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Macroeconomic stabilisation in a volatile inflation environment

Balance sheet policy above the ELB

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Balance Sheet Policy Above the ELB

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Abstract

When the short rate is above the effective lower bound, a central bank does not need a large balance sheet to steer the economy. I discuss factors relevant for setting balance sheet size and composition above the ELB, focusing on the role of convenience yields. Reserves provide a liquid and safe asset suggesting that a central bank may want to supply enough to set their convenience yield of zero. However, if reserves are supplied via purchases of bonds (e.g., Treasuries or Bunds) which themselves have convenience yields, the convenience-maximizing central bank balance sheet size equalizes the convenience yields on reserves and bonds and is smaller than the former prescription would imply. I estimate the convenience-maximizing balance sheet size for the US and the euro area, emphasizing different political constraints across the Atlantic on which types of assets central banks can hold without being perceived as affecting credit allocation or fiscal policy. I clarify the difference between convenience-maximization and maximizing seigniorage from convenience yields.

1   Introduction

The global financial crisis marked a turning point in central bank policymaking in many jurisdictions. When short-maturity policy rates reached the effective lower bound (ELB) and more policy accommodation was needed in the years following the crisis, balance sheet policy and forward guidance were used to a much larger extent than in the past. The Federal Reserve embarked on quantitative easing (QE) starting in 2008 while the ECB purchased sovereign bonds during the European sovereign debt crisis and started its quantitative easing bond purchases in 2015. The COVID crisis saw a resurgence in bond purchases, first for financial stability purposes and later for economic stimulus.

While much has been said about how and how much QE affects asset prices and the economy, much less is known about quantitative tightening (QT), both from a positive and a normative perspective. The Federal Reserve reduced its balance

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sheet by about $700B between April 2017 and August 2019 but reversed course in September 2019 after a spike in various short-term market rates suggested that the reduction in reserve supply led to an undesired scarcity of liquidity. The ECB's balance sheet plateaued in 2019 but did not shrink. With the surge in inflation across the Atlantic, both the Federal Reserve and the ECB have raised their short policy rates substantially and have started balance sheet reduction. However, there appears to be little consensus on how much balance sheet reduction is desirable. The Federal Reserve has announced monthly balance sheet reductions of up to $95B/month but has not announced an end-date to its balance sheet reduction.[2] The ECB has announced balance sheet reduction of €15B/month until the end of June 2023 with the subsequent pace to be determined over time. This is in addition to balance sheet reduction from banks' repayment of TLTRO borrowing.

Against this background, this paper asks what balance sheet size is desirable when a large balance sheet is not needed for policy stimulus, nor for policy tightening. There is no upper bound on short policy rates and while quantitative tightening contributes to tightening the overall policy stance for a given policy rate, it is well-known that a given amount of policy restraint on the economy can be achieved by a variety of combinations of the short policy rate and balance sheet size. I start with a simple graphical exposition of this fact, illustrating what I label the "iso-market rate curve" which is combinations of the interest rate on reserves and reserve supply that achieves the same short market rate. The iso-market rate curve is derived from the demand curve for reserves. The interest rate on reserves affects the demand curve for reserves. Given the demand curve, reserve supply determines reserve scarcity, which maps to the spread between the short market rate and the interest rate on reserves. A central bank wishing to reach a given short market rate can do so with either a low interest rate on reserves, a low reserve supply, and substantial reserve scarcity or with a higher interest rate on reserves, a higher reserve supply and less (or no) reserve scarcity. I extend the iso-market rate curve concept to longer market rates that better capture the overall policy stance.

The central question I ask is: How should central banks choose a point on the iso-market rate curve? In other words, should they choose a small balance sheet and or a large one? Should they target scarce or ample reserve supply? I briefly review factors that may be relevant for this choice, including (a) the central bank's supply of liquid and safe assets, (b) side effects of large central bank balance sheets, (c) interest rate volatility, and (d) central bank profit risk, independence, and headroom for future QE. My analysis then focuses on the first factor, the effect of balance sheet size on the central bank's supply of liquid and safe assets to the economy. This is not to say that only this factor matters. I view my estimates as a benchmark from which policymakers can adjust balance sheet size up or down depending on their view of the importance of other factors (which go in different directions).

The starting point for my analysis is what one could call a Friedman Rule for reserves: Supply reserves to the point that they are no longer scarce. A central bank

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[2] See Federal Reserve Board - Plans for Reducing the Size of the Federal Reserve's Balance Sheet and Monetary policy decisions (europa.eu).

can create reserves at little cost, and this reserve supply maximizes welfare from the liquidity value of reserves. I argue, however, that whether following this approach is desirable or not, depends on how a central bank supplies reserves. If the central bank supplies reserves via purchases of bonds that themselves have a convenience yield from liquidity or safety, maximizing the central bank's net supply of convenient assets should account for both the assets supplied and the assets purchased to supply them. While a central bank able to supply assets via holdings of assets without convenience yields will maximize its convenience supply by setting the convenience yield on reserves to zero, a central bank that supplies reserves via holdings of convenient assets maximizes its net convenience supply by equalizing the convenience yield on reserves and the (average) convenience yield on assets purchased to supply the reserves.

I derive the comparative statics resulting from convenience-maximization to emphasize that the convenience-maximizing reserve supply evolves over time. It is increasing in bank deposits because higher deposits shift the reserve demand curve outward as banks value reserves more when they have more deposits to manage. If reserves are supplied via central bank holdings of convenient assets, the convenience-maximizing reserve supply is furthermore affected by drivers of the convenience yield on these assets. These drivers include the asset supply, the central bank's asset holdings due to its autonomous factors (currency, government deposits etc.), and any shifters of the demand curve for the convenient assets (the size of the economy, which I proxy by GDP).

I extend the analysis to account for bank balance sheet costs, arguing that these should be subtracted from convenience yields for reserves when maximizing the welfare from supplying reserves. I also discuss the extent to which it matters how banks fund reserve holdings.

In my framework, central bank asset mix takes a central role for determining convenience-maximizing balance sheet size because a central bank's holdings of assets with a convenience yield reduces its net supply of convenience. In the US, the Federal Reserve Act grants the Federal Reserve the right to buy and sell obligations that are direct obligations of, or guaranteed by, the United States. The Federal Reserve thus cannot in general transact in non-government guaranteed securities. When it purchased corporate bonds during the COVID crisis, this was done using an emergency lending program set up under Section 13-3 of the Federal Reserve Act to address "unusual and exigent circumstances". Broaddus and Goodfriend (2001) express the common sentiment in the US that the Federal Reserve should continue to focus on holding Treasuries to avoid interfering with credit allocation. They write:

"…the Fed's asset acquisition policy ought to give priority to preserving public support for the Fed's independence by insulating the central bank as much as possible from potentially damaging disputes regarding credit allocation"

"When the Fed purchases Treasury securities, it extends Federal Reserve credit to the Treasury. Doing so, however, leaves all the fiscal decisions to Congress and the Treasury and hence does not infringe on their fiscal policy prerogatives. When the

Fed extends credit to private or other public entities, however, it is allocating credit to particular borrowers, and therefore taking a fiscal action and invading the territory of the fiscal authorities."

The FOMC has stated its intent that the Federal Reserve will primarily hold Treasuries in the longer run "thereby minimizing the effect of Federal Reserve holdings on the allocation of credit across sectors of the economy".[3] By contrast, the ECB has historically supplied reserves via collateralized lending to banks. In the euro area, ECB purchases of substantial amounts of government bonds has been politically sensitive with various ECB programs challenged in court. The ECB only initiated large-scale asset purchases focused on government bonds in March 2015 (under the Public Sector Purchase Programme which accounted for the largest share of purchases under its Asset Purchase Programme). This was much later than the Federal Reserve and several other central banks. Schnabel (2023a) states:

"In the euro area, however, there are [...] additional considerations relevant for the assessment of whether a large bond portfolio is desirable or not. One is that the lack of a consolidated public sector balance sheet raises more fundamental concerns about monetary and fiscal interactions in a currency union with sovereign member states. These concerns may potentially undermine the credibility and independence of the central bank."

As these quotes illustrate, on both sides of the Atlantic, the central bank's assets are viewed as having potential implications for central bank independence. However, what is considered politically sensitive differs, with government bonds a politically safe choice in the US, but a politically risky choice in the euro area. From the perspective of convenience-maximization, the ECB is therefore at an advantage. ECB loans to banks can be collateralized by a range of assets, including assets without convenience yields. In practice, bank borrowing from the ECB is predominantly not backed by central government securities and market participants assess that it is unlikely that many of the securities posted are Bunds.[4] Therefore, in the longer run the ECB would likely be able to return to providing reserves without holdings of convenient assets (and without requiring convenient assets as collateral for lending). By contrast, the high convenience yield on US Treasury securities relative to highly rated corporate bonds is well-documented (e.g., Krishnamurthy and Vissing-Jorgensen (2012), KVJ in what follows). This puts the Federal Reserve in a more difficult position from the perspective of being able to provide liquid and safe assets, on net.

With that observation, I turn to estimating convenience-maximizing liquidity supply for both the US and the euro area. In both cases, estimation of the reserve demand function is crucial. I build on recent work by Lopez-Salido and Vissing-Jorgensen

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3 Federal Reserve Board - Principles for Reducing the Size of the Federal Reserve's Balance Sheet

4 See Eurosystem Collateral Data (europa.eu) and The ECB's collateral conundrum - Central Banking. Schnabel (2023b) provides evidence suggesting that most collateral posted by banks for ECB loans to banks in the TLTROs is rated below Credit Quality Step 1, and thus unlikely to have substantial convenience yields. ECB reserve supply via loans thus does not reduce the availability of convenient asset to the private sector the way ECB purchases of government bonds with convenience yields would.

(2023, LS-VJ). They emphasize three drivers of reserve demand. First, the interest rate spread between the short market rate and the interest rate on reserves. Second, the banking sector's need for liquid assets to manage their liquidity promises. This depends on the amount of bank deposits. Third, bank balance sheet costs (notably capital requirements such as the Supplementary Leverage Ratio (SLR)), which make it costly for banks to borrow to fund reserve holdings. A central finding of their work is that, with the control for deposits, a stable reserve demand curve emerges over the post-GFC period for the US. I update their estimates to April 2023 and extend reserve demand estimation to the euro area, finding a good fit for the period after the European sovereign debt crisis.

In the case of the US, the other input needed to calculate the convenience-maximizing reserve supply is an estimate of the demand for Treasuries, i.e., the convenience yield function for US Treasuries. I update the estimation of this relation from KVJ, focusing on the private (i.e., non-Federal Reserve) sector's Treasury demand. Despite the high Treasury supply in recent years, I estimate that Treasuries still carry a convenience yield. The Aaa corporate – Treasury yield spread in April 2023 averaged 66 bps, of which I estimate that around 31 bps is due to default risk in the corporate bonds (or state-tax differences between types of bonds), leaving a convenience-yield around 35 bps. Since the GFC, Federal Reserve Treasury purchases have reduced the net supply available to the private sector and the private sector's Treasury demand curve has shifted to the right due to Treasury purchases by foreigners, thus making Treasuries scarce even at the current high Debt/GDP ratio.

With these inputs, I estimate convenience-maximizing reserve supplies. For the Federal Reserve, I estimate that, given current deposits, the convenience-maximizing supply of reserves (including holdings in the overnight-reverse repurchase facility) would currently be around $3.3T if the Federal Reserve could supply reserves with holdings of inconvenient assets. However, the convenience-maximizing reserve supply is below $1T with reserves supplied with holdings of Treasuries, as the Federal Reserve has announced is its intention in the longer run. For the ECB, I estimate a convenience-maximizing reserve supply currently of €1.4T, with reserves supplied via inconvenient assets. Careful modelling of reserve demand is crucial in both jurisdictions. Deposits have trended upward (even before COVID). I provide time series estimates of how convenience-maximizing reserve supplies have evolved over time.

The final section of the paper emphasizes that maximizing the welfare created by supplying convenient assets is not the same as maximizing a central bank's seigniorage from issuing liabilities with low interest rates due to their convenience yields. If reserves are supplied via inconvenient asset holdings, the seigniorage from reserves is zero at the welfare maximizing reserve supply. By contrast, the central bank's seigniorage from reserves would be positive at a lower reserve supply. If reserves are supplied via central bank holdings of government bonds with convenience yields, then the convenience-maximizing reserve supply can be above or below that which maximizes the consolidated government's overall seigniorage from reserves and government bonds. However, even a taxpayer narrowly focused

on seigniorage would do better maximizing convenience and taxing the consumer's surplus from convenient assets (if possible) than maximizing seigniorage.

2   Too many tools above the ELB: Iso-market rate curves

It has been well-known since at least Goodfriend (2002), that if a central bank can pay interest on reserves, it can achieve the same equilibrium market rate in various ways.[5] Chart 1, Panel A illustrates this idea. The left figure graphs the reserve demand curve in blue and reserve supply in red. Two points, *A* and *B*, are illustrated at which the equilibrium short market rate equals the central bank's target rate. One possibility, point *A*, is to set a low interest rate on reserves IOR_1, resulting in a low reserve demand curve D_1, and at the same time supply a modest quantity of reserves, S_1. Another possibility, point *B*, is to set a higher interest rate on reserves IOR_2, resulting in a higher reserve demand curve D_2, and at the same time supply a larger quantity of reserves, S_2. These are just two possible combinations that result in the market rate clearing at the desired target. The right figure graphs in blue all possible combinations of reserve supply and the interest rate on reserves which achieve the same target. I will refer to this schedule as the "iso-market rate curve".

The reserve demand curves in Panel A are simplified. In practice, reserve demand is not linear. Furthermore, in recent years it is not uncommon to observe a market rate below the interest rate on reserves in countries with large balance sheets. The standard interpretation of this is that banks must face balance sheet costs (for economic and/or regulatory reasons) when expanding their balance sheet. Lopez-Salido and Vissing-Jorgensen (2023) provide a simple framework for reserve demand that accounts for its key drivers. The reserve demand curve emerges from banks' first-order condition for borrowing to invest in reserves. Their framework has three main ingredients. First, reserves pay interest. Second, reserves have liquidity benefits for banks, captured by a convenience yield function $v(Reserves, Deposits)$. This function captures expected transaction costs savings due to not having to sell bonds/loans when faced with deposits outflows. $v(Reserves, Deposits)$ is positive and increasing in reserves but decreasing in deposits as more deposits imply higher, not lower, expected transactions costs. The marginal value of additional reserves, $v'_R(Reserves, Deposits)$, is decreasing in reserves (as additional reserves are less and less valuable for given deposits) and increasing in deposits (as additional reserves are more valuable when managing a larger amount of deposits).[6] [7] Third, banks face a balance sheet cost φ per dollar of assets (I will discuss this cost more below). Given these ingredients, banks' first-order condition for borrowing at the short market rate r and investing in reserves at the interest rate IOR is

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5 See also Keister, Martin and McAndrews (2008) for a clear exposition of these ideas.

6 The measure of reserves that enters $v(Reserves, Deposits)$ is excess reserves, i.e., reserves minus required reserves. Because reserves cannot go below required reserves, required reserves cannot be used to manage deposits outflows (except to the extent that required reserves go down with deposits).

7 Pre-GFC, reserve requirements were binding in both the US and euro area, generating a regulatory link from deposits to reserve demand. Post-GFC, reserve requirements are not binding and the effect of deposits on reserve demand is due to banks' "economic" demand for reserves as laid out in equation (1).

$$r = IOR + v'_R(Reserves, Deposits) - \varphi. \tag{1}$$

This says that the highest interest rate, r, a bank is willing to pay to borrow to invest in reserves is the net benefit obtained from earning the IOR plus the marginal liquidity benefit from additional reserves, $v'_R(Reserves, Deposits)$, minus the additional balance sheet cost incurred, φ.[8]

Equation (1) is the reserve demand curve illustrated in Chart 1, Panel B, left figure. The reserves demand curve is negatively sloped because $v'_R(Reserves, Deposits)$ is declining in the quantity of reserves held. It shifts up for a higher IOR, with D_1 illustrating reserve demand at IOR_1 and D_2 illustrating reserve demand at IOR_2. The reserve demand curve shifts down for a higher φ. Once reserve demand is satiated, $v'_R(Reserves, Deposits)$ is zero and the reserve demand curve becomes flat at $IOR - \varphi$. I omit this region of the graph for simplicity in Chart 1, Panel B.[9]

Reserve supply now determines reserve scarcity net of balance sheet costs in equilibrium. Points *A* and *B* again illustrate possible ways to achieve a given target rate *r*, with *A* illustrating a low *IOR* and a small balance sheet and *B* illustrating a higher *IOR* and a larger balance sheet. The iso-market rate curve in Chart 1, Panel B, right figure is now curved. From (1), it follows that the iso-market rate curve for a given target rate, r^{Target}, is given by

$$IOR = r^{Target} - [v'_R(Reserves, Deposits) - \varphi]. \tag{2}$$

For a given value of reserve supply, (2) shows how to shade/increase the *IOR* relative to the target rate in order to make the market equilibrium rate clear at the target. At point *A* in Chart 1, Panel B (both left and right figures), reserves are sufficiently scarce (i.e., the expected marginal liquidity value of reserves sufficiently high) that $v'_R(Reserves, Deposits) - \varphi > 0$. The central bank therefore needs to set the IOR below the target to achieve it. Conversely, at point *B* in Chart 1, Panel B (both left and right figures), reserves are so plentiful that $v'_R(Reserves, Deposits) - \varphi < 0$ and the central bank needs to set the *IOR* above the target to achieve it.

While the equilibrium short rate *r* equals the target along all points on the iso-market rate curve, reserves and thus balance sheet size differs. In that sense, the overall policy stance is not constant along the iso-market rate curve. If a central bank buys assets with risk (duration risk, pre-payment risk or credit risk), the spread between the rate on those assets and the short market rate will decline in balance sheet size through a host of channels explored in the literature on quantitative easing. This

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8 Equation (1) assumes competitive markets. An important direction for future work is to incorporate market power by banks in the market for short-term borrowing into my estimations below. For a theory of Nash bargaining in the federal funds market, see Afonso and Lagos (2015).

9 This reserve demand curve shows banks' reserve demand absent any additional central bank facilities. If the central bank has a lending facility (the discount window in the US, the marginal lending facility in the euro area), the rate on this facility will (absent stigma) serve as an upper bound on the equilibrium market rate and the reserve demand curve is cut off at the point it hits the ceiling rate. Similarly, if the central bank has an investment facility accessible to non-banks (the overnight reverse repo facility in the US), the rate on that facility will be a lower bound on the equilibrium market rate and the reserve demand curve is cut off at the point it hits this floor rate. See Lopez-Salido and Vissing-Jorgensen (2023) for further discussion of how borrowing/lending actions by the private sector changes the equilibrium quantity of reserves to ensure that the market rate stays in the corridor between the floor and ceiling rates.

does not change the conclusion that a central bank can achieve a given level of policy tightness with various combinations of reserves and *IOR*. It does, however, lead to a steeper iso-market rate curve for long than short rates. I illustrate this in Chart 1, Panel C. At a smaller reserve supply, both the $r - IOR$ spread for the short rate r and the long-short spread is larger. To hit a given target value for overall policy stance, as summarized by the long rate, the central bank needs to set a substantially lower IOR than would be needed at with a larger reserve supply.

In sum, whether focused on long or short rates, when a central bank is not constrained by the ELB, the same level of policy stance can be achieved with various combinations of the interest rate on reserves and the size of the balance sheet. This is the case even in a realistic setting with convex reserve demand (as suggested by the data in the estimation below) and bank balance sheet costs.

3   Factors relevant for choosing balance sheet size above the ELB

What differs along the iso-market rate curve (for the long market rate) that may guide optimal balance sheet size and composition when the ELB is not binding?

(a) Central bank liabilities: Liquidity/safety supply

From a consolidated government perspective, a larger central bank balance sheet funded with reserves leads to an overall shortening of the duration of the consolidated government's debt. As reviewed by Greenwood, Hanson, Rudolph and Summers (2016), government debt maturity structure does not matter for household resources or welfare unless some of the assumptions underlying Ricardian equivalence fails. Ricardian equivalence assumes away convenience yields (and assumes that taxation is non-distortionary). A larger central bank balance sheet then simply shortens the consolidated government's debt maturity and has no effect on welfare.[10] My analysis will focus on convenience yields, emphasizing that they can be (and in practice are) different for reserves, bills and bonds, and laying out the implication this has for the welfare-maximizing central bank balance sheet size.

With convenience yields, a larger central bank balance sheet is not simply a welfare-neutral shortening of consolidated government debt maturity. A large balance sheet means that the central bank's reserve supply is larger. Reserves are a liquid and safe investment for banks. They facilitate payments (deposit in/outflows) and serve as a very safe interest-bearing store of value. Afonso, Duffie, Rigon and Shin (2022) show, for the US using data from 2010-2020, that even in recent years where

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10 Greenwood et al (2016) state the intuition well: "Consider a government with an initial accumulated deficit and no future expenditures that must decide whether to finance its deficit by issuing short- or long-term bonds. If the government finances itself solely through the issuance of short- term debt, then the government will have to raise taxes if short-term interest rates rise. However, the rise in interest rates will leave a household that is lending short-term to the government with a bit more in its bank account. Since the government's sources of funds (taxes and proceeds from issuing new debt) must equal its uses of funds (paying off maturing debt), the gain in the house hold's bank accounts must precisely offset the increase in taxes."

reserve supply is high, banks rely on incoming payments to fund outgoing payments to conserve liquidity. They study payments in the Fedwire system which add up to around $3T daily in recent years. Incoming payment over the prior 15 minutes is a strong predictor of outgoing payments over the next minute, more so the lower are reserves. The coefficient remains positive and significant even in years of very high reserves such as 2020. Reserves thus appear scarce in that they have value for liquidity management purposes even at high reserve supply (on average $2.6T in 2020). The evidence from Afonso et al (2022) imply that reserve demand is not saturated at any point during their sample.

Supplying plentiful liquidity is a common argument for a large central bank balance sheet. However, from the perspective of supplying liquid and safe assets, it is also relevant how the central bank supplies reserves. If reserves are supplied via central bank purchases of assets that are themselves liquid and safe, then the central bank's net liquidity/safety supply is reduced. Arrata, Nguyen, Rahmouni-Rousseau and Vari (2020) and more recently Schnabel (2023a) emphasize this issue in the euro area, with Eurosystem purchases contributing to a significant scarcity premium on some bonds purchased under ECB asset purchase programs, notably German bunds. Hauser (2022) discusses the relevance of bond scarcity in the UK (especially for medium-term gilts). Similar issues are relevant in the US where the Federal Reserve over the longer run is planning to supply reserves exclusively via Treasury bond holdings. Krishnamurthy and Vissing-Jorgensen (2012) estimate the scarcity premium on Treasuries relative to Aaa-rated corporate bonds (driven by both high liquidity and extremely low default risk of Treasuries, as discussed below).

(b) Side effects of large central bank balance sheets

When a central bank increases reserves, banks need to fund their holding of these reserves. They can do so by lowering their non-reserve assets, or by increasing their liabilities. Central bank reserves may thus crowd out banking lending and may crowd in deposits or other liabilities. To the extent that banks are special and bank borrowers cannot easily replace bank funding with funding from non-banks, *crowding-out* of bank lending can lead to a welfare loss (for empirical evidence, see Diamond, Jiang and Ma (2022) and Chakraborty, Goldstein and MacKinley (2020)). As for liability *crowd-in*, more deposits may be beneficial if they provide liquidity/safety benefits to holders but to the extent the additional deposits are uninsured, they may lead to increased financial stability risks.[11] Financial stability risks may also increase if banks increase the risk of their non-reserve assets in response to larger reserve holdings.

(c) Interest rate volatility

A larger central bank balance sheet tends to be associated with lower short-rate volatility. Reserve demand is typically flatter at higher quantities, implying that a given reduction in reserve supply due to an increase in the autonomous factors will

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[11] Diamond, Jiang and Ma (2022) estimate that in US data covering QE1-QE3, each additional dollar of reserves led to 7 cents of additional deposits, 20 cents of reduction in bank loans (including mortgages), and 73 cents of reduction in banks security holdings (net of wholesale funding and equity).

have a larger effect on the market rate-*IOR* spread at a lower initial reserve supply. The spike in short market rates on September 17, 2019, in the US happened after an increase in autonomous factors (the Treasury General Account increased due to debt issuance and tax receipts, leading to a decrease in reserves) which exacerbated the decline in reserves due to the Federal Reserve's QT. A larger supply of central bank reserves reduces the likelihood of such spikes and associated financial instability risks.

If the central bank has an investment facility for non-banks (like the ONRRP facility in the US) and take-up at this facility is positive, then fluctuations in the autonomous factors may only lead to fluctuations in the take-up at this facility, with no effect on the market interest rate (see Lopez-Salido and Vissing-Jorgensen (2023) for analysis of ONRRP take-up).

(d) Central bank profits and independence; headroom for future QE

A large central bank balance sheet exposes the central bank to duration risk if the assets held are of longer duration than the liabilities supplied (and may expose the central bank to pre-payment risk and credit risk). As discussed above, under Ricardian equivalence this has no effect on welfare. With distortionary taxation, the maturity of the consolidated government debt matters, even aside from convenience yield effects. A large central bank balance sheet may be viewed as imposing too large a risk of a substantial tax increase, should interest rates increase. If sufficiently large, central bank losses may pose a threat to central bank independence. Aside from tax implications, it may also be politically sensitive to pay large amounts of interest to banks (including foreign banks), especially when short market rates are below the interest rate on reserves.[12]. Ex-ante, the concerns imply that a large current balance sheet may be viewed as limiting the room for future QE if needed, as emphasized by Hauser (2022).

In my analysis, I will focus on (a), the effect of central bank balance sheet size and asset composition on the overall supply of liquid and safe assets. I do not attempt to model the effects of (b)-(d) on optimal balance sheet size. For side-effects of QE on banks in terms of loan crowd-out and deposit crowd-in, the net effect on the desirability of a larger balance sheet is unclear. Relative to the convenience-maximizing balance sheet size that I calculate, concerns about interest rate volatility, (c), would push toward a larger balance sheet, while concerns about central bank profit risk and QE headroom, (d), would result in a smaller balance sheet. A central bank interested in these issues could use my framework to calculate the convenience loss from deviating from convenience-maximization. This loss could then be weighed against any benefits from lowering interest rate volatility, or from preserving more room for future balance sheet expansion.

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12 In practice, central banks do appear to care about their profits. Goncharov, Ioannidou and Schmalz (2021) show that central banks are much more likely to report slightly positive than slightly negative profits.

4   Framework: Convenience-maximizing reserve supply and asset choice

The Friedman rule for the optimal supply of central bank money (non-interest bearing cash) says that central banks should supply enough money to drive the market interest rate to zero. Stated in terms of convenience yields, the convenience yield on non-interest-bearing money must equal or exceed the foregone market interest rate for people to hold money. Friedman's prescription entails saturating money demand and setting the convenience yield on non-interest-bearing money to zero. This is based on the underlying observation that producing money has almost no cost to the central bank. Welfare (the consumer's surplus plus the producer's surplus) is then simply the area under the money demand curve, which is maximized for $r = 0$.

Consider what this line of thinking implies about the optimal size of central bank balance sheets. The obvious answer is that the central bank should supply enough reserves that they are not scarce, meaning that the marginal value of reserves for banks in managing payments is zero. Accordingly, many central banks seek to assess the level of reserve supply at which reserves start to become scarce. This is often done via surveys of market participants' estimates of the "lowest comfortable level of reserve balances" (LCLoR), also known as "floor required excess liquidity" (FREL), or the "Preferred Minimum Range of Reserves" (PMRR) see, e.g., Keating, Martinez, Pettit, Rezende, Styczynski and Thorp (2019) for the US, Aaberg et al (2021) for the euro area and Bank of England (2023) for the UK. Lopez-Salido and Vissing-Jorgensen (2023)) develops an econometric approach to estimating the LCLoR that I will use below.

However, the LCLoR ignores central bank asset holdings. If these assets have a convenience yield of their own, the central bank's cost of producing reserves is not zero. It also ignores bank balance sheet costs and bank funding of reserves. After laying out the core argument on how the central bank should trade off convenience yields on reserves and asset holdings, I will extend the analysis to assess the role of bank balance sheet costs and bank funding.

By way of background, by "convenience yield" I mean security benefits over-and-above any interest and principal payments. Krishnamurthy and Vissing-Jorgensen (2012) (KVJ) lay out a framework for convenience yields in which they can arise due to investors placing a special value on the high liquidity of an asset or on its very low default risk (safety).

Convenience yields from liquidity are conceptually straightforward as they simply capture a willingness to earn a lower yield when holding assets that are expected to lead to transactions cost savings. I illustrate convenience yields from safety in Chart 2, which plots the relation between bond yields and default risk. Safety benefits on very low-risk assets are illustrated by the curved part of the relation. Some investors appear willing to accept a yield on very safe assets that is below the yield implied by their low risk and the "normal" yield-risk relation based on higher-risk assets, illustrated with the solid straight line. These also tend to be very liquid assets.

Using Treasuries as an example of a very low default risk asset, the convenience yield is the vertical distance between the red dots on the y-axis. This vertical distance declines with increased Treasury supply as a higher supply moves the curved line up toward the dashed line. With fully saturated convenience demand, the curved and dashed lines would overlap, and the Treasury bond would plot at the top red dot. KVJ exploits this idea to decompose the yield spread between Aaa-rated corporate bonds and Treasuries into the Treasury convenience yield and the default risk component due to default risk on the Aaa-rated corporate bond. They regress the Aaa-Treasury yield spread on Treasury Debt/GDP, allowing for a horizontal asymptote. The estimated horizontal asymptote (i.e., the value of the spread when Treasury debt is plentiful, and safety and liquidity demand satiated) as a measure of the default component of the spread. The remaining spread is then a measure of the convenience yield on Treasuries. They estimate an average Treasury convenience yield relative to Aaa corporate bonds of 46 bps over the period 1919-2008. Aaa-rated corporate bonds may themselves have some appeal to safety investors, implying that Aaa bonds may plot on the curved line a bit to the left of the "Inconvenient corporate bond" point. Using the spread between Baa-rated corporate bonds and Treasuries, adjusted for the expected default component on the Baa bonds, KVJ find a larger Treasury convenience yield averaging 73 bps over the same period.

In terms of the economics underlying convenience yields from safety, some investors face information costs of investing in riskier assets and may be willing to accept a lower yield to not have to analyze and quantify risk (in this case default risk). This is related to the literature on limited participation (e.g., Vissing-Jorgensen (2003)). Assets with low default risk also serve as particularly good collateral for borrowing. Distinguishing between short and long bonds, KVJ furthermore emphasize that for long-maturity bonds, investors like pension funds (and some insurance companies) may have a special demand for low-default risk long-term payoffs to back long-term nominal liabilities. This is a type of preferred-habitat demand, focused on low-default risk assets. Whether convenience yields are due to liquidity effects or safety effects will not matter for my arguments below.

4.1   Trading off convenience yields on reserves and asset holdings

Suppose a central bank (cb) supplies reserves R via bond holdings B^{cb}. Denote the total convenience value from reserves by $v_R(R)$ and the total convenience value from bonds by $v_B(B^{priv})$, where B^{priv} is the private sector's bond holdings and $B^{cb} + B^{priv} = B$, the total bond supply. For now, ignore the fact that $v_R()$ and $v_B()$ have additional arguments because convenient assets are more valuable when the banking sector or the economy is larger. I return to these issues below.[13]

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13 Also, notice that I use separate convenience values for reserves and bonds rather than an aggregate. For the US, empirical proxies for the convenience yield on reserves and government bonds or bills are slightly negative correlated over my sample period from 2009M1-2023M4, as I will discuss below, suggesting that the assets do not enter a common aggregate.

The central bank's balance sheet must balance, so $B^{cb} = R + A$, with A denoting the autonomous factors on the central bank balance sheet (currency, government deposits, etc.):

Assets	Liabilities
Bonds, B^{cb}	Reserves, R
	Autonomous factors (cash, government deposits, etc.), A

Total convenience available to the private sector via its holdings of reserves and bonds is then

$$v_R(R) + v_B(B^{priv}) = v_R(R) + v_B(B - B^{cb}) = v_R(R) + v_B(B - R - A).$$

Should a convenience value for the central bank be added above? In other words, do citizens benefit if their central bank holds assets that are particularly liquid or safe? In the case of a central bank holding domestic bonds to maturity, the central bank, and thus its ultimate owners the citizens, does not benefit from holding liquid assets.[14] Similarly, citizens may not benefit if their central bank holds very safe assets (as the central bank would likely to able to understand and invest in other types of assets without large associated costs). I will thus proceed to assume that total convenience of citizens is as stated above.

As discussed in the introduction, citizens may have reasons to prefer that their central bank hold one type of assets over another. They may have a strong preference for their central bank not interfering with the allocation of credit among private borrowers by buying anything other than government bonds, a case that may fit the Federal Reserve. One could think of this as citizens getting a disutility from the Federal Reserve holding non-Treasuries. Alternatively, citizens may have strong preferences for their central bank not funding the government (or the governments of other countries), perhaps better describing the euro area. Whatever preferences citizens may have over their central banks' holdings, I will model these as a constraint on the central bank's asset choice and analyze how to set reserve supply to maximize convenience given that constraint.

If the central bank (like the ECB) can supply reserves via holding assets *without* convenience value, the central bank's convenience maximization problem is simply

$$\underset{R}{\text{Max}} \ v_R(R)$$

If the central bank (like the Federal Reserve's announced longer-run plans) holds assets *with* convenience value to the private sector, the central bank's convenience maximization problem is

$$\underset{R}{\text{Max}} \ v_R(R) + v_B(B - R - A).$$

Taking first-order conditions, we have the following result. In general, I will denote the marginal convenience values by "convenience yield" (since there marginal

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14 This contrast with a foreign central bank holding these bonds for potential FX interventions.

convenience values are what drives equilibrium yield discounts on convenient assets).

Result 1 (Convenience-maximizing reserve supply).

(a) If a central bank holds assets without convenience yields, then the convenience-maximizing supply of central bank reserves, $\text{R}^{C(a)}$, *solves*

$$v_R'(R) = 0.$$

(b) If the central bank holds bonds (B) with convenience yields (to market participants), then the convenience-maximizing supply of central bank reserves, $\text{R}^{C(b)}$, *solves*

$$v_R'(R) = v_B'(B - R - A).$$

$\text{R}^{C(a)}$ depends on the shape (and shifters) of the convenience yield function $v_R'(.)$. $\text{R}^{C(b)}$ also depends on the shape (and shifters) of the convenience yield function, $v_B'(.)$, as well as the aggregate supply of bonds B and the autonomous factors A. I assume convenience yields are non-increasing in holdings, so $v_R''(R) \le 0$ and $v_B''(B - R - A) \le 0$.

Chart 3 illustrates the moving parts of Result 1. In Chart 3, Panel A, the total convenience value of reserves is the area between the reserve demand curve and the IOR. It has two components. The "consumers' surplus" area indicates how much more banks value reserves than the amount $r*R it costs them to borrow $R at the market rate r to invest in reserves. For a given unit or reserves, the consumers' surplus is $[IOR + v_R'(R) - \varphi - r]$ (ignore the $-\varphi$ for now, I will argue later that it should be subtracted from $v_R'(R)$ even from a welfare perspective). The "producers' surplus" area indicates the central bank's benefit from funding reserves at a rate IOR which is below the short-term market rate r (in the example graphed). For a given unit or reserves, the producers' surplus is $[r - IOR]$. The sum of the consumers' surplus and the producers' surplus is the total convenience value of reserves (net of φ), which is $[IOR + v_R'(R) - \varphi - r] + [r - IOR] = v_R'(R) - \varphi$ for a given unit of reserves.

Chart 3, Panel B illustrates the convenience yield on bonds that have convenience value using Treasuries T as an example (but the same graph results for Bunds or other very safe euro area bonds). The y-axis in this figure is the convenience value for a given unit of Treasuries, defined as the spread between the yield on inconvenient bonds with similar maturity and the yield on the Treasury bonds. The total convenience value on Treasuries is the area between the demand curve and the horizontal axis (the figure focuses on the spread and does not indicate the inconvenient yield and the Treasury yield separately, since both are determined in equilibrium rather than being set administratively like the *IOR*).

The convenience-maximizing reserve supply from Result 1(a) is illustrated in Chart 3, Panel A as the value that makes the market rate *r* clear at the IOR, thus setting $v_R'(R)(-\varphi)$ to zero. This maximized the sum of the consumers' and producers' surplus from reserves. By contrast, in Result 1(b) the convenience-maximizing

reserve supply equates the vertical distances $v_R'(R)(-\varphi)$ equal to $v_B'(B - R - A)$ (where B is Treasuries T in the example graphed). This maximizes the sum of the consumers' and producers' surpluses from reserves and Treasuries combined.

In Chart 3, Panel A, an increase in reserves moves the reserve supply curve S to the *right* and decreases the convenience yield on reserves. In Chart 3, Panel B, an increase in reserve supply leads to an increase in the central bank's bond holdings which shifts the bond supply available to be held by the private sector to the *left* and increases the convenience yield on bonds. The convenience-maximizing reserve supply equates the two convenience yields. Chart 3, Panel C, further illustrates Result 1(a) (left) and Result 1(b) (right), graphing convenience yields as a function of reserve supply.

It should be noted that $v_B(B - R - A)$ refers to the convenience from bond holdings of anyone other than the central bank. This will include foreigners (private and official). Similarly, $v_R(R)$ includes reserve holdings by US branches of foreign banks which may be foreign owned. Implicitly I do welfare optimization assuming foreigners matter equally. If one wanted to calculate convenience-maximizing reserve supply focusing only on the convenience benefits obtained by domestic citizens, one would need to model only holdings by domestic residents/domestically owned banks, $v_{B^{Dom.held}}(B^{Dom.\ held})$ and $v_{R^{Dom.held}}(R^{Dom.\ held})$. The first-order condition in Result 1(b) would change to

$$v_{R^{Dom.held}}'(R - R^{Foreign\ owned}) * \left(1 - \frac{dR^{Foreign\ owned}}{dR}\right)$$
$$= v_{B^{Dom.held}}'(B - B^{Foreign\ owned} - R - A) * \left(1 + \frac{dB^{Foreign\ owned}}{dR}\right).$$

At the margin, all holders of reserves must value them equally so $v_{R^{Dom.held}}'(R - R^{Foreign\ owned})$ equals the overall convenience yield $v_R'(R)$. Similarly for bonds. Therefore, if $\left(1 - \frac{dR^{Foreign\ owned}}{dR}\right)$ is close to $\left(1 + \frac{dB^{Foreign\ owned}}{dR}\right)$ we get back to Result 1(b). I will not pursue this further here. It is likely that countries value the benefits of citizens of other countries to some extent (perhaps fully in that they may expect to get something in return, such as in the US case the fact that these benefits help support the status of the dollar).

Comparative statics

Chart 3, Panel D provides comparative statics. I illustrate what shifts the convenience demand schedules and thus affects the convenience-maximizing reserve supply.

Deposits, $\frac{dR^C}{dD} > 0$: An increase in bank deposits shifts $v_R'(R)(-\varphi)$ up as higher deposits increase the expected marginal value of reserves for managing deposit outflows with low transactions costs. The top left graph shows how the deposit increase leads to an increase in the convenience-maximizing reserve supply in Result 1(a). The top right graph shows how the deposit increase affects the convenience-maximizing reserve supply in Result 1(b). The convenience-maximizing reserve supply increase is smaller in Result 1(b) than in Result 1(a), because of the

positive slope of the $v_B'(B - R - A)$ (with Treasuries again used as the example of B). Increasing reserves is less attractive when reserves are increased by buying bonds with a convenience yield.

GDP, $\frac{dR^C}{dGDP} < 0$: The bottom left graph in Chart 3, Panel D illustrates the effect of higher GDP. When the economy is larger, convenient bonds are scarcer at each level of supply, so the $v_B'(B - R - A)$ function shifts up. This reduces the convenience-maximizing reserve supply in Result 1(b) which moves from point A to point B.

Convenient bond supply, B, $\frac{dR^C}{dB} > 0$: The bottom right figure in Chart 3, Panel D shows the effect of a higher convenient bond supply or an increase in the autonomous factors on the convenience-maximizing reserve supply. Starting from point A, an increase in convenient bond supply decreases the convenience yield from convenient bonds for each level of reserves, $v_B'(B - R - A)$. At the initial reserve level, the marginal convenience yield from reserves now exceeds that of convenient bonds. Equating the convenience yield on reserves and on convenient bonds requires more reserves as this will increase $v_B'(B - R - A)$ (along its new curve) and decrease $v_R'(R)$. A convenience-maximizing central bank should therefore increase reserves (and thus its convenient bond holdings), but less than one-for-one due to the negative slope of $v_R'(R)$, moving the optimum to point B. We can use Result 1(b) to derive the magnitude of the convenience-maximizing reserve-response to increased convenient bond supply, considering changing R and B such that Result 1(b) still holds and convenience yields are equalized. The reserve-response will depend on how sensitive each of the convenience yields are to changes in their arguments:

$$\frac{dR^{C(b)}}{dB} = \frac{v_B''(B - R - A)}{v_R''(R) + v_B''(B - R - A)} \leq 1.$$

Both $v_B''(B - R - A)$ and $v_R''(R)$ are negative. If $v_R''(R)$ is large in absolute value, the convenience yield on reserves declines fast with additional reserve supply, and the convenience-maximizing reserve increase in response to additional convenient bond supply is more muted. If $v_B''(B - R - A)$ is large in absolute value, the convenience yield on bonds declines a lot with additional convenient bond supply, and the convenience-maximizing reserve increase in response to additional convenient bond supply is larger.

Autonomous factors, $\frac{dR^C}{dA} < 0$: In contrast to the effect of a higher supply of convenient bonds, an increase in the autonomous factors increases the marginal convenience yield from convenient bonds for each level of reserves (because the central bank now holds more convenient bonds, leaving fewer for the private sector). With higher autonomous factors A, the marginal convenience yield from reserves is below that of convenient bonds at the initial reserve level. In response, a convenience-maximizing central bank should decrease reserves (and thus its convenient bond holdings), as illustrated in point C, but less than one-for-one.

Two maturities of convenient bonds

A central bank that supplies reserves via purchases of convenient bonds has access to a range of bond maturities. Consider a case with two bond maturities that each have their own convenience function because they differ in maturity and some investors have preferred habitat demand for very safe long bonds (bond 1) while others have preferred habitat demand for very safe short bonds (bond 2). Using B_1^{priv} and B_2^{priv} to denote the private sector's holdings of each type of convenient bonds, this could be captured as follows:

$$\underset{R,B_1^{cb},B_2^{cb}}{\text{Max}} \ v_R(R) + v_{B_1}\left(B_1^{priv}\right) + v_{B_2}\left(B_2^{priv}\right)$$

$$st. \ B_1^{priv} = B_1 - B_1^{cb}$$

$$B_2^{priv} = B_2 - B_2^{cb}$$

$$B_1^{cb} + B_2^{cb} = R + A$$

Substituting in the constraints:

$$\underset{B_1^{cb},B_2^{cb}}{\text{Max}} \ v_R(B_1^{cb} + B_2^{cb} - A) + v_{B_1}(B_1 - B_1^{cb}) + v_{B_2}(B_2 - B_2^{cb})$$

The first-order conditions are:

$$v_R'(B_1^{cb} + B_2^{cb} - A) = v_{B_1}'(B_1 - B_1^{cb})$$

$$v_R'(B_1^{cb} + B_2^{cb} - A) = v_{B_2}'(B_2 - B_2^{cb})$$

Convenience-maximization thus implies equalizing the convenience yield on reserves, bond 1 and bond 2. For example, if the Federal Reserve supplied reserves with purchases of Treasury bills and Treasury bonds, and it observed that Treasury bills had a lower convenience yield than Treasury bonds, then it could increase the supply of convenience to the private sector by reallocating its portfolio from bonds toward bills. Similarly, if the Federal Reserve observed a higher convenience yield on both bills and bonds than on reserves, it could increase convenience by reducing its reserve supply.

One may wonder whether reserves and bills should enter the same convenience measure, e.g., $v_{short}(R + kB_1^{cb})$ for some constant k. This may be appropriate if reserves and bills were both held by the same investor type. However, d'Avernas and Vandeweyer (2023) argue theoretically and empirically that for high reserve supply and modest bill supply, banks tend to not hold bills which are instead held by investors without access to reserves (shadow banks such as money market funds). The pricing of reserves then disconnects from the pricing of bills, with the spread between reserves and inconvenient assets driven by banks' convenience yield from reserves and the spread between bills and inconvenient assets driven by shadow banks' convenience yield from bills.

The prescription of this sub-section emphasizes the need to think about not only liquidity but also safety effects. Since Treasury bills tend to be more liquid than Treasury notes and bonds, maximizing the overall liquidity services to the private

sector would suggest supplying reserves by buying notes or bonds. However, these have large convenience yields due to safety effects and buying them may be more detrimental to overall convenience supply to the private sector than buying bills! Optimally, the central bank (and the fiscal authority) maximizes convenience supply by equalizing convenience yields on all securities supplied, i.e., reserves, bills and notes/bonds.

What if the ECB supplies reserves via a mix of inconvenient and convenient assets?

I have argued that pre-QE, and thus a feasible option going forward, the ECB supplied reserves via loans to banks. Such loans are unlikely to appeal to safety or liquidity investors (even if collateralized). The ECB's convenience-maximization problem then maps to Result 1(a). How would the ECB's convenience-maximization problem change if, for reasons outside my framework, the ECB decided to supply reserves with a mix of bank lending and government bond purchases?

Suppose the ECB's assets were a pre-set mix of government bonds and loans to banks with weights ω and $1-\omega$ and that bank loans carry no convenience yields (to the ECB or others). Assume that the ECB's government bond holdings were a pre-set mix of the N euro area countries' government bonds. Denote portfolio weights within the government bond portfolio by $\alpha_1, \dots, \alpha_N$, where $\Sigma_{i=1}^N \alpha_i = 1$. Suppose that a subset M of the N countries' government bonds had a convenience yield, and the rest did not. Assume a convenience yield function $v_B\big(B_1^{priv} + k_2 B_2^{priv} + \cdots + k_M B_M^{priv}\big)$, where $M \leq N$. The first-order condition for convenience-maximization would be

$$v_R'(R) = v_B'\big(B_1^{priv} + k_2 B_2^{priv} + \cdots + k_M B_M^{priv}\big) * \omega * (\alpha_1 + k_2 \alpha_2 \dots + k_M \alpha_M)$$

For example, if bond 1 was German bunds, and these were the only ones with a convenience yield, the first-order condition for convenience-maximization would set the convenience yield on reserves equal to the convenience yield on bunds times $\omega \alpha_1$

$$v_R'(R) = v_B'\big(B_1^{priv}\big) * \omega * \alpha_1.$$

4.2 Effects of bank balance sheet costs and bank funding on optimal reserve supply

For most of the post-GFC period, the effective federal funds rate and a host of other short-term rates have been below the interest rate on reserves. From equation (1) this implies that φ must be substantial. If not, competition between banks for raising funds from investors without access to interest-bearing reserves would push up short-term rates to equal the interest rate on reserves plus $v_R'(R)$ which is non-negative. While balance sheet costs are thus clearly relevant for banks, how do we think of these costs from a welfare perspective? Is φ a social cost? If it is, should it be netted out from $v_R'(R)$ when implementing Result 1(a) and Result 1(b)?

Start with the question of whether φ a social cost. From banks' perspective, the balance sheet cost is typically thought to emerge from capital requirements such as the Supplementary Leverage Ratio that requires banks to use a minimum fraction of equity financing when expanding their balance sheets. Equity financing is more expensive for banks because equity, unlike short-term bank debt, does not appeal to safety or liquidity investors. Capital requirements are, in turn, imposed on banks because there is an externality from banks funding themselves with short-term debt.[15] Stepping back and thinking about how regulators set capital requirements, if they are set optimally, the banks' perceived balance sheet cost (the cost of using more equity financing than they would prefer) is set to equal social cost of larger bank balance sheets (from the short-term debt externality). Given this argument, I proceed under the assumption that φ equals the social cost of larger bank assets.

There is a related debate about whether capital relief should be given for reserve holdings given their safe and liquid nature. Both the Federal Reserve and the ECB gave SLR relief for reserves during the COVID crisis but have since ended such relief.[16] Implicitly, this suggests that regulators perceive a social cost of larger bank balance sheets even for reserves. An argument supporting this view is that while a narrow bank that funded reserve holdings with short-term debt would always be able to cover funding outflows with reserves, aggregate reserves must be held by someone. Therefore, an aggregate funding outflow from the overall banking sector (to something other than another central bank liability, notably cash) necessitates either sales of non-reserve assets by banks or by the central bank. This makes SLR relief less attractive from a macro-prudential than a micro-prudential perspective.

I proceed to assume that φ represents a social cost and therefore should be subtracted from $v_R'(R)$ when maximizing convenience supply net of balance sheet costs. The following result emerges. It will hold regardless of how banks fund reserves; I provide the argument for that claim after stating the result.

Result 2 (Convenience-maximizing reserve supply accounting for bank balance sheet costs)

If there is a social cost φ of bank balance sheets, then in competitive markets, regardless of how banks fund reserves:

(a) If a central bank holds assets without convenience yields, then the convenience-maximizing supply of central bank reserves accounting for bank balance sheet costs φ, $\mathrm{R}^{C(a)}$, solves

$$v_R'(R) - \varphi = 0.$$

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15 Stein (2012) explains how this externality can emerge from banks not internalizing the effect of their fire-sales of assets (in response to short-term debt rollover problems) on the value of other banks' assets. This results in too much short-debt debt issuance, too large fire-sale discounts, and a social welfare loss because patient investors who buy in a fire-sale could alternatively invest in new investment projects with positive welfare. The higher the fire-sale discount, the higher the hurdle for such investments, thus leading to underinvestment.

16 By contrast, the Bank of England gives capital relief for reserves.

(b) If the central bank holds bonds (B) that have a convenience yield (to market participants), then the convenience-maximizing supply of central bank reserves accounting for bank balance sheet costs φ, $R^{C(b)}$, solves

$$v_R'(R) - \varphi = v_B'(B - R - A).$$

To see why these conditions hold regardless of how banks fund the reserves, start from the overall welfare generated by reserves, bonds, loans/bank security holdings, and bank deposits:

$$v_R(R, D) + v_{B,banks}(B_{banks}, .) + v_{B,nonbanks}(B_{nonbanks}, .) - \varphi * (R + L) + \lambda(L)$$

where R is reserves, D is deposits, B_{banks} and $B_{nonbanks}$ are holdings by banks and non-banks (including households) of the type of convenient bonds the central bank is purchasing, L is loans (including bank holdings of other types of securities). $R + L$ equal total bank assets which equal total bank liabilities. $\lambda(L)$ denotes the welfare (producers and consumers surplus) from bank loans (above and beyond the balance sheet cost). I note that there is not a separate term for deposits in the welfare expression since φ is the social cost of bank liabilities net of any benefits households may get from holding the deposit part of these liabilities.

By the first welfare theorem, competitive markets will lead to welfare maximization, assuming capital regulation is set optimally so banks bear the full social cost φ of their liability creation. Consider therefore how banks may fund reserve holdings and whether this leads to different conditions for welfare-maximization or not.

Bank can fund reserves by (a) raising more liabilities, (b) reducing lending (or selling other assets than those purchased by the central bank), or (c) selling convenient assets to the central bank.

In case (a) (more bank liabilities), banks' reserve holdings increase, banks liabilities increase, and convenient bond holdings of non-banks decrease. The first-order condition for welfare maximization is:

$$v_R'(R, D) - v_{B,nonbanks}'(B_{nonbanks}, .) - \varphi = 0 \qquad (3)$$

where $v_R'(R, D)$ is the derivative with respect to R for given D. Non-bank balance sheet size is unchanged overall, with lower holdings of convenient bonds, but higher holdings of bank liabilities.

In case (b) (reduced bank lending/securities holdings), banks' reserve holdings increase, banks' lending decreases, and convenient bond holdings of non-banks decrease. The first-order condition for welfare maximization is:

$$v_R'(R, D) - v_{B,nonbanks}'(B_{nonbanks}, .) - \lambda'(L) = 0 \qquad (4)$$

Non-bank balance sheet size is unchanged overall, with lower holdings of convenient bonds, but higher holdings of other assets, purchased from banks. Importantly, $\lambda'(L) = \varphi$, so (4) becomes the same as (3). To see this, observe that in the competitive equilibrium (with r^{Loan} denoting the interest rate on bank loans), households' FOC for borrowing is:

$$r^{Loan} = marginal\ benefit.$$

Banks' FOC for lending is:

$$r^{Loan} = marginal\ cost + \varphi$$

Therefore, $\lambda'(L)(= marginal\ benefit - marginal\ cost) = \varphi$.

In case (c) (banks sell convenient bonds to the central bank), banks' reserve holdings increase, and banks convenient bond holdings decrease. Banks incur no balance sheet costs from additional reserves since their assets and liabilities are unchanged overall. The first-order condition for welfare maximization is:

$$v'_R(R, D) - v'_{B,banks}(B_{banks},.) = 0 \tag{5}$$

For the last unit held, banks and non-banks must value convenient bonds the same. Since banks incur balance sheet costs on all their assets, their bond holdings must in equilibrium be sufficiently small that their convenience yield on the marginal unit held exceeds that of non-banks by φ, so (5) becomes (3) since

$$v'_{B,banks}(B_{banks},.) = v'_{B,nonbanks}(B_{nonbanks},.) + \varphi.$$

Mapping Result 2 to data, I will measure $v'_{B,nonbanks}(B_{nonbanks},.)$ using a spread between inconvenient bonds and convenient bonds that are both held by nonbanks.[17] $v'_R(R) - \varphi$ will be measured using a spread between reserves and an inconvenient security that is a liability for banks (federal funds since banks post-GFC borrow from government sponsored enterprises in this market).

Result 2 implies that we can assess whether welfare is maximized by measuring whether $v'_R(R) - \varphi = 0$ (Result 2,a) or $v'_R(R) - \varphi = v'_B(B - R - A)$ (Result 2,b), regardless of how banks fund reserves. That does not mean that it is irrelevant from a welfare perspective how reserves are financed, but that we can assess welfare-maximization based on the same conditions regardless. From a welfare perspective, if reserves crowd out lending or holdings of securities this affects welfare negatively. If reserves crowd in bank liabilities this may increase welfare to the extent that some of these liabilities have safety and liquidity benefits to their holders (notably deposits), but it may decrease welfare to the extent that bank liabilities then crowd out other investments that buyers of bank liabilities would hold instead.

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[17] A recent literature including Klingler and Sundaresan (2019) and Du, Hebert and Li (2022) compares Treasury yields to yields on interest rate swaps and finds that long-maturity Treasury yields are above swap rates in the post-GFC period. This is reconciled by banks (or dealer banks) being short swaps and long Treasuries, with a spread opening up to cover balance sheet costs. This is different from measurement of $v'_{B,nonbanks}(B_{nonbanks},.)$ in my setting which measures whether bonds purchased by the central bank have convenience yields relative to other assets held by nonbanks, not relative to derivatives.

5 Estimating the convenience-maximizing reserve supply for the US

This section estimates the convenience maximizing reserve supply for the US as of April 2023 based on both Result 2(a) (reserves supplied via inconvenient asset holdings) and Result 2(b) (reserves supplied via convenient asset holdings). The Federal Reserve plans to supply reserves via Treasury holdings over the longer run. My estimates imply that the convenience-maximizing balance sheet size of the Federal Reserve is therefore much smaller than if the Federal Reserve was able (politically) to supply reserves via inconvenient asset holdings. The next section turns to estimates for the ECB for which supply of reserves via inconvenient asset holdings appears preferable from a political perspective.

5.1 Yield spreads

I illustrate the yield inputs to my analysis for the US in Chart 4 which is based on monthly average data. In the top left figure, the red line shows the Effective federal funds rate (EFFR) – IOR spread. This spread is negative over most of the post-GFC sample, except around September 2019 which includes the infamous September 17, 2019, spike in repo rates and in the effective federal funds rate. The EFFR-IOR spread is a measure of $v_R'(.) - \varphi$.[18] We can infer that the balance sheet cost φ must be substantial and at least as large in absolute value as the most negative value of the EFFR-IOR spread. This lowest value is around -20 bps. While the balance sheet cost may vary somewhat over time, to get a sense of the level of $v_R'(.)$, I illustrate EFFR-IOR+20 bps in the top left figure in orange. The line suggests that reserves had a scarcity value of around 25 bps in September 2019, making it less puzzling that yields spiked in the sense that reserves were less plentiful than the EFFR-IOR spread may suggest.

Chart 4, top right, graphs the yield spread between corporate bonds and Treasuries, using long-maturity bonds (dark blue line) or 3-month maturity securities (light blue line). The long-maturity spread is based on Aaa-rated corporate bonds with 20 or more years to maturity and the Treasury yield is the 20-year yield. This spread is large across the sample, much larger than the short maturity spread which is positive but modest over most of the sample. Some of the Aaa-Treasury (and Commercial paper – T-Bill) spread reflects default risk in the corporate securities. I will account for this below by estimating the default component as the asymptote of the Aaa-Treasury spread for large Treasury supply (and thus saturated Treasury demand). The Aaa-Treasury spread is 66 bps in April 2023. I will estimate an asymptote of about 31 bps, thus implying a Treasury convenience yield from this approach of 35 bps for long-maturity Treasuries at the end of the sample. This is substantially larger

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18 This assumes that federal funds are inconvenient. As illustrated with the brown line in Chart 4 top left, the commercial paper-IOR spread is similar to the EFFR-IOR spread over the sample. Both federal funds and commercial paper have a small amount of credit risk and are likely to have only modest appeal to safety-focused investors. To the extent that the EFFR-IOR spread underestimates $v_R'(.) - \varphi$, the convenience-maximizing reserve supply will be larger than what I estimate here.

than the currently negative convenience yield on reserves, net of the balance sheet cost.

Chart 4, bottom left, graphs the EFFR – IOR spread along with the two corporate-Treasury spreads. The EFFR – IOR spread has correlation of -0.04 with the Commercial paper – T-bill spread and a correlation of -0.35 with the Aaa corporate – Treasury spread, consistent with my use of separate convenience yield functions (v) for reserves and bills/bonds.

As an alternative approach to measuring the Treasury convenience yield, one can use the fact that measures of the credit risk in corporate bonds are available in recent years from credit default swap markets. The Treasury convenience yield can then be measures as follows:[19]

(Corporate yield - Treasury yield) - (Credit default swap rate for corporate bonds).

This approach is due to Longstaff, Mithal and Neis (2005) and assumes that a spread position plus CDS protection is a package which is neither liquid nor appeals to safety investors. In recent years one can also adjust the corporate-Treasury yield spread for the fact that some corporate bonds may be callable. This is done by using option-adjusted spreads (OAS) of corporates relative to Treasuries, instead of the raw spread (Corporate yield - Treasury yield). I pursue this alternative approach in Chart 4, bottom right, using OAS and CDS data for investment-grade corporate bonds. The OAS-CDS spread can be constructed for bonds of various maturities. For the OAS on bonds of 3 to 5-year maturity, I subtract the average of the 3-year and 5-year CDS rates. Similarly, for the OAS of bonds of 5 to 7-year maturity (subtracting the average of the 5-year and 7-year CDS rates) and for the OAS of bonds of 7 to 10-year maturity (subtracting the average of the 7-year and 10-year CDS rates). The three OAS-CDS spreads are between 51 and 62 bps in April 2023 and generally similar across maturities. This suggests that Treasuries have a convenience yield not only for long maturities but also down to at least the 3-year maturity.[20] The slightly lower Treasury convenience yield when using Aaa-rated corporate bonds (35 bps in April 2023) than all investment-grade corporate bonds is consistent with Aaa-rated corporate bonds themselves having some convenience yield.

Below, I will focus on the long-maturity Aaa-Treasury spread (adjusted for the asymptote), but this spread is a stand-in for the convenience yield of Treasuries in general, given the apparent lack of term structure of the convenience yield past the 3-year point. Of course, given that the Commercial paper – T-bill spread is lower than the Aaa-Treasury spread (and OAS-CDS spreads), one may wonder whether the Federal Reserve could increase its convenience supply by shifting some of its assets from medium/long maturity Treasuries to Treasury bills, given their lower spreads and lower convenience yields. As noted in section 4.1, for given overall

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19 I do not adjust for the CDS-rate on Treasuries. While it is not zero, it is generally thought to mainly reflect the risk of delayed payments due to debt ceiling negotiations, as opposed to outright default.

20 The weighted average maturity of marketable Treasury debt outstanding is 74 months as of 3/31/2023, according to the US Treasury.

government debt, convenience supply is maximized when convenience yields on bills, bonds and reserves are equalized. Therefore, it is currently the case that the convenience-maximizing reserve supply would be larger with reserves supplied via bills than bonds. Below, I will calculate convenience-maximizing reserve supply for the case with reserves supplied via inconvenient assets and the case with reserves supplied via Treasury bonds. I do not attempt to calculate the intermediate value with reserves supplied via a mix of Treasury bills and bonds. As the Federal Reserve allocated from bonds to bills, the convenience yield on bills would increase from currently close to zero but it is difficult to assess by how much. I leave it to future work to estimate the current Treasury bill demand function to assess this intermediate case.[21]

To estimate convenience-maximizing reserve supply, we need estimates of the functional forms of $v_R'(.) - \varphi$ and $v_T'(.)$. I turn to this next based on two earlier papers of mine, updated with the latest available data.

5.2   Estimating reserve demand

Lopez-Salido and Vissing-Jorgensen (2023) estimate reserve demand for the US over the post-GFC period. They assume a log-linear functional form for $v_R' - \varphi$ as a function of reserves and deposits (where reserves as noted are excess reserves over reserve requirements):

$$v_R'(Reserves, Deposits) - \varphi = a + b * \ln(Excess\ Reserves) + c * \ln(Deposits) + u$$

where u is an unobserved reserve demand shock. Using the effective federal funds rate for the short market rate r, the reserve demand from equation (1) then takes the form[22]

$$EFFR - IOR = a + b * \ln(Excess\ Reserves) + c * \ln(Deposits) + u. \quad (6)$$

LS-VJ instrument excess reserves with the sum of reserves and take-up at the overnight reverse repo (ONRRP) facility. This is needed because a shock to reserve demand affects the split between reserves and ONRRP but not the sum which thus serves as a useful instrument. The reduced form of the IV estimation regresses the EFFR-IOR spread directly on the exogenous variables

$$EFFR - IOR = A_R + B_R * \ln(Reserves + ONRRP) + C_R * \ln(Deposits) + U. \quad (7)$$

LS-VJ emphasize the need to control for deposits to achieve a stable reserve demand function because of strong growth in deposits over the post-GFC sample.[23]

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21 From the fact that the Federal Reserve has stated its intent to not hold MBS in the longer-run, I infer that such holdings are also politically sensitive and do not explore backing reserves with MBS. Government-backed MBS are generally thought to have some convenience yield but one that is lower than Treasuries. He and Song (2022) estimate an MBS convenience yield about half as large as that on Treasuries.

22 Expressing reserve demand as a function of reserve demand drivers, this corresponds to a reserve demand function given by $Excess\ Reserves = \alpha\ Deposits^\beta e^{\gamma*[EFFR-IOR]}\varepsilon$, where $\alpha = e^{-a/b}$, $\beta = -c/b$, $\gamma=1/b$, and $\varepsilon = e^{-u/b}$. LS-VJ use total bank deposits (of all commercial banks in the US), but results are similar when using only liquid deposits.

Chart 5, Panel A, left illustrates the expected negative correlation between EFFR-IOR and excess reserve supply, consistent with a downward sloping demand curve. However, the relation is unstable as the EFFR-IOR spread is much larger in September 2019 than it was the last time excess reserves had a similar value to that in this month. Accounting for growing deposits reconciles this instability and results in a stable reserve demand function. Updating LS-VJ's estimates to cover the period 2009M1-2023M4 results in $\widehat{A_R} = -2.186, \widehat{B_R} = -0.172, \widehat{C_R} = 0.367$, with all parameter estimates significant at the 1% level (accounting for autocorrelation up to monthly 12 lags). The fit is good with an R^2 of 0.89. Reorganizing (7), they express the EFFR-IOR spread as a function of the quantity of Reserves+ONRRP, adjusted for the demand shifter (deposits) as follows

$$EFFR - IOR = A_R + B_R * \left[\ln(Reserves + ONRRP) + \frac{C_R}{B_R} * \ln(Deposits)\right] + U. \quad (8)$$

Chart 5, Panel A, right graph, shows this relation. I graph the EFFR-IOR spread and its fitted value against the deposit-adjusted reserve+ONRRP supply, i.e., the exponential of $\left[\ln(Reserves + ONRRP) + \frac{C_R}{B_R} * \ln(Deposits)\right]$. I take the exponential to emphasize that reserve demand is a convex function of reserves+ONRRP. The fitted line is the empirical version of $v'_R(.) - \varphi$, expressed as a function of its exogenous drivers.

5.3   Estimating Treasury demand

Krishnamurthy and Vissing-Jorgensen (2012) estimate v'_T(Treasuries/GDP) by relating the Aaa-Treasury spread to Treasuries/GDP. They consider both a log-linear functional form and a piecewise-linear functional form with a horizontal asymptote. The latter allows for estimation of the (typical) credit risk component of the spread as the horizontal asymptote as Treasury supply gets very large. An asymptote can also be imposed when using a log-linear functional form and I follow that approach here so both the reserve demand and Treasury demand estimations are based on log-linear functional forms.[24]

KVJ's estimate of $v'_T(\frac{Treasuries}{GDP})$ is an estimate of the aggregate demand for Treasuries. What is needed for convenience-maximization is the relation between the Aaa-Treasury spread and private holdings of Treasuries to GDP,

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23 See Lopez-Salido and Vissing-Jorgensen (2023) for results that instrument for deposits. Results do not change much so, for simplicity, I do not instrument for deposits here. What matters is controlling for deposits, but not instrumenting for deposits.

24 One could also impose an asymptote on the estimation of reserve demand. It would capture the default risk in federal funds, minus φ. It could potentially result in a better fit if reserve demand becomes fully satiated over the estimation period. I have estimated EFFR-IOR=$\max(A_R + B_R * \ln(Reserves + ONRRP) + C_R * \ln(Deposits), D_R) + U$. The fit is even better than for equation (7) with an R^2 of 0.97. Estimates of A, B and C do not differ much and the lower bound D (which is estimated to be -16 bps) only binds in 10 months all in 2013 and 2014. Accounting for potential saturation of reserve demand therefore does not affect convenience-maximization results substantially.

$v'_T(\frac{Treasuries^{Private}}{GDP})$.[25] The two are not the same. For example, suppose the Federal Reserve purchases a lot of Treasuries for macroeconomic reasons unrelated to Treasury convenience. This will appear as an aggregate shock to the demand for Treasuries and will shift v'_T(Treasuries/GDP) to the right. By contrast, the function $v'_T(\frac{Treasuries^{Private}}{GDP})$ would not be directly affected by the Federal Reserve's purchases but the input $\frac{Treasuries^{Private}}{GDP}$ would be reduced by larger holdings of the Federal Reserve. I illustrate these patterns in Chart 5, Panel B. The top left graph shows the main figure from KVJ relating the Aaa-Treasury spread to the total supply of Treasury debt relative to GDP. The top right graph adds data for 2009-2023. The new data points lie far to the right of the earlier relation, partly due to large Federal Reserve purchases over the later period. As shown in the bottom left graph, $v'_T(\frac{Treasuries^{Private}}{GDP})$ appears somewhat more stable over time, though the post-GFC points still plot somewhat to the right of the pre-GFC relation, suggesting that even $v'_T(\frac{Treasuries^{Private}}{GDP})$ may have shifted to the right. The rightward shift of $v'_T(\frac{Treasuries^{Private}}{GDP})$ is mainly due to larger Treasury holding by foreigners, official and private (remember that "private" refers to anyone but the Federal Reserve). This is clear from the bottom right figure in which I have subtracted both Federal Reserve and foreign Treasury holdings on the horizontal axis.

The magnitude of Federal Reserve plus foreign holdings of Treasuries is shown in Chart 5, Panel C, left. Federal Reserve plus foreign holdings of Treasuries grew from 17% of GDP in 2007 to 59% in 2021 before falling somewhat by 2023. As a result, Treasury holdings to GDP for other sectors grew less dramatically than total Treasury debt to GDP, illustrated in the right graph.

With only aggregate data to work with, it is not possible to statistically estimate precisely how the $v'_T(\frac{Treasuries^{Private}}{GDP})$ function (mapping to Chart 5, Panel B, bottom left) may have changed over time. I assume the reason that the post-GFC points plot further to the right is due to a rightward shift in the relation driven by shocks to foreign demand (or that of other non-Fed sectors), with no change in the default component of the spread (the asymptote).

Using annual data for 1919-2023, I estimate the following relation using non-linear least squares:[26]

$$y^{Aaa} - y^{Treasury}$$

$$= \max \left(A_T + B_T * ln\left(\frac{Treasuries^{Private}}{GDP}\right) + \sum_{i=2009}^{2023}\beta_i D(year = i), C_T\right) + U. \qquad (9)$$

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25 From section 3.2, what is needed is the convenience yield function for the non-bank part of the private sector. I use the Treasury holdings of the overall private sector because the split between banks and non-banks may be correlated with the error term. One can think of this approach as reduced form of an instrumental variable approach where non-bank Treasury holdings are instrumented by overall private sector holdings.

26 The regression uses fiscal year-end data, September in recent years. Data for $\frac{Treasuries^{Private}}{GDP}$ is not yet available for September 2023. For 2023, I use the value of $\frac{Treasuries^{Private}}{GDP}$ as of the end of 2023Q1 which is 0.759 and the average Aaa-Treasury spread in April 2023 which is 0.658.

The max operator accounts for saturation of Treasury demand, with C_T the estimated default component of the spread. By including year dummies for 2009-2023 (thus perfectly matching data for these years), I capture the rightward shift post-GFC. This results in $\widehat{A_T} = -0.219$, $\widehat{B_T} = -0.933$, $\widehat{C_T} = 0.306$ and $\hat{\beta}_{2023} = 0.620$, with $R^2 = 0.908$ ($R^2 = 0.889$ even on the original sample from 1919-2008 where no data points are fitted perfectly by construction). Accounting for autocorrelation up to 10 annual lags, $\widehat{B_T}$, $\widehat{C_T}$ and $\hat{\beta}_{2023}$ are significant at the 1% level while $\widehat{A_T}$ is not significant.

I note that by estimating the Treasury convenience yield from the Aaa-Treasury spread rather than the Baa-Treasury spread or OAS-CDS measures, my approach is conservative in assessing the magnitude of the Treasury convenience yield. To the extent the Treasury convenience yield is larger than what I estimate, this would imply a lower convenience-maximizing reserves supply than what I estimate below (for the case with reserves backed with Treasuries).

Working in the other direction, if the Federal Reserve was to lend out most of the Treasuries it held, this would return some of the liquidity services of those Treasuries. Consider three possibilities: (1) If the Federal Reserve lent Treasuries against cash (like in the ONRRP facility), this would reduce reserves, with an ambiguous effect on overall convenience supply. (2) If the Federal Reserve lent Treasuries out against other less special Treasuries (as in the SOMA Securities Lending Program), this could result on some increase in convenience supply. (3) It is unclear whether the Federal Reserve could -- politically -- lend out large amounts of Treasuries against non-cash non-Treasury collateral. Which collateral would be accepted, on what terms, and how large benefits would issuers of such securities get? This would raise many of the same issues that makes it difficult for the Federal Reserve to hold non-Treasury assets directly. Furthermore, regardless of the collateral posted, securities lending would not transfer the safety benefits of the Treasuries borrowed to those borrowing them (since ownership and principal/coupon payments would remain with the Federal Reserve). Therefore, the convenience-maximizing reserve supply would still be lower with reserves supplied via Treasury purchases with Treasury lending than via inconvenient assets.[27] [28]

5.4   Convenience-maximizing reserve supply

Reserves supplied via inconvenient assets

Chart 6, Panel A shows what the convenience-maximizing Reserve+ONRRP supply for the US would be as of April 2023, if the Federal Reserve supplied reserves with

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[27] See Roh (2022) for an analysis of securities lending in the context of QE. Roh shows that for both the Federal Reserve and the ECB, on average less than 2% of bonds held during government bond QE have been lent out.

[28] One could use the (default-adjusted) Baa-Aaa spread as a lower bound measure of the safety component of the Treasury convenience yield (see Krishnamurthy and Vissing-Jorgensen (2012)) and estimate the convenience-maximizing reserve supply under the assumption that 100% of Federal Reserve Treasury holdings were lent out against inconvenient collateral. I leave this for future work as this strategy is unlikely to be politically feasible currently.

inconvenient assets. I graph the fitted value for the EFFR-IOR spread based on equation (7)

$$A_R + B_R * ln(Reserves + ONRRP) + C_R * ln(Deposits)$$

with $\widehat{A_R} = -2.186, \widehat{B_R} = -0.172, \widehat{C_R} = 0.367$, and using the value of deposits for April 2023 which is \$17.190T. A value of Reserves+ONRRP of \$3.257T sets the predicted spread, and thus the predicted value of $v'_R(R, .) - \varphi$ to zero. The gray shaded area in the figure indicates the range of data over which the regression was estimated. Chart 6, Panel B graphs the time series evolution of the convenience-maximizing Reserve+ONRRP value in this case (reserves and ONRRP supplied via inconvenient assets). The value grows over time as deposits grow. The actual Reserve+ONRRP supply exceeded the convenience-maximizing Reserve+ONRRP value throughout the sample, except for a period in 2019 and early 2020.

These calculations do not account for the fact that deposits may themselves be a function of Reserve+ONRRP supply. A more ambitious approach would model this dependence. To the extent that a smaller balance sheet size causes lower deposits, the convenience-maximizing Reserve+ONRRP supply will be lower than calculated here. This effect is likely to be modest given the estimates of Diamond, Jiang and Ma (2022) that only 7 cents of a dollar of extra reserves is funded with additional deposits.

Reserves supplied via Treasury holdings

Chart 7 turns to the more realistic case for the US in which reserves are supplied via Federal Reserve holdings of Treasuries. In Chart 7, Panel A, I graph in red the estimated convenience yield of reserves as a function of Reserves+ONRRP as of April 2023 (this is the same line as the one in Chart 6, Panel A). In blue, I graph the estimated convenience yield on long Treasuries as a function of the private sector's holdings of Treasuries as of April 2023. This is based on the estimated version of equation (9). Specifically, the blue line is

$$\max \left(A_T + B_T * ln \left(\frac{Treasuries^{Private}}{GDP} \right) + \beta_{2023} - C_T, 0 \right)$$

with $\widehat{A_T} = -0.219$, $\widehat{B_T} = -0.933$, $\widehat{C_T} = 0.306$, $\hat{\beta}_{2023} = 0.623$, and using GDP for the four quarters 2022Q2-2023Q1 of \$25,899T. It is apparent that the demand for Treasuries is much larger than the demand for reserves in the sense that Treasuries remain convenient for much larger supply (presumably because a much wider set of entities can hold Treasuries than can hold reserves and because Treasuries are available for many maturities). Private long-maturity Treasury demand is saturated for a total Treasury supply (minus Federal Reserve holdings) a bit below \$30T.

If Chart 7, Panel A, point A on the red line indicates the predicted EFFR-IOR spread in the reserve market, given current reserves+ONRRP supply as of April 2023 of \$5.554T. Point A on the blue line indicates what the predicted Aaa-Treasury spread would be, if the Federal Reserve exclusively held Treasuries, so $Treasuries^{Private} = Treasuries - Treasuries^{Federal\ Reserve}$. Current total Treasury supply is \$24.614T (end of 2023Q1 from the Z1 release, L.210, line 1 minus line 10). Federal Reserve

holdings are $4.961T, but they would be $8.601T if the Federal Reserve held only Treasuries (as current MBS holdings are large). This would result in private Treasury holdings of $16.013T at point A on the blue line. The black vertical line indicates current $Treasuries^{Private}$ given that the Federal Reserve currently holds a mix of Treasuries and MBS, unlike in its longer-run plans.

Since point A on the red line is a lot lower than point A on the blue line, convenience-maximization implies lowering the supply of reserves+ONRRP thereby increasing $Treasuries^{Private}$ correspondingly. Point B on the red and blue lines illustrate the resulting outcome. Convenience yields are now equalized at 29 bps. This happens for reserves+ONRRP=$593B, resulting in private Treasury holdings of $20,974T. The convenience-maximizing supply of reserves+ONRRP is calculated as the value x that solves

$$\widehat{A_R} + \widehat{B_R} * ln(x) + \widehat{C_R} * ln(Deposits)$$

$$=\max\left(\widehat{A_T} + \widehat{B_T} * ln\left(\frac{Treasuries-[x+Autonomous\ factors]}{GDP}\right) + \hat{\beta}_{2023} - \widehat{C_T}, 0\right)$$

given the values of deposits, Treasuries, autonomous factors, and GDP (with April 2023 data for deposits and autonomous factors, end of 2023Q1 data for Treasuries, and 2022Q2-2023Q1 data for GDP). Chart 7, Panel B, illustrates how the convenience yield on reserves (net of balance sheet costs) and on Treasuries change with the supply of reserves+ONRRP. Chart 7, Panel C, illustrates the time-series evolution of the convenience-maximizing supply of reserves+ONRRP. The value is tiny in early years because of high Treasury convenience yields when Debt/GDP was lower. In recent years, the series fluctuates due to fluctuations in the four inputs (deposits, Treasuries, autonomous factors, and GDP) and due to shifts in the Treasury convenience yield curve which, as explained above, is assumed to shift left/right to perfectly fit the points from 2009-2023.

6 Estimating the convenience-maximizing reserve supply for the euro area

This section repeats the convenience-maximization exercise for the euro area. A key difference will be that, because it appears politically feasible for the ECB to provide reserves via inconvenient asset holdings (bank loans, typically collateralized with inconvenient assets), I will mainly focus estimation of the ECB's convenience-maximizing reserve supply on this case.

6.1 Yield spreads

Chart 8 provides information on yield spreads for the euro area. The top left figure shows the spread between measures of banks' (inconvenient) funding costs and the ECB's deposit facility rate. These spreads are similar to the EFFR-IOR spread for the US. The EONIA-DFR spread is just over 100 bps up to the financial crisis, a reminder that the ECB differed from the Federal Reserve over this period by not

setting the interest rate on reserves to zero and instead operating with a reserve scarcity of around 100 bps. The ECB moved to less scarce reserves around the start of the financial crisis. There is a spike in EONIA-DFR around the European sovereign debt crisis, likely related to bank default-risk. Rather than trying to estimate this risk, I will estimate reserve demand for the euro area for the 2013M1-2023M4 period.

The ECB started publishing data on the euro short-term rate (ESTR) in October 2019. ESTR includes transactions between banks and non-banks. EONIA was discontinued in January 2022 and, from October 2019 onward, EONIA was set at a fixed spread of 8.5 bps over ESTR (by the administrator of EONIA). I assume that had ESTR been quoted in earlier years it would have been 8.5 bps below EONIA. By this measure, ESTR (directly measured or assumed 8.5 bps below EONIA) – DFR has been negative about 80% of the time period since 2013M1. This series is shown in red in Chart 8, bottom left.

Chart 8, top right, provides measures of the convenience yield on German government bonds, measured relative to euro-denominated KfW bonds. The KfW bonds are real estate-related bonds that have government guarantees. They may thus appeal to safety-investors. Therefore, the KfW-Bund spreads likely understate the convenience yield on bunds, capturing mainly the liquidity part. That said, the KfW-Bund spreads are substantial, more so for longer bonds as in the case of US Treasuries. At the 10-year maturity, the KfW-Bund spread is 60 bps as of April 2023, while it is 30 bps at the 2-year maturity. Chart 8, bottom right shows that positive convenience yields in the euro area is not entirely unique to German government bonds. As an example, 10-year Dutch government bonds also carry a convenience yield, while the convenience yield on 10-year French government bonds is slightly positive in some recent years.

The combination of a negative ESTR-DFR spread and positive KfW-Government bond spreads for Germany and (to a lesser extent) a few other euro area countries suggests that, from a convenience-maximization perspective, the ECB's asset choice matters. The convenience-maximizing ECB reserve supply is larger if the ECB supplies reserves exclusively via inconvenient assets.

6.2   Estimating reserve demand

Using data for 2013M1-2023M4, I repeat the estimation from Lopez-Salido and Vissing-Jorgensen (2023) using data from the euro area.

$$ESTR - DFR = a + b * \ln(Excess\ Liquidity) + c * \ln(Deposits) + u \qquad (10)$$

$$= a + b * [\ln(Excess\ Liquidity) + \frac{c}{b} * \ln(Deposits)] + u$$

where I measure excess liquidity (ECB terminology for excess reserves) as banks' total holdings with the ECB (current account holdings+deposit facility holdings) minus required reserves. The variable $[\ln(Excess\ Liquidity) + \frac{c}{b} * \ln(Deposits)]$ is the ECB's deposit-adjusted excess liquidity supply in logs. I note that since the ECB

does not operate an ONRRP facility, the issue of endogeneity of reserves in response to reserve demand shocks does not emerge. Therefore, I do not instrument for excess liquidity (nor deposits).

For the euro area, the fit is slightly better when controlling for overnight deposits rather than total deposits, so I use overnight deposits. I estimate $\hat{a} = -0.428$, $\hat{b} = -0.064$, and $\hat{c} = 0.097$. a is significant at the 10% level, while b and c are significant at the 1% level, accounting for autocorrelation up to 12 monthly lags. The R^2 is 0.87.

Chart 9, top right, graphs the ESTR-DFR spread and the fitted value against the deposit-adjusted excess liquidity supply. The fit is good, though one could experiment with other functional forms to get more curvature for low deposit-adjusted excess liquidity supply.

6.3 Convenience-maximizing reserve supply

Reserves supplied via inconvenient assets

Chart 10, Panel A estimates the convenience-maximizing liquidity supply as of April 2023, given overnight deposits of € 9.379T and assuming that liquidity is supplied via inconvenient asset holdings. I estimate a value of excess liquidity of € 1.251T. Adding current required reserves of € 165B gives a convenience-maximizing liquidity estimate of € 1.416T.

This estimate is likely somewhat higher than the true convenience-maximizing value since the blue fitted line in Chart 9, top right, is a bit to the right of the data around a spread of zero. Again, working on the functional form more would improve the accuracy of the estimate.

Chart 10, Panel B, left shows in blue how the estimated convenience-maximizing excess liquidity has evolved since the start of 2013. The estimate increases from under € 400B in 2013M1 to around € 1.25T in 2023M4. The time-series variation in the estimate is driven by the time-series variation in reserve demand due to changing overnight deposits, shown in Chart 10, Panel B, right in bright green. These increase from € 4.3T in 2013M1 to € 9.4T in 2023M4. By contrast, omitting ln(Deposits) from the regression in (10) would incorrectly lead to an estimated convenience-maximizing excess liquidity of around € 600B for all years.

In the case of the euro area, omitting ln(Deposits) from the regression in (10) has only a modest effect on the R^2. This is likely due to ln(Excess liquidity) and ln(Deposits) being highly correlated over the 2013M1-2023M4 sample, with a correlation of 0.94.[29] However, as discussed, omitting ln(Deposits) leads one to mis the fact that the convenience-maximizing excess liquidity grows over time with the size of the banking sector. While I have focused my estimations on the Federal Reserve and the euro area, Bank of England (2023) implements the LS-VJ reserve

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[29] This compares to a correlation of 0.86 of ln(Reserves+ONRRP) and ln(Deposits) for the US over the 2009M1-2023M4 period.

demand estimation for the UK and also find an important role for deposits. For the UK, deposit growth from 2018 to 2022 leads to sharply rising estimates over time in the value of reserves at which reserve scarcity turns positive.

Reserves supplied via convenient assets

Since the ECB can supply reserves via inconvenient assets, it is less relevant to calculate the convenience-maximizing reserve supply for the euro area if reserves were to be supplied mainly via holdings of government bonds. Furthermore, attempting to estimate this number is complicated by the fact that we do not have a long time series of data with which to estimate the convenience yield function for euro-denominated convenient government bonds, notably Bunds (their convenience yield may have changed with the introduction of the euro). Government bond supplies move slowly and there is little QE-induced variation in the supply of bunds to the private sector given that the ECB started QE only in 2015.

That said, since I was asked for this number, here is a back-of-the-envelope attempt at calculating the convenience-maximizing reserve supply for the euro area with reserves supplied via government bond purchases.

Suppose the ECB supplied reserves solely via holdings of euro area government bonds. Then, from section 4.1, and accounting for balance sheet costs, the convenience-maximizing reserve supply solves

$$v'_R(R) - \varphi = v'_B\left(B_1^{priv} + k_2 B_2^{priv} + \cdots + k_M B_M^{priv}\right) * \omega * (\alpha_1 + k_2\alpha_2 \ldots + k_M\alpha_M)$$

where I set the weight for government bonds $\omega = 1$ for this hypothetical scenario. Suppose the ECB purchased government bonds in proportion to the ECB capital key. This key is 21.4% for Germany. Suppose as an approximation that only German bonds have convenience yields. Letting Germany be country 1 in the formula above, we have

$$v'_R(R) - \varphi = v'_B\left(B_1^{priv}\right) * \alpha_1 \tag{12}$$

with $\alpha_1 = 0.214$. From Chart 8, top right, the convenience yields on bunds vary by horizon, but as an average suppose it is around 40 bps. Then $v'_B\left(B_1^{priv}\right) * \alpha_1$ is around 8 bps. Therefore, the value of $v'_R(R) - \varphi$, proxied by the ESTR-DFR spread, that correspond to convenience-maximization in this scenario is at most 8 bps. Why at most? Consider Chart 7, Panel A as applied to this case. The blue line would be $v'_B\left(B_1^{priv}\right) * \alpha_1$. If point A on the blue line was at 8 bps, then the points B on the red and blue lines are below 8 bps. Point B on the red line, corresponding to a spread of 8 bps based on the reserve demand estimation for the euro area, can be calculate from the red line in Chart 10, Panel A. It solves

$$0.08 = a + b * \ln(Excess\ Liquidity) + c * \ln(Deposits)$$

using current overnight deposits of € 9.4T. The resulting value for excess liquidity is € 356B. Adding required reserves of € 165B, we get € 521B (of which € 111B in bunds). That is an underestimate since the convenience yield on bunds would decline as the ECB's bund holdings were reduced from the current level of about €

750B. Therefore, the convenience-maximizing liquidity supply for the euro area for this case equalizes $v_R'(R) - \varphi$ and $v_B'\left(B_1^{priv}\right) * \alpha_1$ at a number below 8 bps, corresponding to a liquidity supply above € 521B but below the value of € 1.4T from the baseline case where the ECB supplies reserves via inconvenient assets.

7   Fiscal effects of balance sheet size and composition via convenience yields

This section clarifies that maximizing convenience supply is not the same as maximizing the seigniorage generated from convenience yields. The latter ignores the consumer's surplus from reserves and from any convenient assets used to supply them. Maximizing seigniorage is thus different from maximizing welfare.

7.1   Central bank seigniorage and fiscal authority seigniorage

Consider the case where central bank liabilities are supplied via purchases of government bonds with a convenience yield. I will use Treasuries (T) for concreteness. The central bank's profits are

$$Central\ bank\ profits = \tilde{r}^T T^{cb} - \tilde{r}^R R - \tilde{r}^A A$$

where $\tilde{r}$ denotes realized returns, R denotes reserves, and A is the autonomous factors. Rewriting each of the three returns as the return on the inconvenient asset with the same maturity, minus the convenience yield, we get

$$Central\ bank\ profits$$

$$= \left(\tilde{r}^{Inconv,T} - v_T'(T^{priv})\right)T^{cb} - \left(\tilde{r}^{Inconv,short} - v_R'(R)\right)R - \left(\tilde{r}^{Inconv,short} - v_A'(A)\right)$$

$$= \left[\left(\tilde{r}^{Inconv,T} - \tilde{r}^{Inconv,short}\right)(R+A)\right] + \left[v_R'(R)R + v_A{}'(A)A - v_T'(T^{priv})T^{cb}\right] \quad (13)$$

where "Inconv, T" refers to the inconvenient asset of the same maturity as T^{cb} and I have used $T^{cb} = R + A$.[30] The central bank's portfolio thus has both duration risk and alpha. The profit is the sum of:

(1) The return on a portfolio of inconvenient bonds funded with short-term inconvenient debt and of size R+A (the first term in (13)).

(2) The central bank's "alpha" in dollar terms. This stems from convenience yields on liabilities as well as assets (the second term in (13)).

The alpha term, (2), is the central bank's seigniorage. It measures the net convenience yield the central bank earns, accounting for both the benefit to the central bank of being able to issue liabilities with lower interest rates than similar-

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30 If T, R and A appeal the same investors, one can link the convenience yields, as discussed in section 4.1 above.

maturity inconvenient liabilities, and the loss to the central bank from holding government bonds earning a lower return than similar-maturity inconvenient assets (if government bonds have a convenience yield). This term expands on the concept of seigniorage from cash, $r^{Inconv,short}M$, which is the value to the central bank from funding its assets with non-interest-bearing money. If the autonomous factors consisted only of currency, then $v_A'(A)A = r^{Inconv,short}M$, since $A = M$ and $v_A'(A) = r^{Inconv,short} - r^A = r^{Inconv,short}$ (from currency holder's first-order condition and $r^A = 0$). This simply states that households/firms hold cash to the point where their marginal convenience value from cash equals the foregone nominal interest rate $r^{Inconv,short}$ on a short-maturity inconvenient asset without money-like properties. The central bank's profits due to convenience yields is the generalization of the concept of seigniorage, accounting for all parts of the central bank's balance sheet.

$$Central\ bank\ seigniorage = v_R'(R)R + v_A'(A)A - v_T'(T^{priv})T^{cb}$$

To understand the effect of central bank balance sheet size on seigniorage, it is important to account for the fact that the fiscal authority earns its own seigniorage if it can issue debt that carries a convenience yield.

$$Fiscal\ authority\ seigniorage = v_T'(T^{priv})T$$

where $T^{priv} = T - R - A$ is the private sector's holdings of government bonds. If the central bank supplies liabilities via government bond purchases, then the consolidated government's seigniorage (the sum of central bank and fiscal authority seigniorage) is

$$Seigniorage = v_T'(T^{priv})T^{priv} + v_R'(R)R + v_A'(A)A.$$

In Chart 3, Panel B, Treasury seigniorage $v_T'(T^{priv})T^{priv}$ is the area of the rectangle indicating the producer's surplus from government bonds. Larger central bank holdings of Treasuries lower private sector holdings $T^{priv} = T - R - A$, which makes the rectangle narrower, but increase Treasury scarcity $v_T'(T^{priv})$, which makes the rectangle taller). Similarly, in Chart 3, Panel A, reserve seigniorage $v_R'(R)R$ is the producer's surplus rectangle.

A central bank that maximizes welfare would focus on convenience-maximization as laid out in Result 1 and 2. These results set reserve supply to maximize the sum of the producer's and consumer's surplus from reserves (Result 1(a), 2(a)) or from both reserves and Treasuries (Results 1(b), 2(b)). From the perspective of taxpayers, if it is feasible to impose lump sum taxes on banks who get the consumer's surplus from reserves and investors who get the consumer's surplus from Treasuries, then even a taxpayer narrowly interested in central bank profits and tax revenues would be better off with convenience-maximization plus taxation than with seigniorage-maximization. If taxing the consumer's surplus from reserves and Treasuries is not feasible, a narrowly focused taxpayer would want to set R to maximize seigniorage (from reserves and Treasuries combined). This is clearly not the same as setting R to maximize convenience supply, as maximizing seigniorage disregards the consumers surplus from reserves and Treasuries.

7.2   How do the reserve supplies from convenience maximization and seigniorage maximization compare?

To get a sense of how the reserve supply differs between convenience-maximization and seigniorage maximization, this sub-section provides a few analytical results.

For simplicity, when answering this, I will subtract φR in the seigniorage maximization problem. This assumes that the fiscal authority fully internalizes these costs, for example because it can tax banks less when more costs are incurred (this is an approximation but keeps the math more tractable).

In the case where the central bank only holds assets without a convenience yield ($T^{cb} = 0$), consolidated government seigniorage is

$$Seigniorage = v_T'(T)T + [v_R'(R) - \varphi]R + v_A'(A)A.$$

Assuming T and A are not affected by the chosen reserve supply, the reserve value that maximizes the consolidated government seigniorage is the value that maximizes reserve seigniorage (producer's surplus from reserves):

$$\underset{R}{\text{Max}} \; [v_R'(R) - \varphi]R$$

In the case where the central bank holds convenient assets ($T^{cb} > 0$),

$$Seigniorage = v_T'(T - R - A)(T - R - A) + [v_R'(R) - \varphi]R + v_A'(A)A$$

Assuming T and A are again not affected by the chosen reserve supply, the reserve value that maximizes the consolidated government's seigniorage maximizes the sum of seigniorage from Treasuries and reserves

$$\underset{R}{\text{Max}} \; v_T'(T - R - A)(T - R - A) + [v_R'(R) - \varphi]R$$

Chart 11, Panel A, graphs reserve seigniorage against reserve supply. Reserve seigniorage (net of φ) is zero for $R = 0$ and for the value of R that sets $v_R'(R) - \varphi = 0$, but positive for values of R between these two values. Similarly, Chart 11, Panel B, illustrates Treasury seigniorage $v_T'(T^{priv})T^{priv}$ as a function of private sector Treasury holdings $T^{priv} = T - R - A$.

From the first-order conditions, in each of the two cases, we have the following result.

Result 3 (Seigniorage-maximizing reserve supply).

(a) If the central bank only holds assets without convenience yields, then:

(a,i) The supply of central bank reserves $\mathrm{R}^{S(a)}$ that maximizes seigniorage for the consolidated government budget solves

$$[v_R'(R) - \varphi] + v_R''(R)R = 0. \tag{14}$$

(a,ii) The seigniorage-maximizing reserve value is below the convenience-maximizing value (that both account for φ) iff $v_R''(R)R < 0$ evaluated at the convenience-maximizing reserve supply $\mathrm{R}^{\mathrm{C}(a)}$.

(b) If the central bank holds bonds (B) that have a convenience yield (to the private sector), then:

(b,i) The supply of central bank reserves $\mathrm{R}^{\mathrm{S}(b)}$ that maximizes seigniorage for the consolidated government budget solves

$$[v_R'(R) - \varphi + v_R''(R)R] - [v_T'(T^{priv}) + v_T''(T^{priv})T^{priv}] = 0 \qquad (15)$$

with $T^{priv} = T - R - A$.

(b,ii) The seigniorage-maximizing reserve supply exceeds the convenience-maximizing value (that both account for φ) iff $v_R''(R)R - v_T''(T^{priv})T^{priv} > 0$ evaluated at the convenience-maximizing reserve supply $\mathrm{R}^{\mathrm{C}(b)}$.

(c) If $v_T'(T - R^{S(a)} - A) + v_T''(T - R^{S(a)} - A)(T - R^{S(a)} - A) < 0$, then a higher level of reserve plus government bond seigniorage is feasible if the central bank supplies reserves via government bonds than inconvenient assets, and the seigniorage-maximizing reserve value exceeds $\mathrm{R}^{\mathrm{S}(a)}$.

In Result 3(a,i), when the central bank only holds assets without convenience yields, the seigniorage-maximizing reserve supply maximizes the area $[v_R'(R) - \varphi]R$ of the grey producers' surplus rectangle in Chart 3, Panel A. This reserve value leads to the peak value for reserve seigniorage in Chart 11, Panel A. Result 3(a,ii) follows from equation (14) since $v_R'(R) - \varphi = 0$ at the convenience-maximizing reserve supply. Therefore, if $v_R''(R)R < 0$ at $\mathrm{R}^{\mathrm{C}(a)}$ then $\mathrm{R}^{\mathrm{C}(a)} < \mathrm{R}^{\mathrm{S}(a)}$. This is the case when reserve demand is downward sloping at $\mathrm{R}^{\mathrm{C}(a)}$ as illustrated in Chart 3, Panel A. Maximizing seigniorage then involves a lower reserve supply to increase the producer's surplus.

In Result 3(b,i), the seigniorage-maximizing reserve supply maximizes reserves seigniorage (producer's surplus) in Chart 3, Panel A, plus government bond seigniorage (producer's surplus) in Chart 3, Panel B. In equation (15), the first term, $[v_R'(R) - \varphi + v_R''(R)R]$, is the change in reserve seigniorage from an increase in reserves, i.e., the slope of the reserve-seigniorage curve in Chart 11, Panel A. The second term, $-[v_T'(T^{priv}) + v_T''(T^{priv})T^{priv}]$ is the change in government bond seigniorage from an increase in reserves leading to a corresponding decrease in T^{priv}. $[v_T'(T^{priv}) + v_T''(T^{priv})T^{priv}]$ is the slope of the government bond-seigniorage curve in Chart 11, Panel B. The seigniorage-maximizing reserve supply in Result 3(b,i) equates the two seigniorage curve slopes, as illustrated in Chart 11. Result 3(b,ii) follows from (15) and the fact that $v_R'(R) - \varphi = v_T'(T^{priv})$ at the convenience-maximizing reserve supply. In terms of the rectangles for producers' surplus in Chart 3, the intuition for Result 3(b,ii) is as follows.

Adding $1 of reserves makes the reserve seigniorage (producers' surplus) rectangle in Chart 3, Panel A wider (generating a marginal reserve convenience yield of $v_R'(R) - \varphi$) but less tall (lowering the marginal reserve convenience yield and thus

the seigniorage revenues from all the infra-marginal units of reserves; these revenues fall by $v_R''(R)R$).

Adding \$1 of reserves lowers T^{priv} by \$1 which makes the Treasury seigniorage rectangle in Chart 3, Panel B narrower (losing a marginal Treasury convenience yield of $v_T'(T^{priv})$) but taller (increasing the marginal Treasury convenience yield and thus the seigniorage revenue from all the infra-marginal units of Treasuries which increases by $-v_T''(T^{priv})T^{priv}$).

Since $v_R'(R) - \varphi = v_T'(T^{priv})$ at the convenience-maximizing reserve supply $R^{C(b)}$, what matters for whether the seigniorage-maximizing reserve value exceeds the convenience-maximizing value is the height effects. If $v_R''(R)R - v_T''(T^{priv})T^{priv} > 0$ evaluated at $R^{C(b)}$ then when the central bank increases reserves (and thereby lowers T^{priv}), the increase in the seigniorage revenue from the infra-marginal units of Treasuries exceeds the decrease in seigniorage revenue from the infra-marginal units of reserves, and seigniorage-maximization involves a larger reserve supply than convenience-maximization.

Finally, Result 3(c) compares seigniorage across the scenarios of reserves being supplied via inconvenient or via convenient asset holdings. If $v_T'(T - R^{S(a)} - A) + v_T''(T - R^{S(a)} - A)(T - R^{S(a)} - A) < 0$, then Treasury supply is "too plentiful" in the sense that at $T^{priv} = T - R^{S(a)} - A$ the slope of the Treasury seigniorage curve in Chart 11 Panel B is negative. In that circumstance, central bank Treasury purchases increases Treasury seigniorage and at the margin (i.e., evaluated at $R^{S(a)}$) has no effect on reserve seigniorage, thus implying $R^{S(b)} > R^{S(a)}$.

The bottom line from this section is thus that maximizing convenience and maximizing seigniorage driven by convenience yields is not the same. Maximizing seigniorage can prescribe a smaller or larger reserve supply than maximizing convenience depending on whether reserves are supplied via inconvenient assets or via government bonds with convenience yields and depending on the shape of the reserve and government bond convenience functions as well as the supply of government bonds. I present these results to distinguish welfare maximization from seigniorage maximization and emphasize the importance of maximizing welfare.

8   Conclusion

The paper provides a simple framework for thinking about central banks' provision of reserves in terms of convenience yields. The framework is intended to guide central bank balance sheet choice above the ELB, where the balance sheet is not needed for stimulus or contractionary purposes. Reserves constitute a liquid and safe asset to the public suggesting that a central bank may want to supply enough reserves to set their convenience yield (net of balance sheet costs) to zero. However, if the central bank supplies reserves via holdings of government bonds that themselves carry a convenience yield due to liquidity or safety, then the convenience-maximizing reserve supply equalizes the convenience yields on reserves and government

bonds. This implies a positive value for the scarcity of reserves (even net of balance sheet costs).

Political constraints on which assets a central bank can hold without it creating threats to its independence therefore affect its convenience-maximizing reserve supply. I provide estimates of the convenience-maximizing reserve (or reserve+ONRRP) supply for the US and the euro area and clarify that overall welfare from convenient assets is maximized by convenience-maximization, not by maximizing the central bank's seigniorage from issuing liabilities with convenience yields.

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Chart 1
Too many tools above the ZLB: The iso-market rate curve

Panel A. Simple setting with linear demand and no balance sheet costs

Reserve demand and supply

Iso-market rate curve for short rate
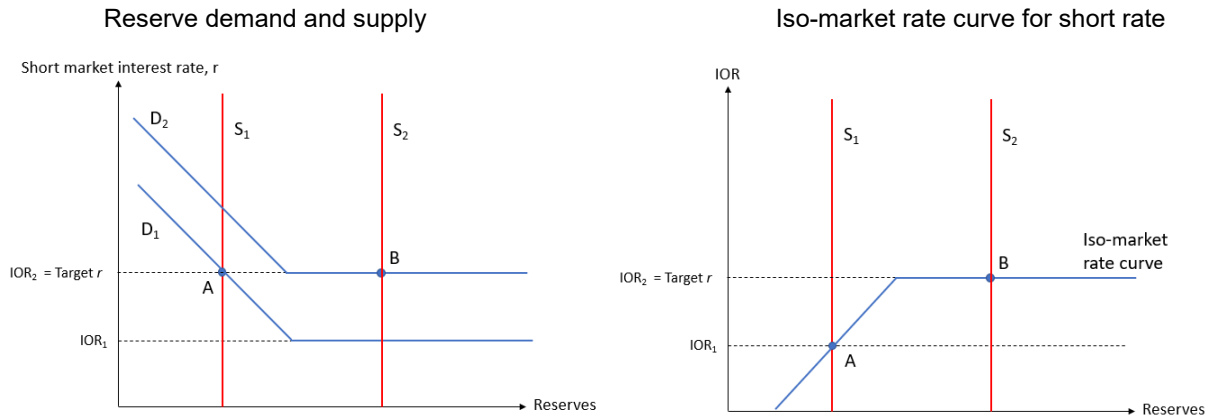
Panel B. More realistic setting with curved demand and balance sheet costs

Reserve demand and supply

Iso-market rate curve for short rate
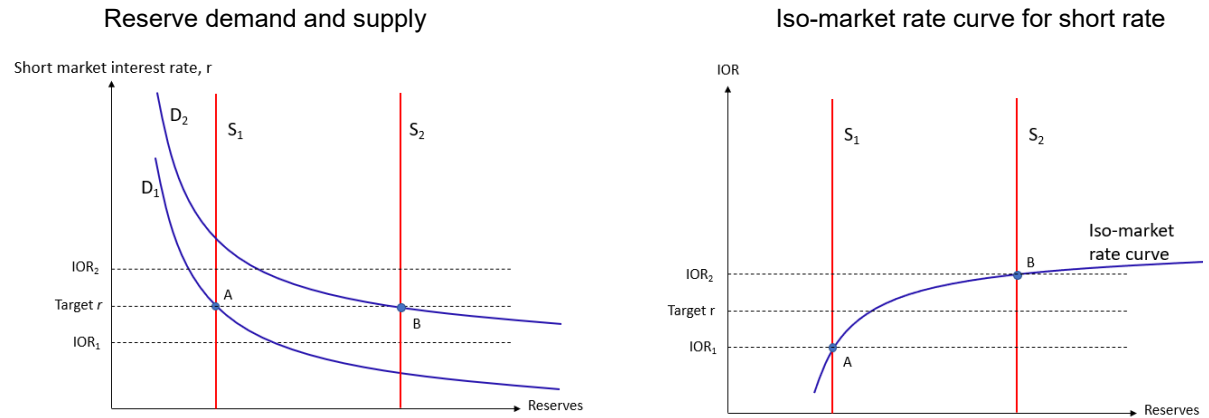
Panel C. Iso-market rate curve for long market rate
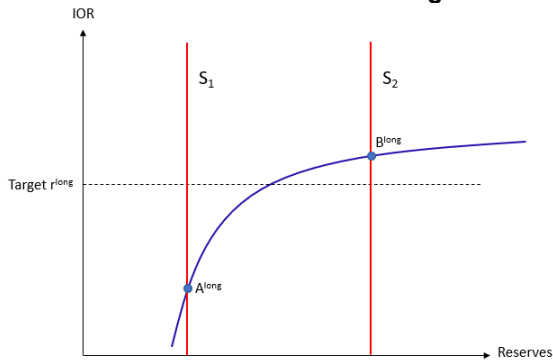
Chart 2
Convenience yields on bonds -- illustration for Treasuries
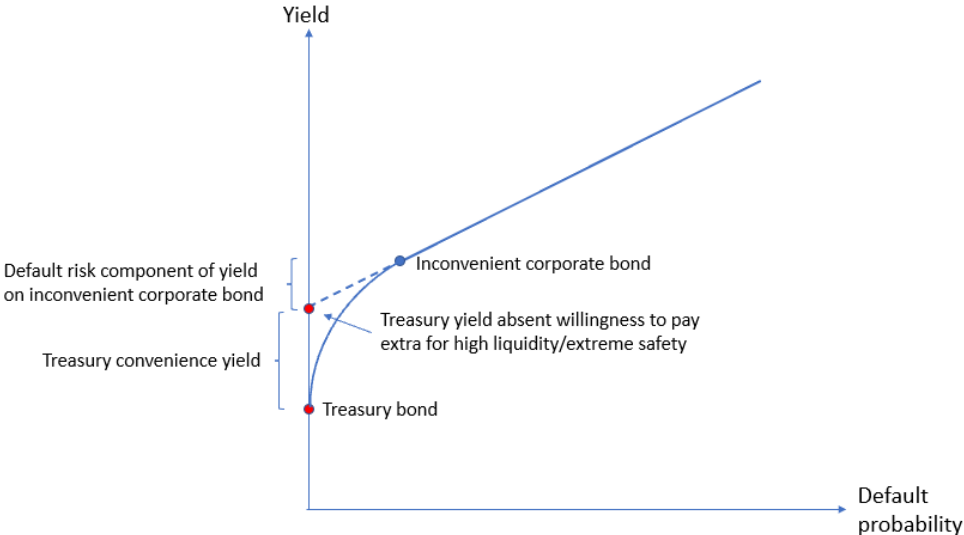
Note: For simplicity. I illustrate the relation between yield and default probability absent convenience effects as a straight line. In practice it is slightly convex: Thinking of a 1-period zero-coupon bond with face value F, yield y, default probability p, and zero recovery in default, the price is $P=F/(1+y)=(1-p)*F/(1+E(r))$. This implies $1+y=(1+E(r))/(1-p)$ which means that y is approximately, but not exactly, equal to $E(r)+p$.

Chart 3
Convenience-maximizing reserve supply

Panel A. Reserve market

Market interest rate (inconvenient, short maturity), r
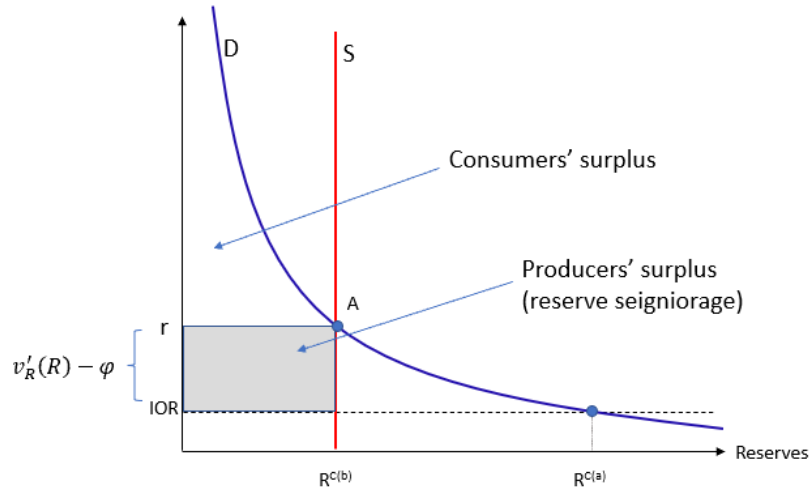
Panel B. Bond market

[Market interest rate (inconvenient, same maturity as Treasuries), r^L] − [Treasury yield, r^T]
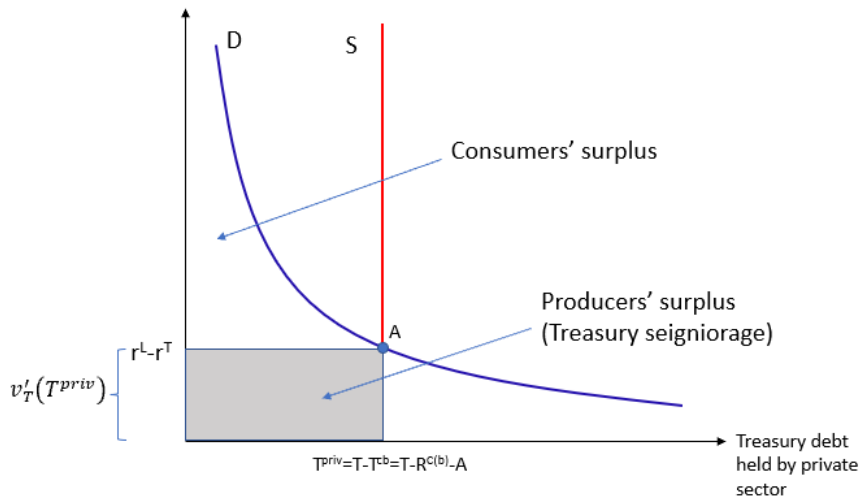
Panel C. Optimum
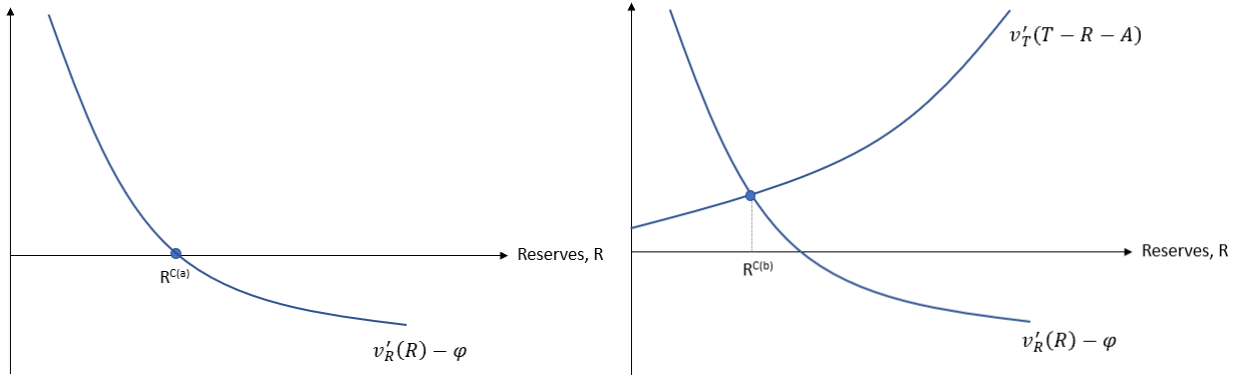
Panel D. Comparative statics: Effects of deposits, GDP, bond supply, autonomous factors
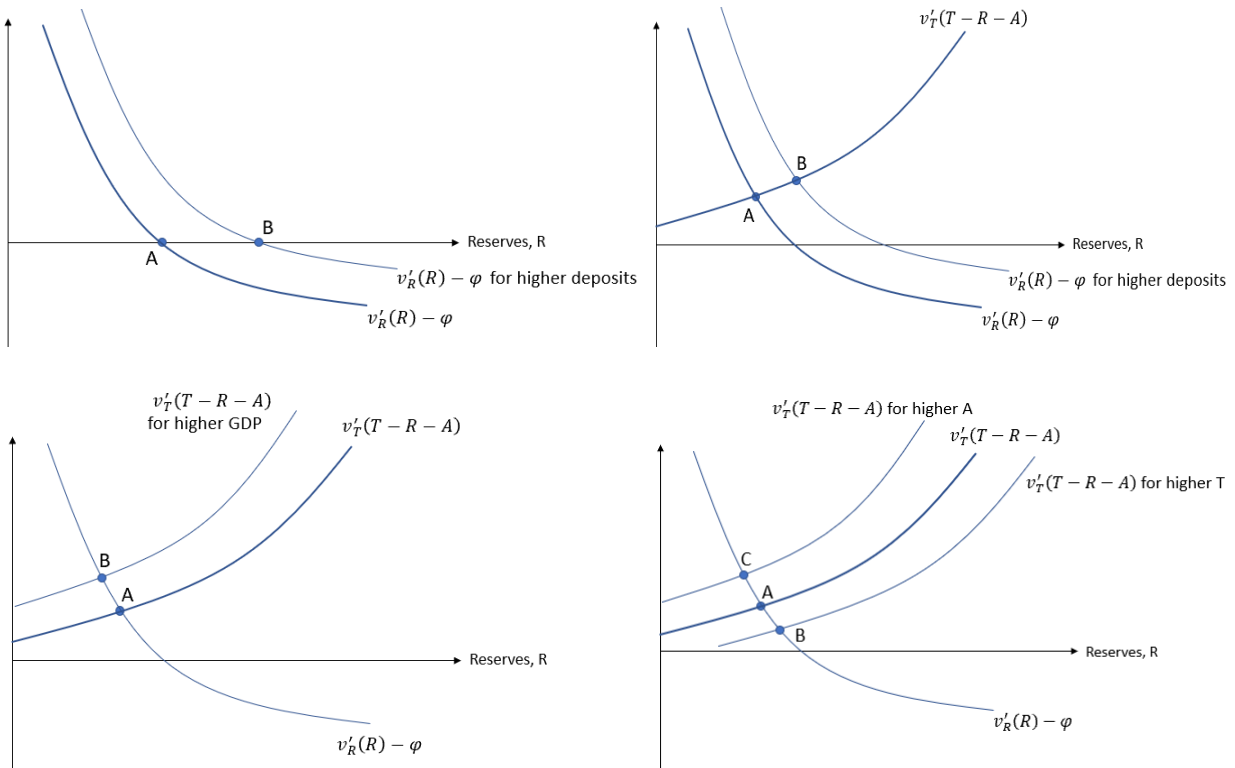
Chart 4

US: Yield spreads on reserves and Treasuries (relative to inconvenient assets)

Monthly data, 2009M1-2023M4.

Spreads on reserves
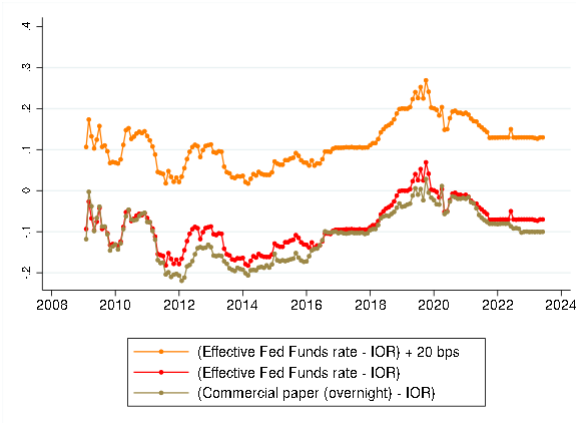
Spreads on Treasuries relative to corporate securities
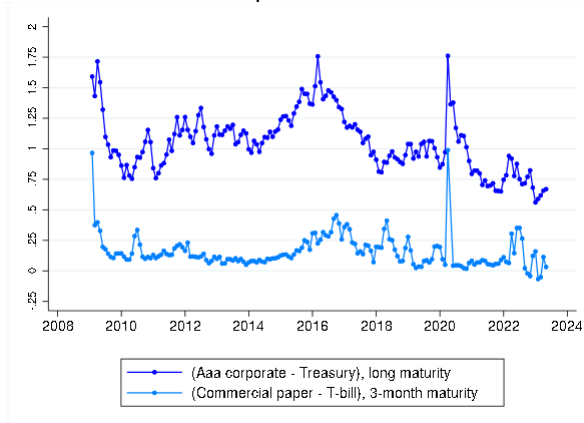
Combined chart
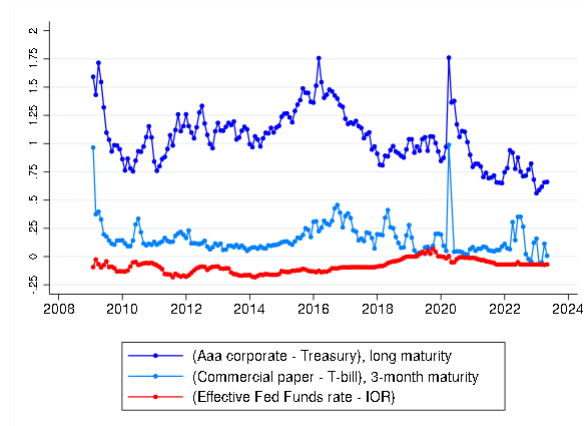
Spreads on Treasuries relative to corporate securities adjusted for corporate CDS (and callability)
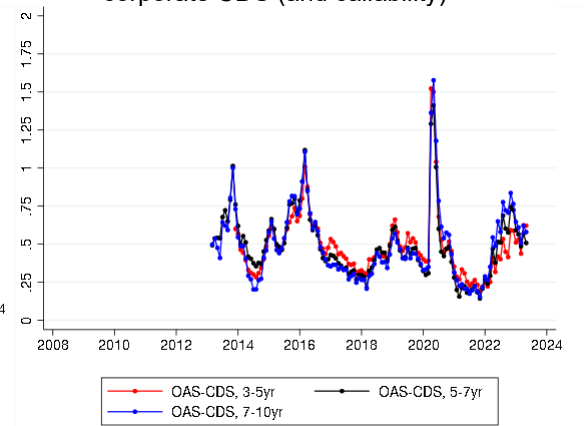
Note: All data in the top left, top right, and bottom left graphs are from FRED. In the top left graph, commercial paper is AA-rated, financial. In the bottom right graph, OAS and CDS data are from Bloomberg. Both refer to investment-grade corporate bonds. OAS data are for the ICE BofA US Corporate Index (which is investment grade). CDS data are for Markit's investment grade index (MARKIT CDX.NA.IG).

Chart 5

US: Estimating reserve and Treasury demand functions

Panel A. Reserve demand (monthly data, 2009M1-2023M4)
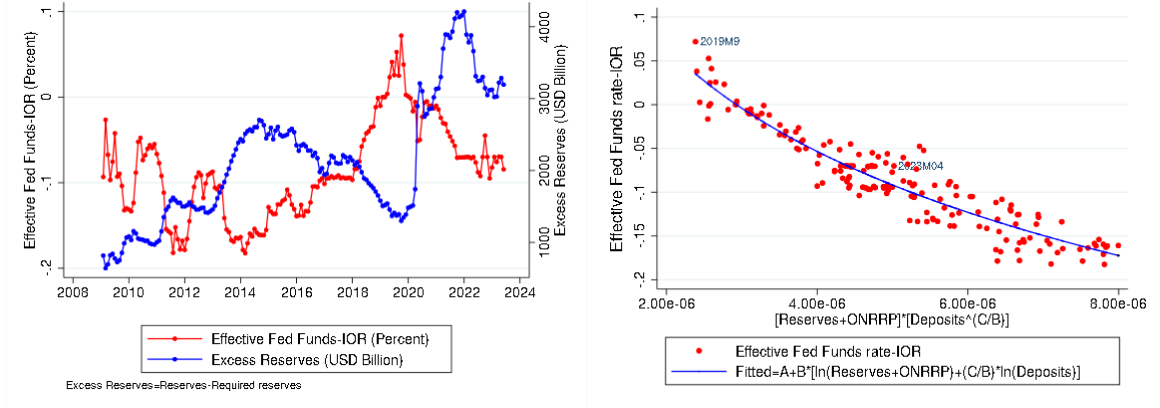
Note: All data are from FRED.

Panel B. Treasury demand (annual, 1919-2023)
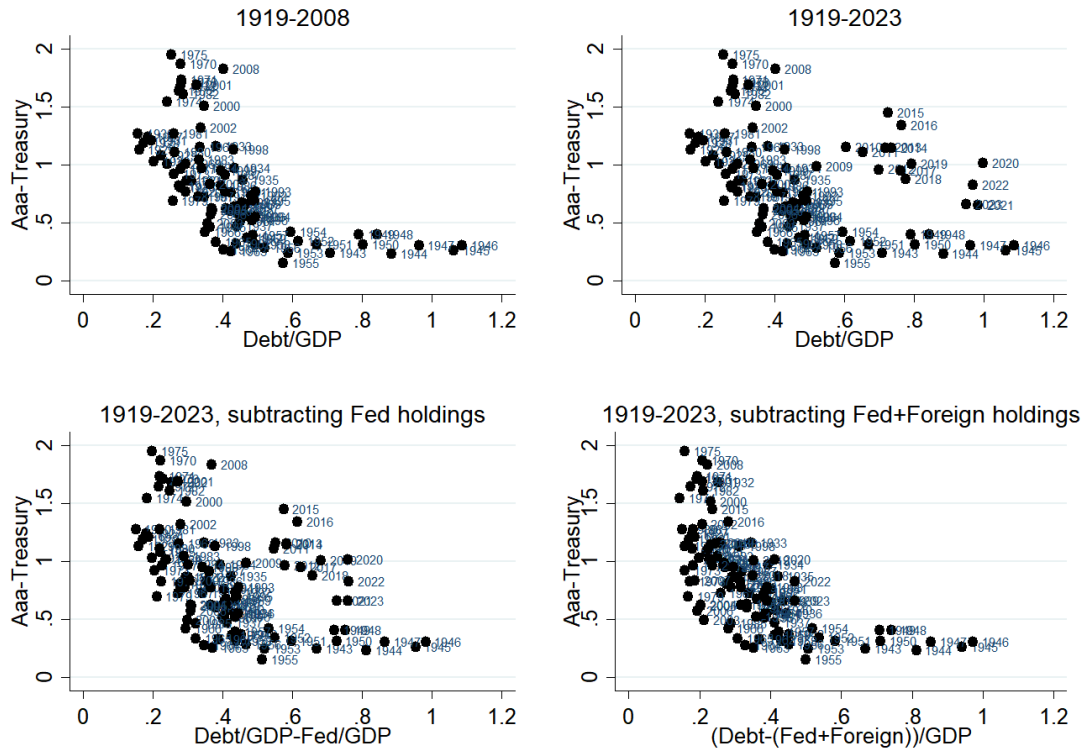
Note: For simplicity and ease of replication, in this paper, I use Debt/GDP at book value, as opposed to market value as in KVJ. The Debt/GDP series used is from Henning Bohn's homepage up to 2008. I update it using data on Treasury debt from the Federal Reserve's Z1 release, Table L.210, line 1 minus line 10. Line 10 refers to Treasury holdings by federal government defined benefit pension plans. I subtract these for comparability with Bohn's data. See Krishnamurthy and Vissing-Jorgensen (2012) for data sources for the Aaa-Treasury spread. Data on Treasury debt and holdings of the Federal Reserves and foreigners are as of the government's fiscal year end (June or September depending on the year), with GDP measured as the 4-quarter value leading up to the fiscal year end. For 2023, quantity data are for 2023Q1. Spread data are as of the following month.

Panel C. Treasury holdings of the Federal Reserve and foreigners

Annual data, 2019-2023
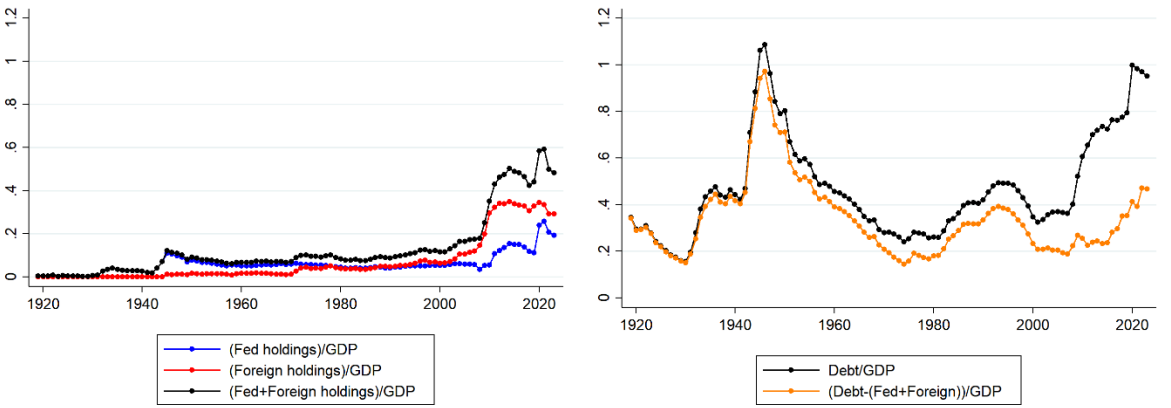
Note: Data on Federal Reserve Treasury holdings are from the Z1 release from 1945 onward and from Banking and Monetary Statistic pre-1945. Data on foreign Treasury holdings are from the Z1 release from 1945 and assumed close to zero before 1945.

Chart 6

US: Convenience-maximizing reserve supply if reserves are supplied via holdings of inconvenient assets

Panel A. April 2023
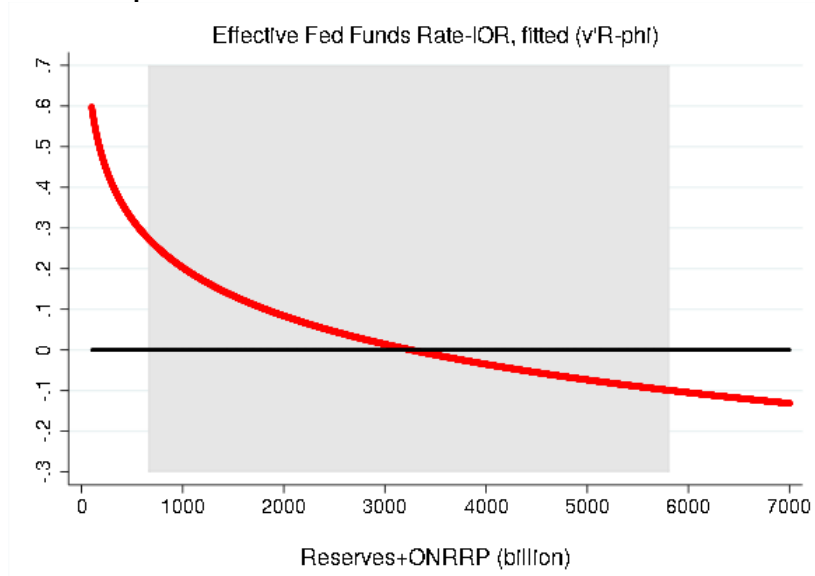
Note: Grey shading indicates range of data used in estimation. Red line is constructed using Reserves+ONRRP values between $100B and $7T.

Panel B. 2009M1-2023Q4
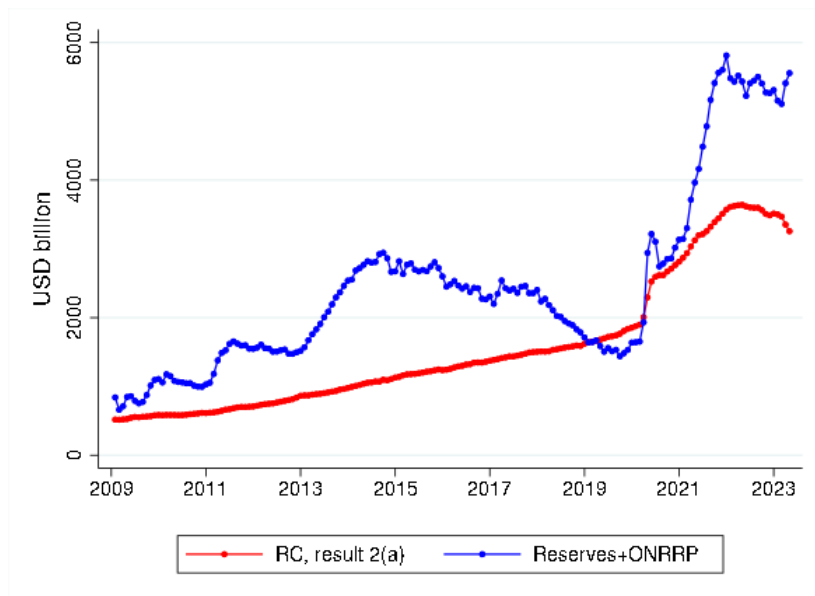
Chart 7

US: Convenience-maximizing reserve supply if reserves are supplied via holdings of Treasuries

Panel A. Reserves and Treasury convenience yields using latest available data
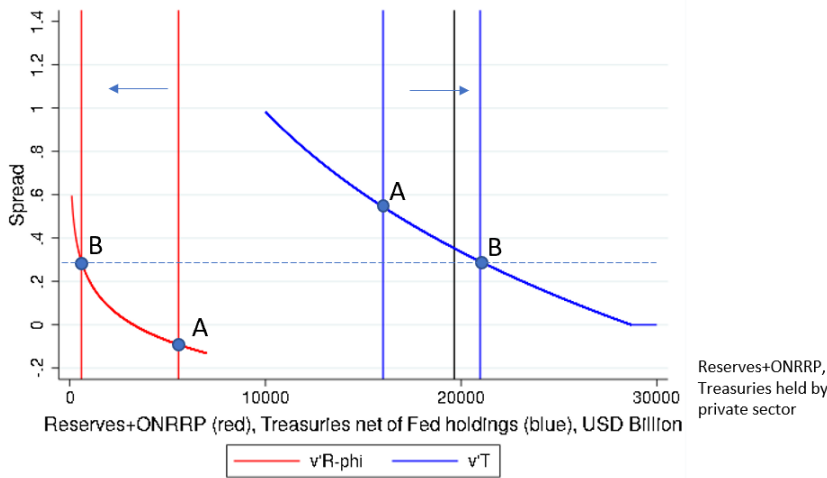
Panel B. Reserves and Treasury convenience yields using latest available data, graphed against Reserves+ONRRP supply
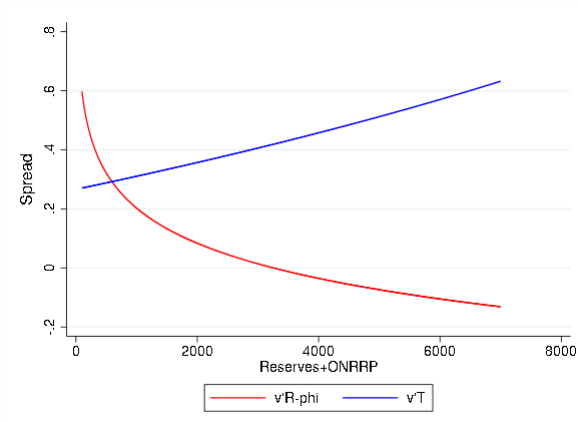
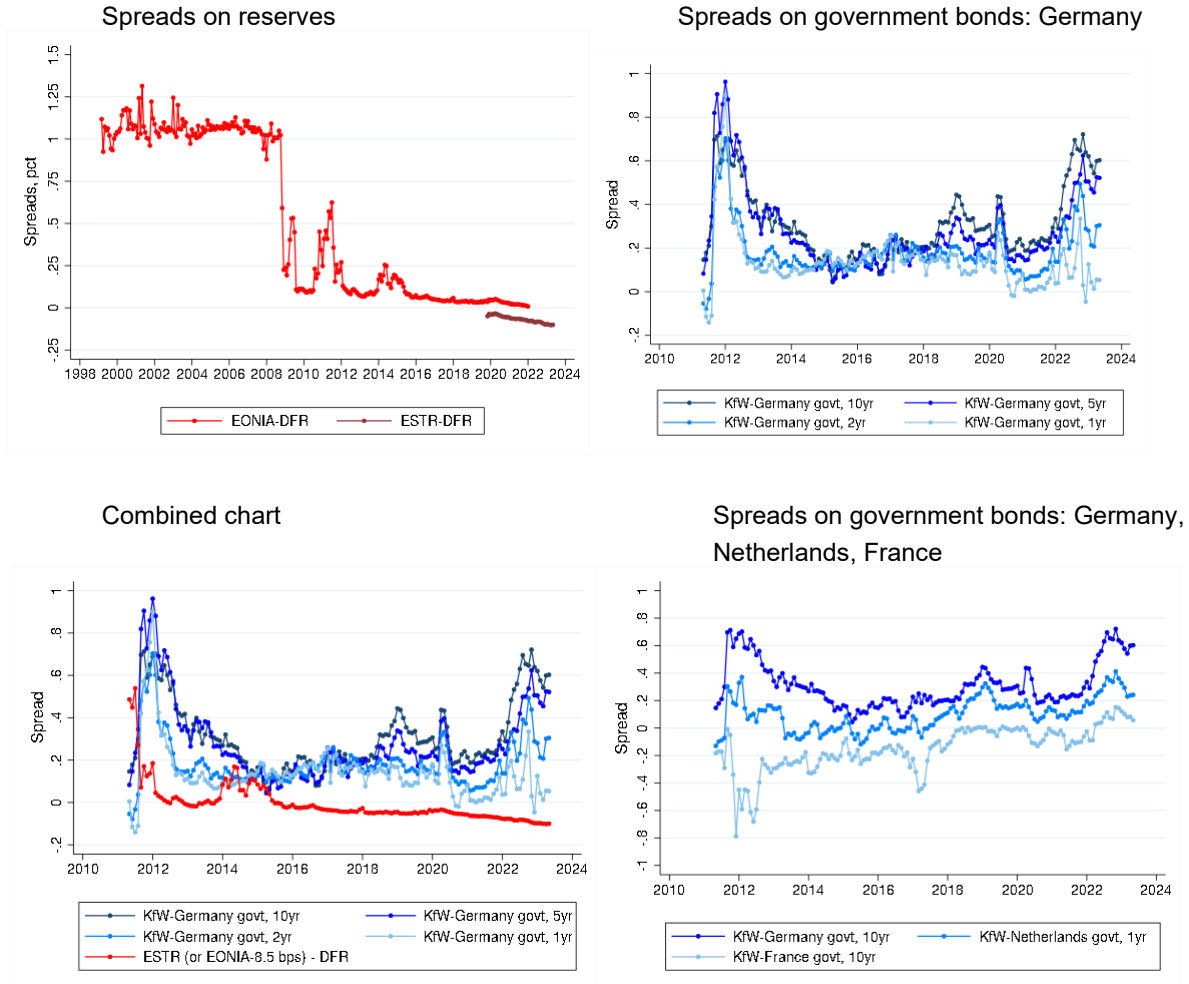**Panel C. Convenience-maximizing reserves+ONRRP supply, 2009-2023**
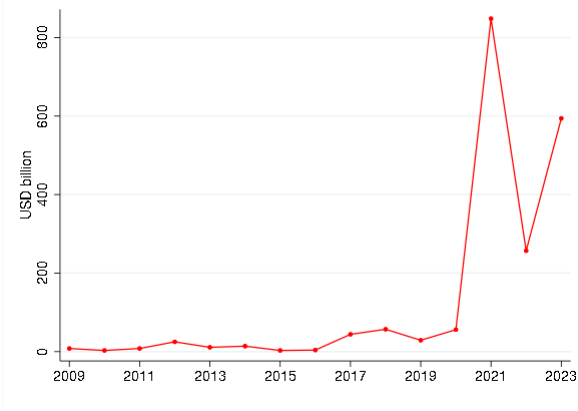
Chart 8

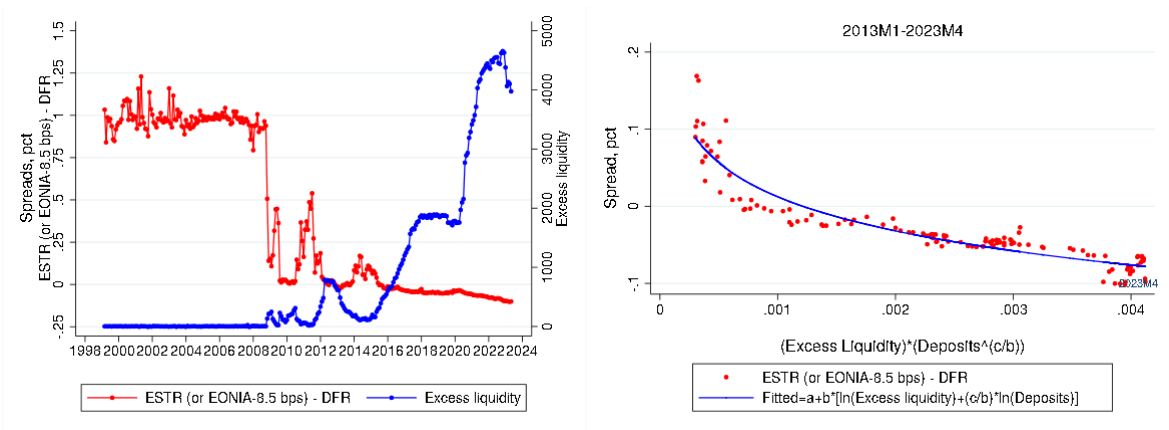Euro area: Yield spreads on reserves and government bonds (relative to inconvenient assets)

Monthly data (1999M2 to 2023M4)

Note: Data for EONIA, ESTR and DFR are from FRED. Data for KfW yields and government bond yields are from Bloomberg.

Chart 9

Euro area: Estimating reserve demand function

Monthly data (1999M2-2023M4)

Note: Excess liquidity=Current account balances+Deposit facility-Required reserves. Data on deposits and the inputs for calculating excess liquidity are from the ECB's webpage.
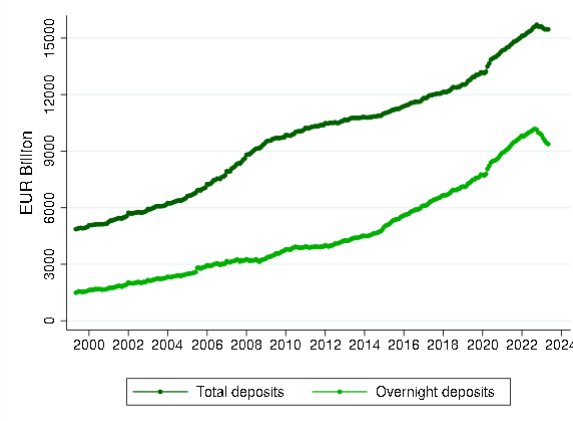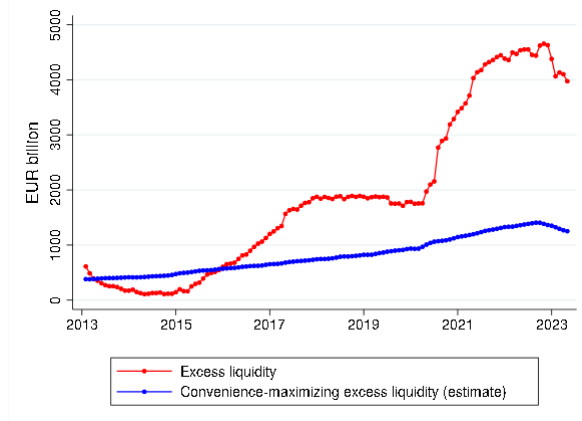
Chart 10

Euro area: Convenience-maximizing reserve supply if reserves are supplied via holdings of inconvenient assets

Panel A. April 2023

ESTR - DFR, fitted, 2023M4
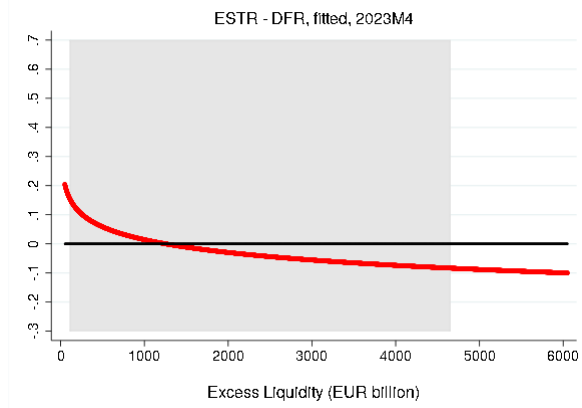
Note: Grey shading indicates range of data used in estimation. Red line is constructed using excess liquidity values between EUR 50B and EUR 6T.
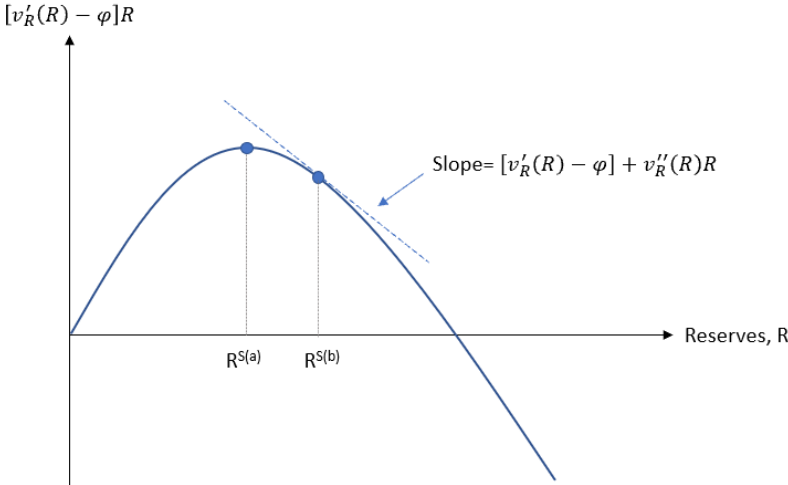
Panel B. 2013M1-2023M4

Note: Excess liquidity=Current account balances+Deposit facility-Required reserves. Deposit data are from the ECB's webpage.

Chart 11
Seigniorage effects of central banking

Panel A. Seigniorage from reserves

$[v'_R(R) - \varphi]R$

Slope= $[v'_R(R) - \varphi] + v''_R(R)R$

Reserves, R

$R^{S(a)}$ $R^{S(b)}$

Panel B. Seigniorage from Treasuries

$v'_T(T^{priv})T^{priv}$

Slope= $v'_T(T^{priv}) + v''_T(T^{priv})T^{priv}$

T^{priv}

$T^{priv}=T-R^{S(b)}-A$