

# Government Debt, The Zero Lower Bound and Monetary Policy

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*The views expressed are those of the authors and do not necessarily those of the ECB, the Board of Governors of the Federal Reserve System or any other person associated with the Eurosystem or the Federal Reserve System.*

# Background

- The financial crisis and subsequent world-wide recession triggered:
  - reductions of policy rates to effective lower bound.
  - expansions of central bank balance sheets.
  - ballooning of gov. debt and increases of risk premia on gov. debt, especially in the euro area.
- Question: how should monetary policy be conducted optimally with rising gov. debt and risk premia?

# What We Do

- Develop a monetary Blanchard-Yaari model.
  - OLG implies non-trivial role for government debt.
  - Assume risk premium on gov. debt depends on debt-to-GDP ratio.
- Examine optimal monetary policy to large recessionary shock with binding ZLB.
- Parameterize model to the euro area.

# What We Do

- Find that optimal monetary policy:
  - reduces risk premium on gov. debt.
  - expands its balance sheet.
  - relies less on forward guidance.
- Study variations of Taylor rule-based policies that replicate optimal policy prescriptions.

# The Model

# Firms

- Final goods firms:
  - Perfect competition.
  - Dixit-Stiglitz production using intermediate input goods.
- Intermediate goods firms:
  - Production linear in labor.
  - Hire on competitive labor market.
  - Monopolistic competition; set prices as markup over marginal cost.
  - May not reset price every period due to Calvo (1983) sticky prices.

# Households

- Households die with probability  $1 - \delta$
- Newborn generation  $j$  fraction  $1 - \delta$  of total pop.

$$\max_{c_t^j, M_t^j, B_t^{H,j}, n_t^j} E_0 \sum_{t=0}^{\infty} (\beta\delta)^t \sigma_{t-1} \left[ \log c_t^j - \frac{\nu}{2} \left( \max \left\{ \frac{\overline{M}_t^j}{P_t} - \frac{M_t^j}{P_t}, 0 \right\} \right)^2 + A \log(1 - n_t^j) \right]$$

subject to

$$P_t c_t^j + \frac{B_t^{H,j}}{R_t^{gov}} + M_t^j = (1 - \tau \cdot) W_t n_t^j + \Theta_t^j + TR_t^j + \frac{1}{\delta} (B_{t-1}^{H,j} + M_{t-1}^j)$$

# Households

- Ricardian equivalence does not hold
- Aggregate household real money demand:

$$m_t = \bar{m} - \left[ \frac{R_t^{gov} - 1}{R_t^{gov}} \right] \frac{1}{\nu y_t}$$

- Aggregate Euler equation in steady state:

$$R^{gov} = \frac{\Pi}{\beta} + \frac{(1 - \delta)(1 - \delta\beta)}{\delta\beta} \left[ \frac{b^H}{y} + \frac{m}{y} \right]$$



# Risk Premium on Gov. Debt

- Assume a risk premium,  $\gamma_t$ , that drives a wedge between gov. debt and policy interest rates:

$$R_t^{gov} = \gamma_t R_t$$

- Functional form:  $\gamma_t = \max \left\{ \exp \left( \varkappa \left[ \frac{B_t^G}{4P_t y_t} - \frac{b^G}{4y} \right] \right), 1 \right\}$
- Choose  $\varkappa$  such that 1pp increase of Debt/GDP from 60% increases  $R_t^{gov}$  by 10 basis points:
  - Laubach (2009): 3-4 basis points.
  - Corsetti et al (2011): 13-15 basis points.

# Government

- Budget constraint:

$$B_{t-1}^G + TR_t = \frac{B_t^G}{R_t^{gov}} + \tau W_t n_t + S_t$$

- Fiscal rule:

$$\frac{TR_t}{P_t} = tr - \theta_{TRB,t} \left( \frac{B_{t-1}^G}{P_{t-1}} - b^G \right) - \theta_{TRY} \left( \frac{y_t}{y} - 1 \right)$$

- Feedback coefficients:

- $\theta_{TRB,t} = 0$  for  $t=0, \dots, T$  and  $\theta_{TRB,t} > 0$  for  $t > T$ .
- $\theta_{TRY} > 0$ .

# Central Bank

- Budget constraint:  $\frac{B_t^M}{R_t^{gov}} + S_t = B_{t-1}^M + M_t - M_{t-1}$

- Transfers to government set according to:

$$\frac{S_t}{P_t} = s + \theta_C \left( \frac{B_t^M}{P_t} - b^M \right)$$

- Nominal interest rate determined by:

- Taylor rule:  $R_t = \max \left\{ R + \phi_\pi (\Pi_t - \Pi) + \phi_y \left( \frac{y_t}{y} - 1 \right), 1 \right\}$

- Optimal policy:  $L = \min \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[ (\Pi_t - \Pi)^2 + \lambda \left( \frac{y_t}{y} - 1 \right)^2 \right]$

subject to non-linear equilibrium conditions

and  $R_t \geq 1$ . Study equilibrium under commitment.

# Parameterization

# Parameterization (Euro Area)

Table 1: Parameters and Imposed Steady States

Parameter	Value	Description
$\beta$	0.999	Discount factor
$\delta$	0.97	Survival probability of households
$\xi_p$	0.95	Calvo price stickiness
$\omega$	1.35	Gross price markup
$\phi_\pi$	1.5	Taylor rule coefficient on inflation
$\phi_y$	0.5	Taylor rule coefficient on output
$\lambda$	0.001	Weight on output in loss function
$\theta_C$	0.01	CB to gov. transfer rule coefficient
$\theta_{TR,B}$	0.1	Gov. to households transfer rule coef.
$\theta_{TR,Y}$	0.45	Gov. to households transfer rule coef.
$\nu$	0.1	Level parameter utility of real money
$\varkappa$	0.025	Slope coefficient sovereign risk premium
$\rho_\sigma$	0.8	AR(1) of discount factor shock
$\varepsilon_\sigma$	2	Initial shock to discount factor, in percent
<i>Imposed steady states</i>		
$\tau$	0.5	Distortionary tax rate (tax wedge on labor)
$\pi$	1.9	Annual inflation rate
$m/y$	0.25	Annual money to GDP ratio
$b^G/y$	0.6	Annual total gov. debt to GDP ratio
$b^H/y$	0.5	Annual gov. debt held by public to GDP ratio
$n$	1/3	Hours worked
$\chi$	0	Subsidy to firms
$\sigma/\sigma_{-1}$	1	Discount factor shock

# A Great Recession Type Shock

- Similar to e.g. CER (2011), assume large and persistent rise in household discount factor:
  - Stand-in for tightening of credit constraints; Precautionary savings due to higher uncertainty.
  - Euler equation for gov. bonds:

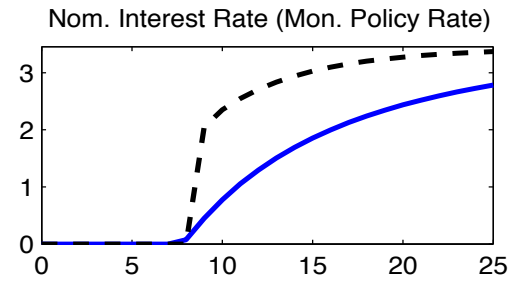
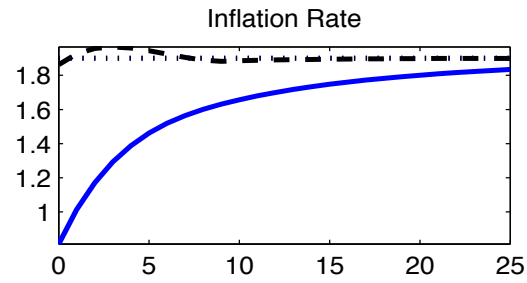
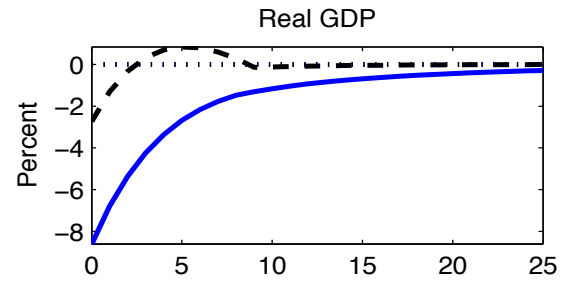
$$\beta \frac{\sigma_t}{\sigma_{t-1}} c_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1 - \delta}{\delta \mu_{t+1} \Pi_{t+1}} \left[ \frac{B_t^H}{P_t} + \frac{M_t}{P_t} \right] + c_{t+1}$$

- Assume ratio  $\frac{\sigma_t}{\sigma_{t-1}}$  increases by 2% initially and has AR(1) coefficient 0.8.
- Assume initial government debt to GDP ratio of 70% and no debt-stabilization for first 8 quarters.

# Results

# Baseline Results

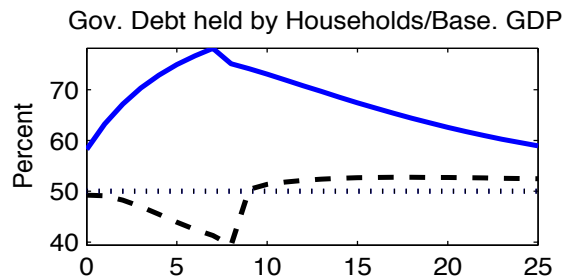
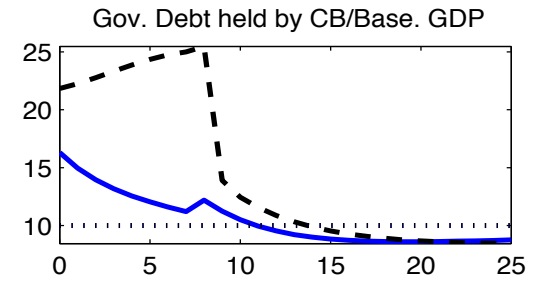
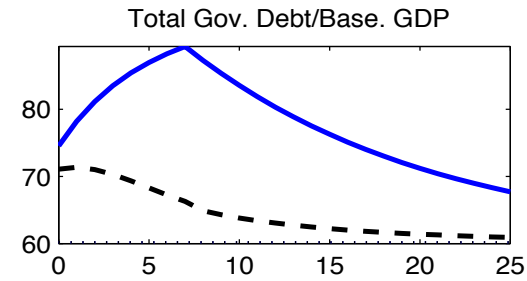
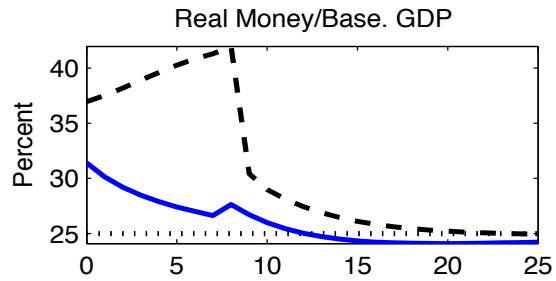
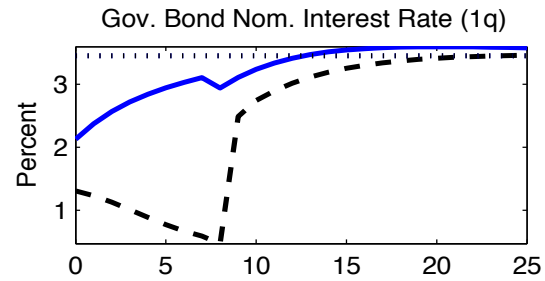
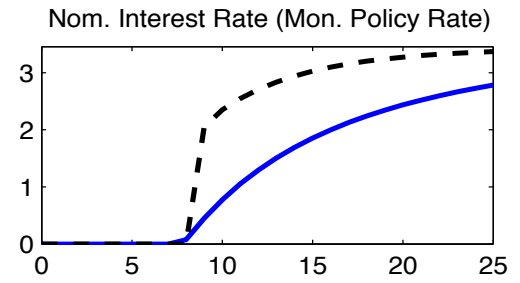
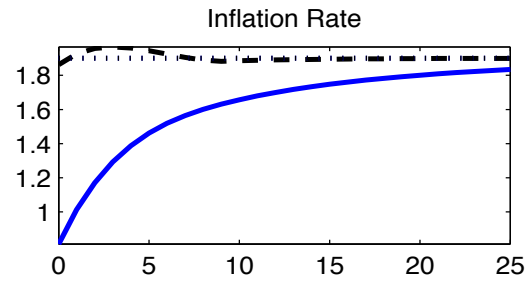
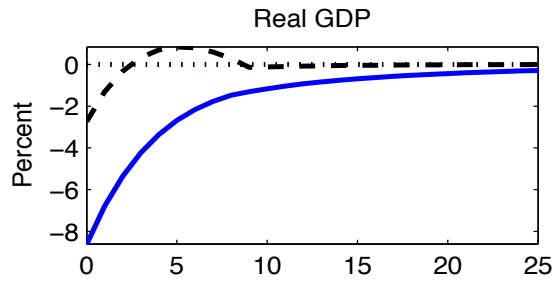
— Taylor rule based monetary policy    - - - Optimal monetary policy (loss-function based )





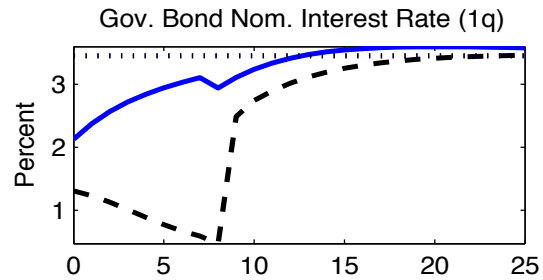
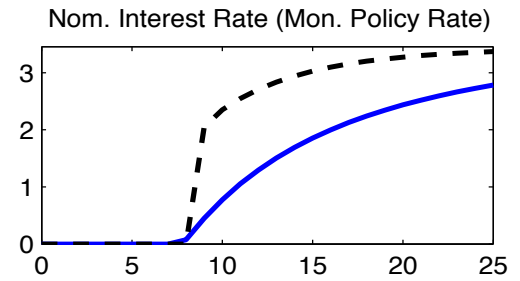
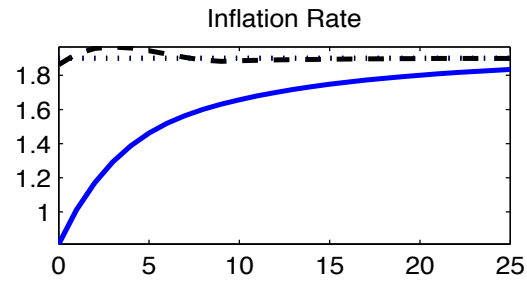
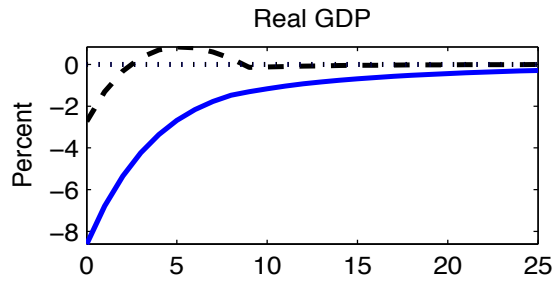
# Baseline Results

— Taylor rule based monetary policy    - - - Optimal monetary policy (loss-function based)



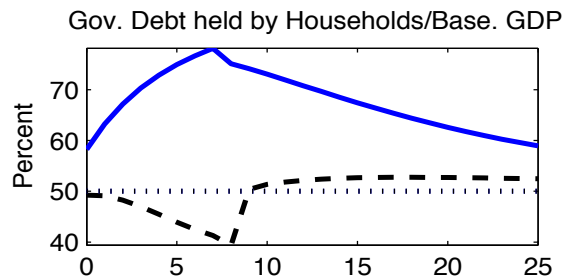
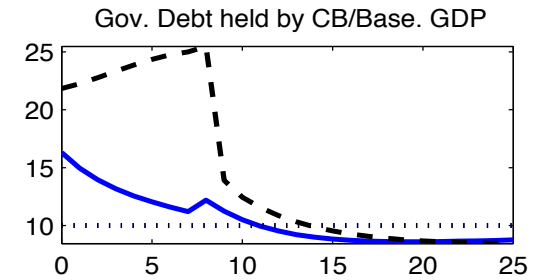
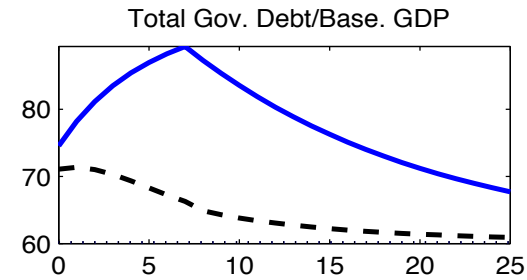
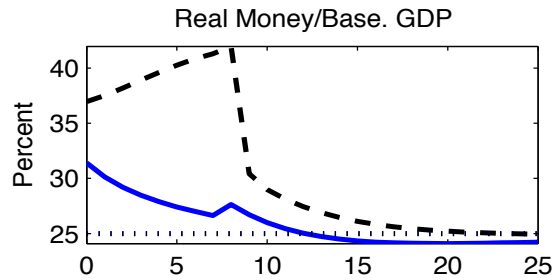
# Baseline Results

— Taylor rule based monetary policy   
 - - - Optimal monetary policy (loss-function based )



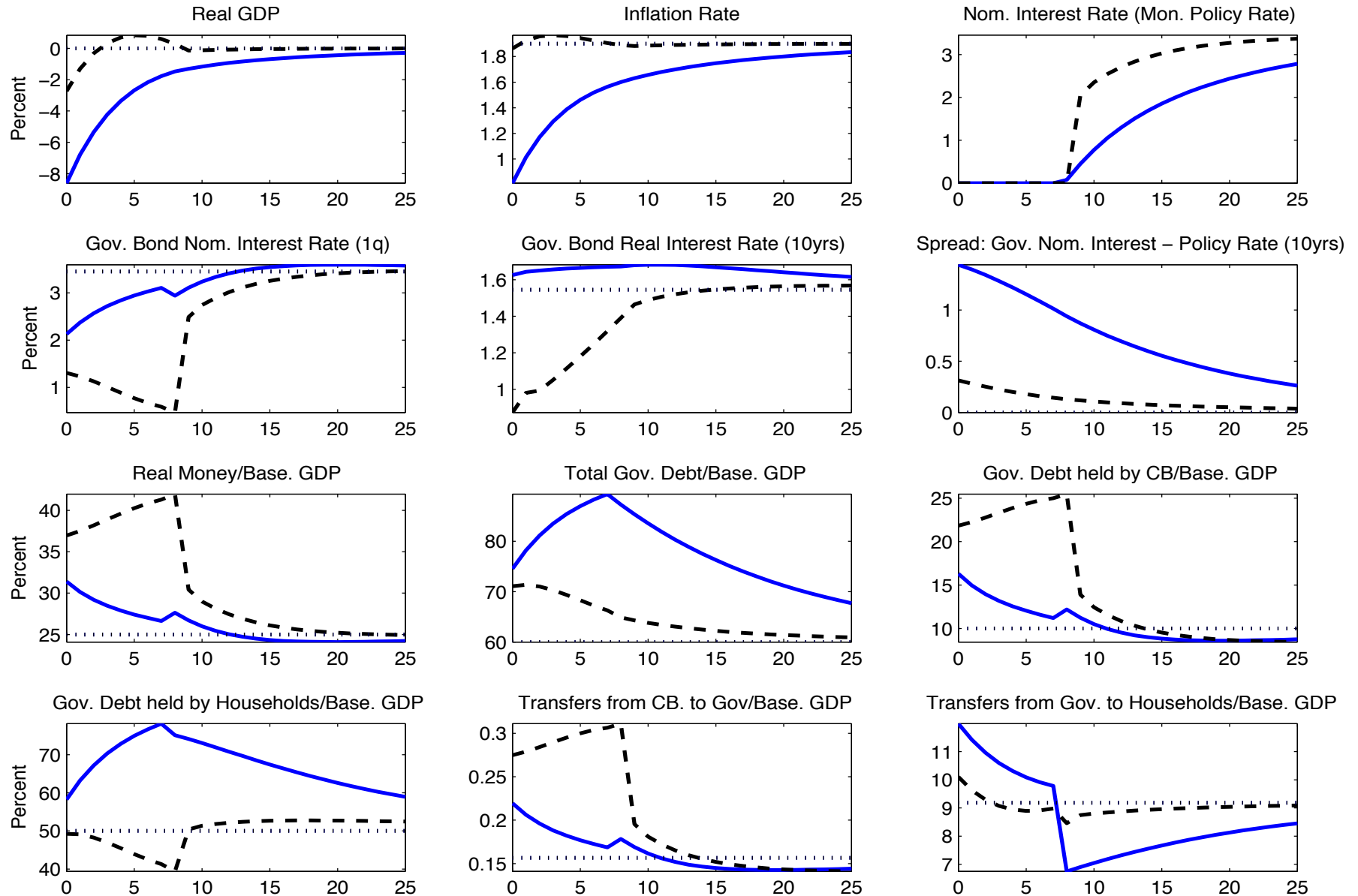
Under optimal monetary policy:

- Lower risk premium
- Larger balance sheet
- Same ZLB lift-off as under Taylor rule



# Baseline Results

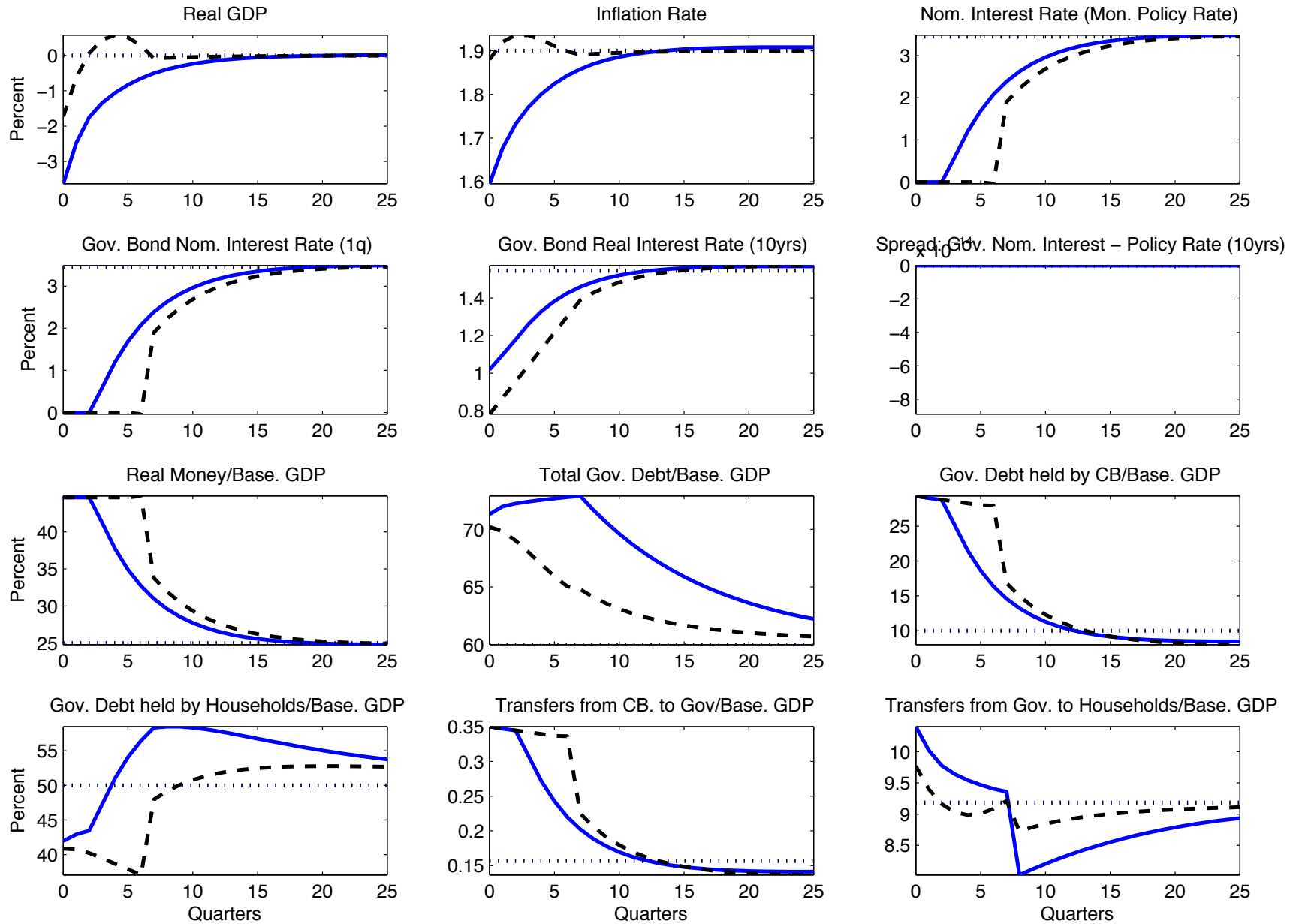
— Taylor rule based monetary policy    - - - Optimal monetary policy (loss-function based)



Sensitivity

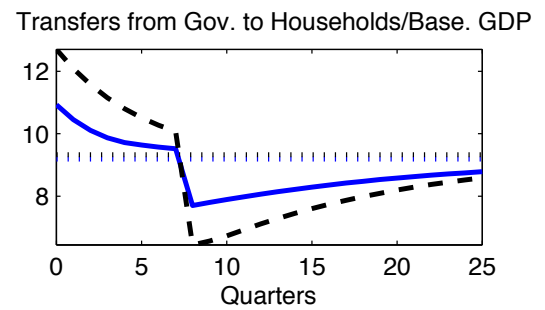
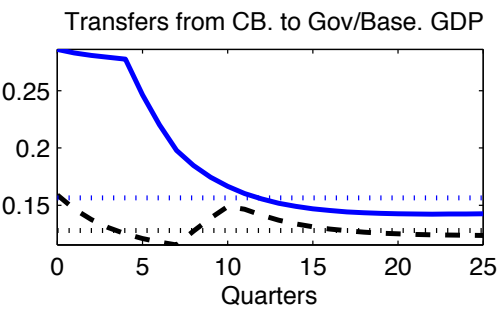
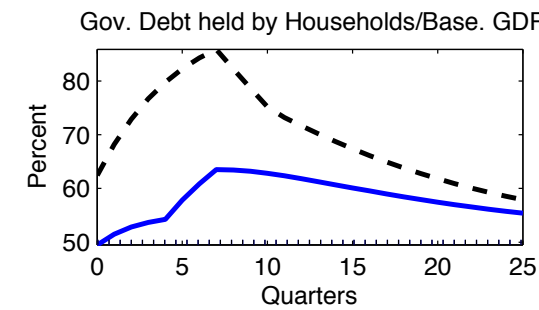
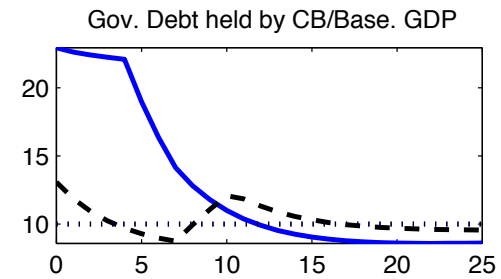
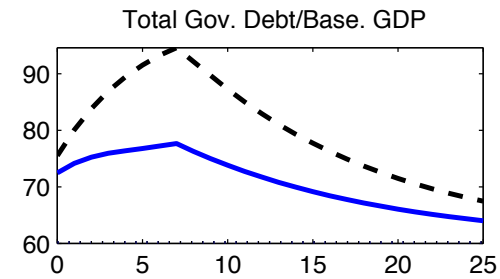
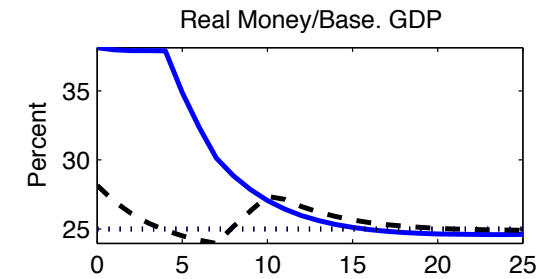
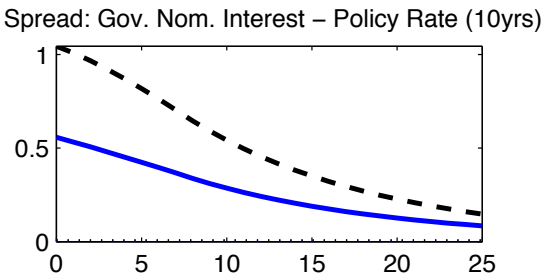
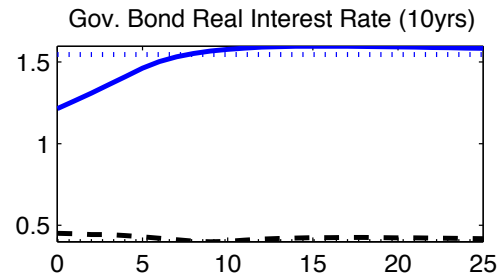
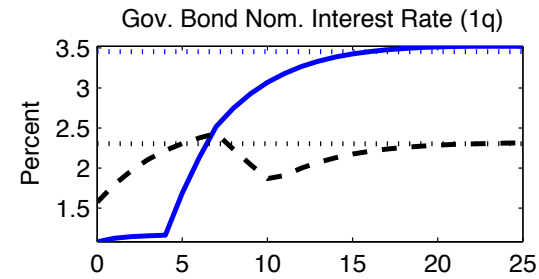
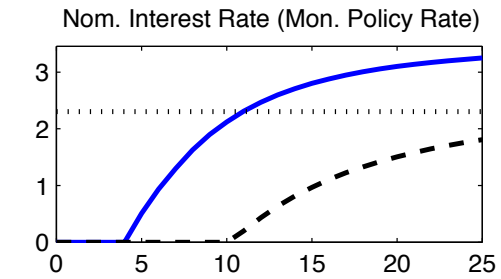
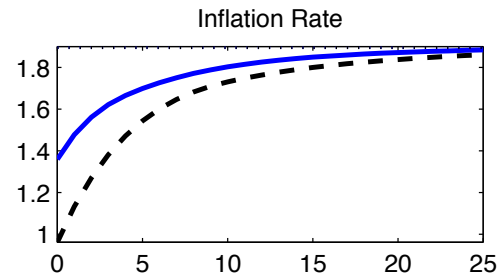
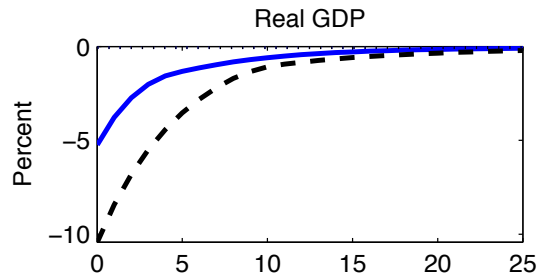
# No Gov. Risk Premium, OLG

— Taylor rule based monetary policy    - - - Optimal monetary policy (loss-function based)



# OLG vs. Infinitely Lived HH, Taylor Rule

— Blanchard Yaari Households (OLG) - - - Infinitely Lived Households



# Extensions

# Alternative Taylor Rule-Based Policies

- What does it take to get the Taylor rule-based equilibrium close to the one under optimal policy?
- Examine and compare three alternatives:
  1. Reaction to spread: augment Taylor rule by spread between long-term gov. debt and policy rate.
  2. Forward guidance: central bank pre-commits to longer ZLB under Taylor rule.
  3. Money boost (in progress): while being at the ZLB, provide more liquidity via buying up government debt.



# Reaction to Spread

- Strong differences of Taylor and optimal policy for spread between long-term interest rate on gov. bonds and policy rate.
- Augment the Taylor rule as follows:

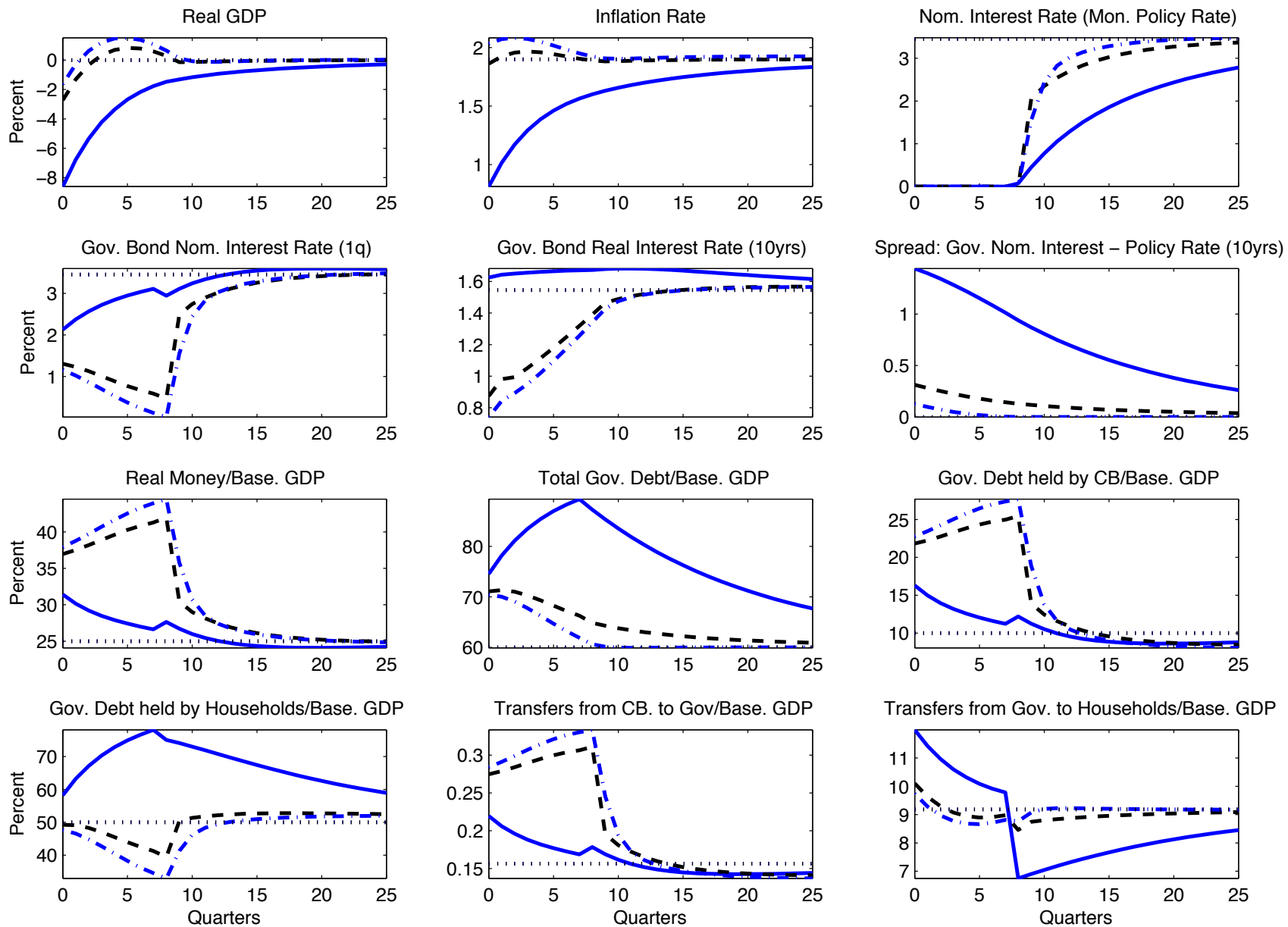
$$R_t = \max \left\{ R + \phi_\pi (\Pi_t - \Pi) + \phi_y \left( \frac{y_t}{y} - 1 \right) + \phi_s \Theta_t, 1 \right\}$$

where

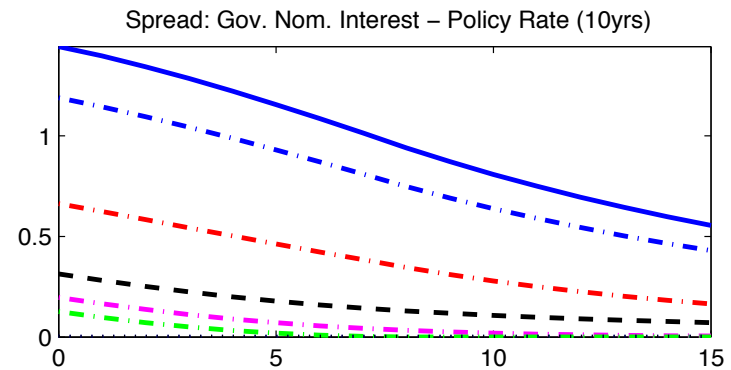
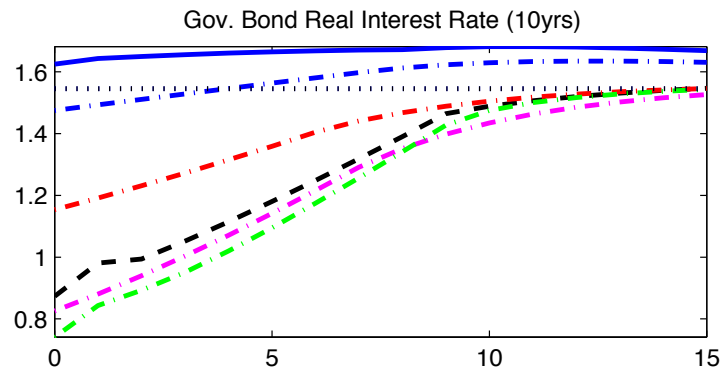
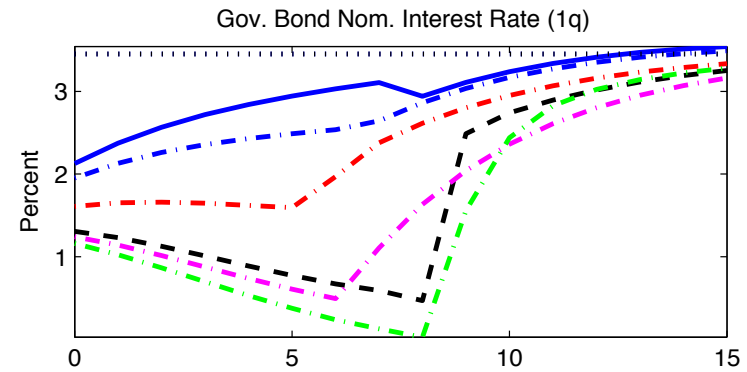
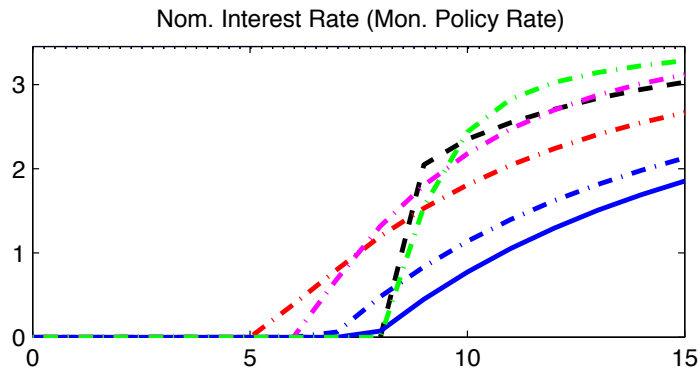
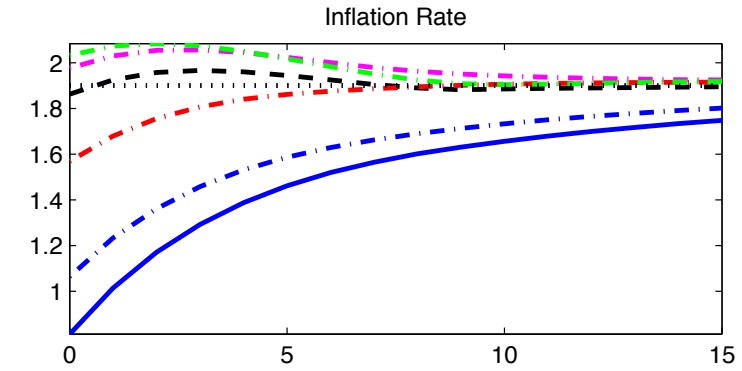
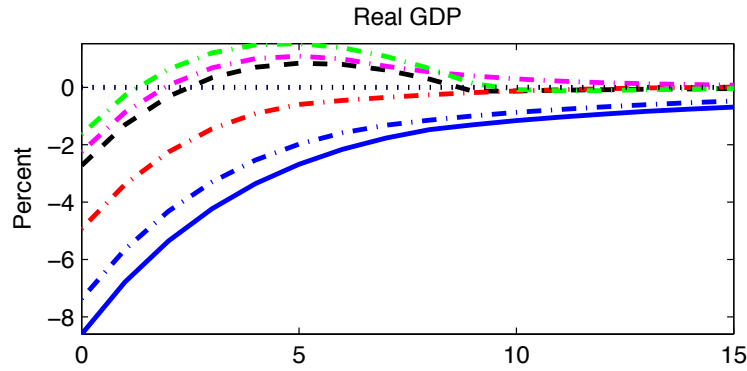
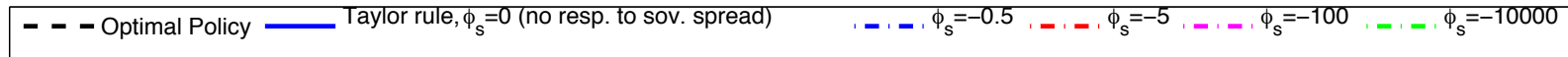
$$\Theta_t = \left[ \prod_{i=0}^{39} R_{t+i}^{gov} \right]^{\frac{1}{40}} - \left[ \prod_{i=0}^{39} R_{t+i} \right]^{\frac{1}{40}} .$$

# Effects of Augmenting the Taylor Rule

— Taylor rule based monetary policy 
 - - - Optimal monetary policy (loss-function based ) 
 - · - · Taylor rule with reaction to sovereign spread (10yrs),  $\phi_s = -10000$



# Augmenting Taylor Rule: Sensitivity

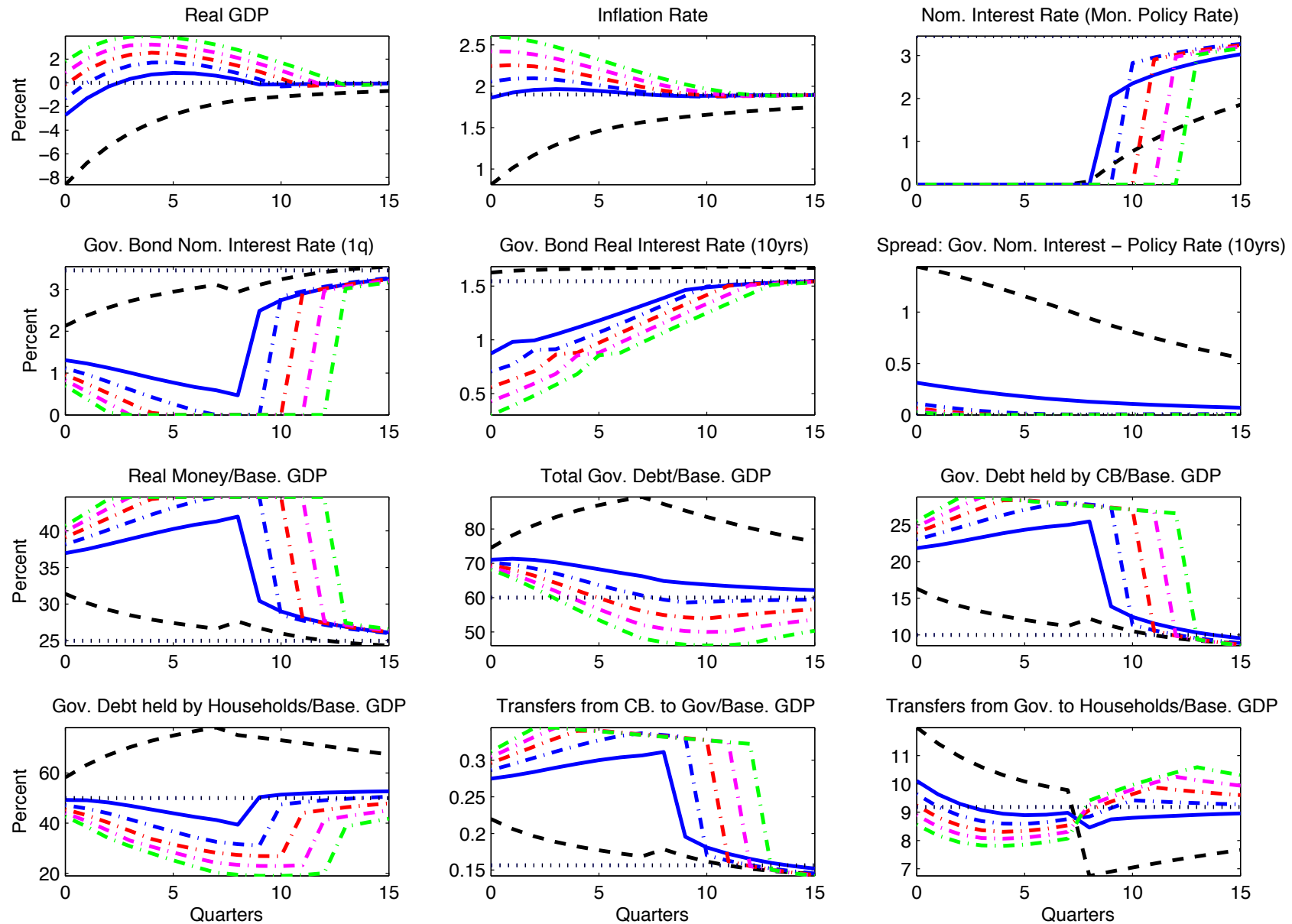


# Forward Guidance

- In the baseline results, the ZLB lasts for about 8 quarters under the standard Taylor rule.
- Assume the central bank pre-commits to keep interest rates low for 9, 10, 11 or so quarters at the onset of the recession....

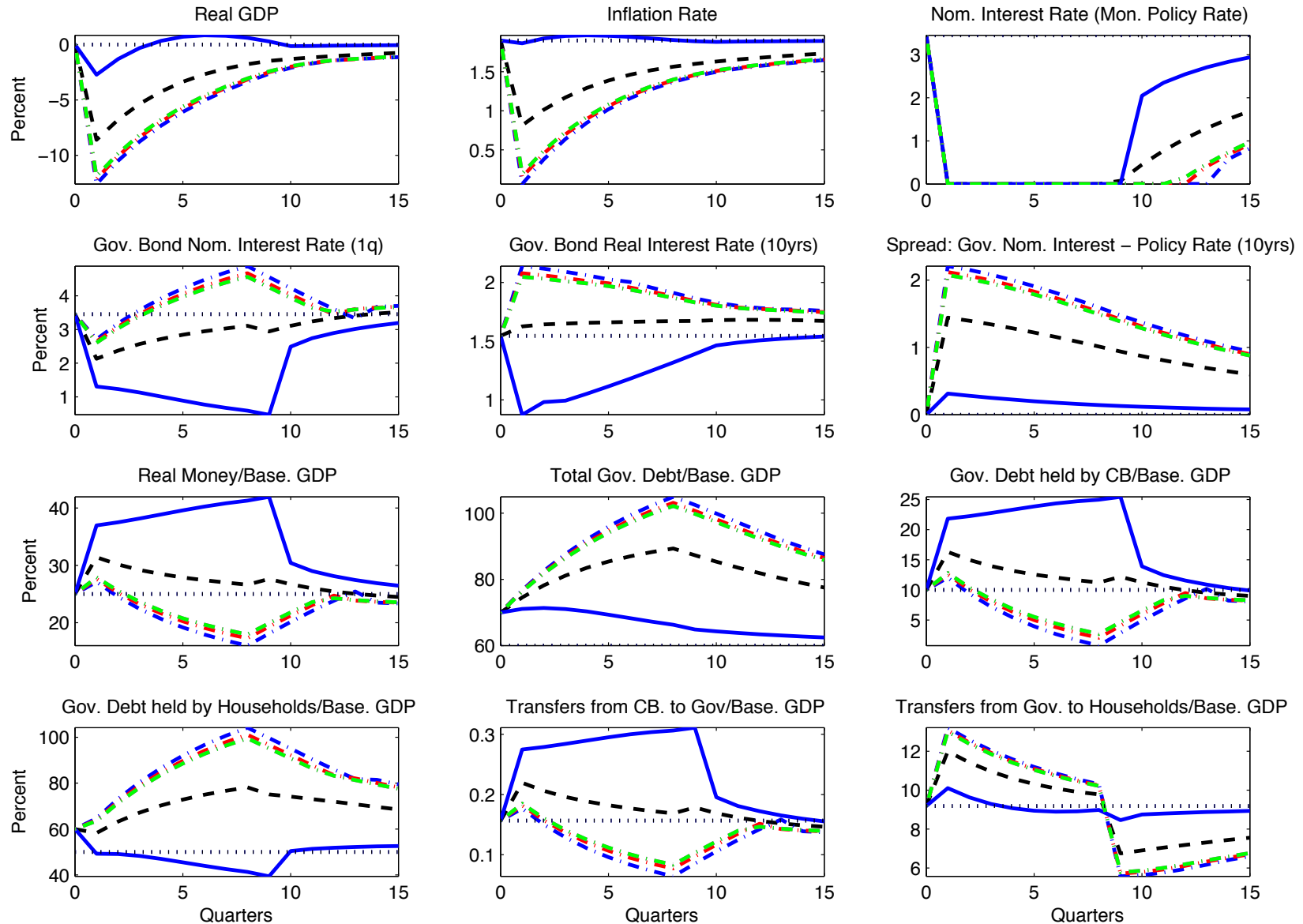
# Effects of Forward Guidance I

- - Taylor rule    — Optimal policy    - - Taylor rule + 1Q ZLB    - - Taylor rule + 2Q ZLB    - - Taylor rule + 3Q ZLB    - - Taylor rule + 4Q ZLB



# Effects of Forward Guidance II

- - - Taylor rule   
 — Optimal policy   
 - - - Taylor rule + 1Q ZLB   
 - - - Taylor rule + 2Q ZLB   
 - - - Taylor rule + 3Q ZLB   
 - - - Taylor rule + 4Q ZLB



# Conclusions

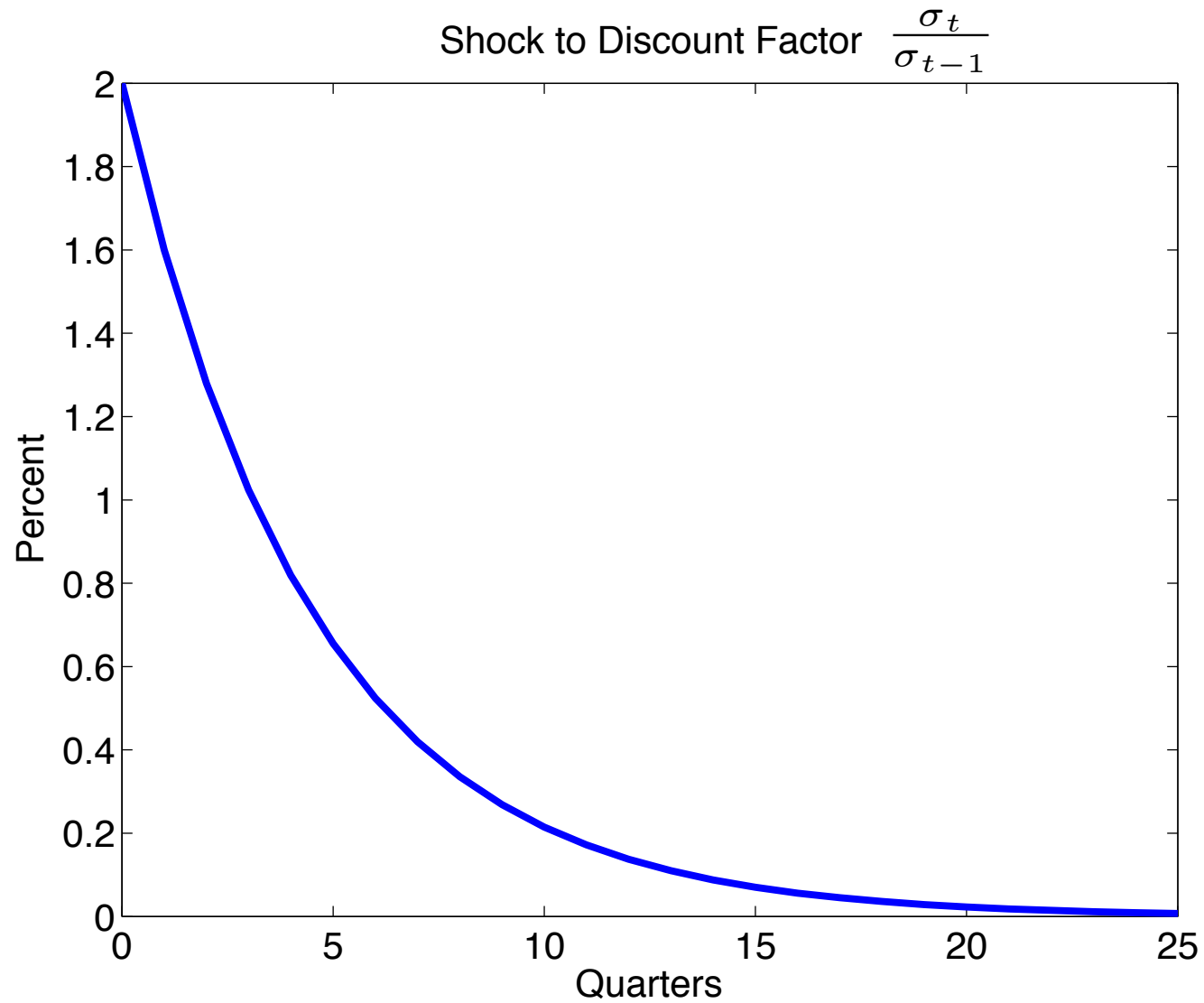
# Conclusions

- Studied optimal monetary policy at ZLB to large recessionary shock in Blanchard-Yaari model with gov. debt risk premium:
  - Central bank reduces risk premium on gov. debt.
  - Central bank expands balance sheet and relies less on forward guidance.
- Augmented Taylor rule reacting to long-term spread virtually replicates optimal policy.
- Mechanical forward guidance under Taylor rule subject to multiple equilibria.



# Annex

# Discount Factor Shock



# Households

- First order conditions:

$$\frac{M_t^j}{P_t} = \overline{\frac{M_t^j}{P_t}} - \left[ \frac{R_t^{gov} - 1}{R_t^{gov}} \right] \frac{1}{\nu \cdot c_t^j}$$

$$\frac{Ac_t^j}{1 - n_t^j} = (1 - \tau) \frac{W_t}{P_t}$$

$$1 = \beta \frac{\sigma_t}{\sigma_{t-1}} E_t \left[ \frac{c_t^j}{c_{t+1}^j} \frac{R_t^{gov}}{\Pi_{t+1}} \right]$$

## Equilibrium Equations

Bond Market Clearing (e1) :  $b_t^G = b_t^M + b_t^H$

Central Bank Budget (e2) :  $\frac{b_t^M}{R_t^{gov}} + s_t = \frac{b_{t-1}^M}{\Pi_t} + m_t - \frac{m_{t-1}}{\Pi_t}$

Transfer from CB to Gov. (e3) :  $s_t = s + \theta_C (b_t^M - b^M)$

Government Budget (e4) :  $\frac{b_{t-1}^G}{\Pi_t} + tr_t = \frac{b_t^G}{R_t^{gov}} + \tau w_t n_t + s_t$

Fiscal Rule for Transfers (e5) :  $tr_t = tr - \theta_{TR,B} (b_{t-1}^G - b^G) - \theta_{TR,Y} \left( \frac{y_t}{y} - 1 \right) + \varepsilon_t$

Leisure/Labor Tradeoff (e6) :  $\frac{A y_t}{1 - n_t} = (1 - \tau) w_t$

Gov. Bond Interest Rate (e7) :  $R_t^{gov} = \gamma_t R_t$

Sovereign Risk Premium (e8) :  $\gamma_t = \max \left\{ \exp \left( \varkappa \left[ \frac{b_t^G}{4y_t} - \frac{b^G}{4y} \right] \right), 1 \right\}$

Euler Equation Bonds (e9) :  $\beta \frac{\sigma_t}{\sigma_{t-1}} y_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1 - \delta}{\delta \mu_{t+1} \Pi_{t+1}} [b_t^H + m_t] + y_{t+1}$

Recursive Discounting (e10) :  $\mu_t = 1 + \frac{\sigma_t}{\sigma_{t-1}} \delta \beta \mu_{t+1}$

Real Money Demand (e11) :  $m_t = \bar{m} - \left[ \frac{R_t^{gov} - 1}{R_t^{gov}} \right] \frac{1}{\nu_t y_t}$

Optimal Price Setting 1 (e12) :  $F_t = \sigma_{t-1} + \delta \xi_p \beta \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\frac{1}{\omega-1}} F_{t+1}$

Optimal Price Setting 2 (e13) :  $K_t = \sigma_{t-1} \omega w_t + \delta \xi_p \beta \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\frac{\omega}{\omega-1}} K_{t+1}$

Optimal Price Setting 3 (e14) :  $\frac{K_t}{F_t} = \left[ \frac{1 - \xi_p \left( \frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega-1}}}{1 - \xi_p} \right]^{1-\omega}$

Inv. Price Dispersion (e15) :  $\hat{p}_t^{1-\frac{\omega}{1-\omega}} = (1 - \xi_p) \left( \frac{1 - \xi_p \left( \frac{\Pi_t}{\Pi} \right)^{\frac{1}{\omega-1}}}{1 - \xi_p} \right)^\omega + \xi_p \left( \frac{\Pi}{\Pi_t} \hat{p}_{t-1} \right)^{\frac{\omega}{1-\omega}}$

Production (e16) :  $y_t = n_t \hat{p}_t^{\frac{\omega}{\omega-1}}$

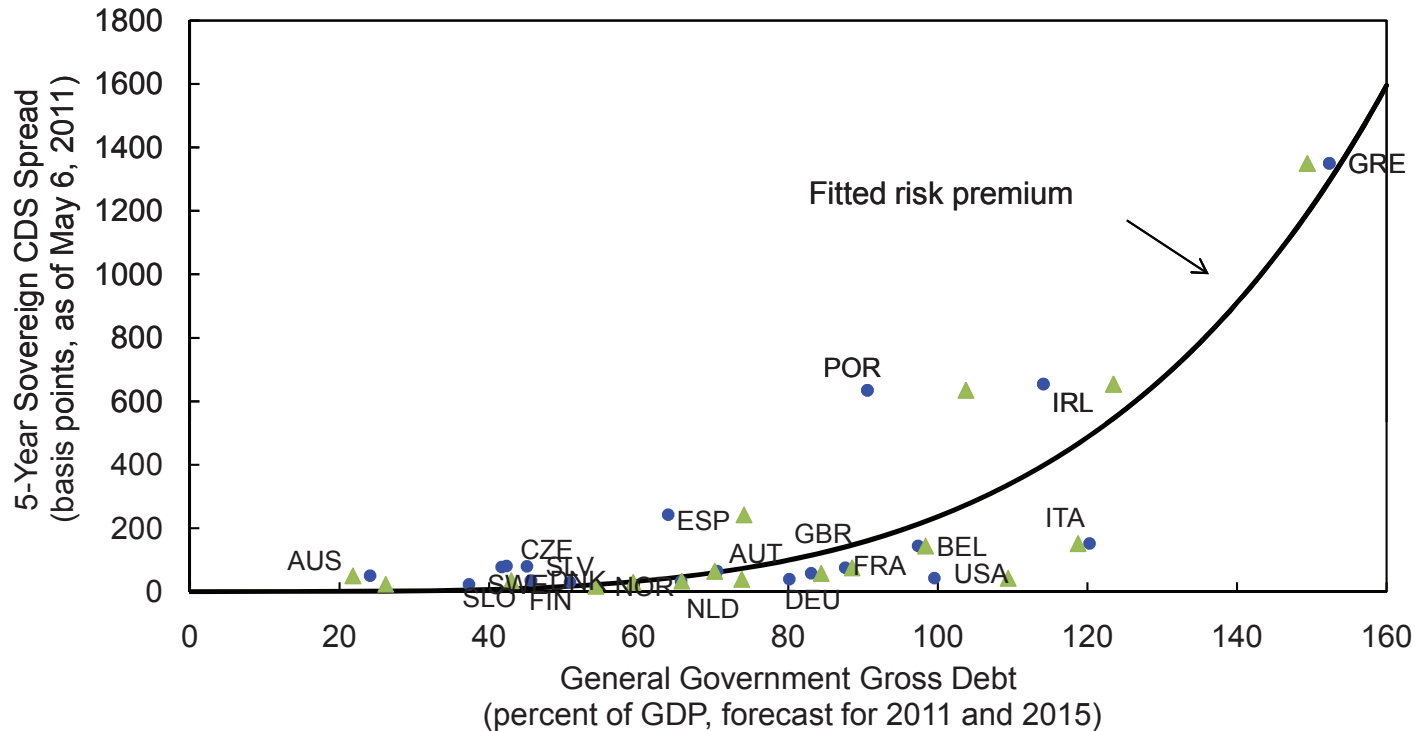
# Steady State Comparison

Table 2: Steady States and Implied Parameters for Different Households (HH)

Variable	Infinitely lived HH ( $\delta = 1$ )	Blanchard/Yaari HH ( $\delta = 0.97$ )	Description
$r$	2.3	3.45	Nominal interest rate
$\bar{m}/y$	0.3814	0.4465	Satiation: annual money to GDP ratio
$tr/y$	0.0933	0.0919	Annual transfer to GDP ratio
$w$		0.74	Real wage
$A$		0.75	Level parameter disutility of labor
$y$		0.33	Real GDP
$b^M/y$		0.1	Annual gov. debt to GDP ratio held by CB
$s/y$		0.001	Annual CB transfers to gov. as ratio to GDP

# Corsetti et al (2011)

Figure 2: Sovereign risk premia vs. debt



*Notes:* The figure shows 5-year sovereign CDS spreads for industrialized countries against forecasts for end-2011 gross general government debt/GDP (blue circles) and end-2015 debt to GDP (green triangles). The countries shown are Australia, Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden, United Kingdom, and United States. Note: Excludes Japan. The forecasts are taken from the IMF World Economic Outlook April 2011.