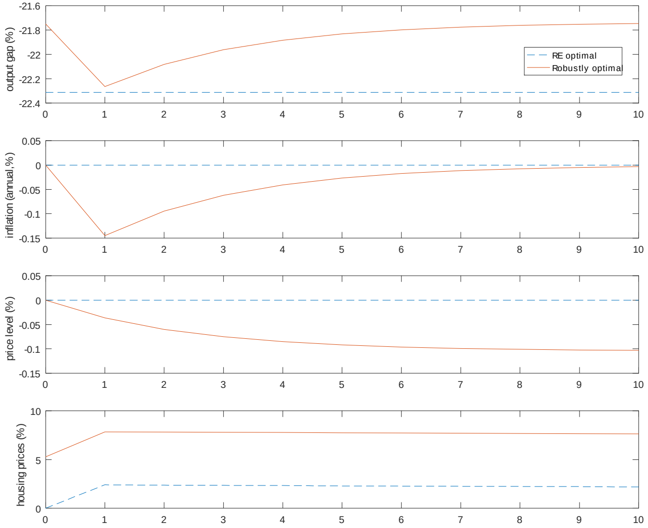
	RE steady state	Worst-case steady state
Output gap $(\widehat{Y} - \widehat{Y}^*)$	-22.3%	-21.8%
Inflation (π)	0%	0%
Housing price gap $(\widehat{q}^u - \widehat{q}^{u*})$	15%	20.3%

Numerical Illustration: Optimal Targeting Rule

Coefficient on		RE optimal	Robustly optimal
Change in output gap	$\frac{\Lambda_y}{\Lambda_\pi \kappa_y}$	0.1292	0.1292
Inflation surprises	$\frac{\theta^{-1}}{\Lambda_\pi} E_{\varphi\varphi}^{new}$	0	0.0414
Housing price surprises	$\frac{\theta^{-1}}{\Lambda_\pi}(1-\delta) E_{\varphi\psi}^{new}$	0	0.0406

Table 3: Optimal targeting rule coefficients

Impulse Response to +1 Std.Dev. housing pref. shock



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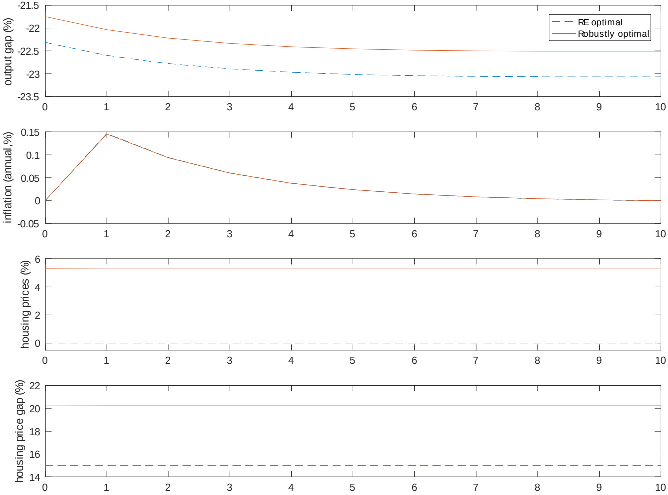

Conclusions

- In the presence of a housing subsidy and inefficiently low output (empirically relevant case) CB concerned with the robustness of PS expectations should "lean against the wind"
  - target lower inflation and/or output gap when housing prices unexpectedly high
  - target higher inflation and/or output gap when housing prices unexpectedly low
- Result obtained in a setting where CB is implementing fully optimal commitment policy
- Optimal target criterion does not require CB to establish a view on which price movements are due to fundamentals and which ones are due to expectational errors

Conclusions

- Would have required raising interest rates more or sooner in the mid-2000's?
- Present model still very stylized: concern for housing prices arises solely from concern for oversupply in housing
- In practice many additional concerns: effects on balance sheets of banks and amount of private borrowing. Do these concerns push policy in the same direction?